Farm Business Management

Analysis of Farming Systems

FSC
www.fsc.org
MIX
Paper from
responsible sources
FSC® C013604

This page intentionally left blank

Farm Business Management

Analysis of Farming Systems

Peter L. Nuthall

Lincoln University, New Zealand

www.cabi.org

CABI is a trading name of CAB International

CABI Head Office
Nosworthy Way
Wallingford
Oxfordshire OX10 8DE
UK

Tel: +44 (0)1491 832111
Fax: +44 (0)1491 833508
E-mail: cabi@cabi.org
Website: www.cabi.org

CABI North American Office
875 Massachusetts Avenue
7th Floor
Cambridge, MA 02139
USA

Tel: +1 617 395 4056
Fax: +1 617 354 6875
E-mail: cabi-nao@cabi.org

A catalogue record for this book is available from the British Library, London, UK.

Library of Congress Cataloging-in-Publication Data

Nuthall, P.L. (Peter Leslie)
Farm business management : analysis of farming systems / Peter L. Nuthall.
p. cm.
Includes bibliographical references and index.
ISBN 978-1-84593-839-0 (alk. paper)
1. Farm management--Decision making. 2. Agricultural systems--Mathematical models. I. Title.

S561.N87 2011
630.68--dc22

2011001257

ISBN-13: 978 1 84593 839 0

Commissioning editor: Sarah Hulbert
Editorial assistant: Alexandra Lainsbury
Production editor: Fiona Chippendale

Typeset by SPi, Pondicherry, India.
Printed and bound by CPI Group (UK) Ltd, Croydon, CR0 4YY.

Contents

The Author

Peter Nuthall has spent many years teaching and researching aspects of farm management. In addition he has developed and managed a team involved in producing and supporting computer aided management systems used by large numbers of farmers. While most of his time has been at Lincoln University in Canterbury, New Zealand, he has also researched and/or taught at the University of Queensland, Purdue University (Indiana), the University of Kent and the University of Edinburgh. He has also worked for the UK Meat Marketing Board while based at the University of Nottingham (Sutton Bonnington). Many institutions involved in researching and teaching farm management have been visited to gather ideas in both the developed and developing world. He also has experience of agriculture in a diverse range of situations including Russia, India, Fiji, Australia, NZ, USA and the UK. Nuthall has published widely in scientific journals throughout the world, and in monographs as well as having many research results taken up by the popular farming press.

This page intentionally left blank

Acknowledgements

Many people and systems have contributed to the material in this book through the author's general reading and research. They include researchers around the world, farmers similarly, consultants and text book authors. Grateful thanks is offered to all these people.

This page intentionally left blank

1 Introduction

1.1 Introduction to Problem Solving

Every farmer likes to think s/he operates an optimal farming system, a system that more than likely will be different from any other farmer's optimal systems. Each farm is unique in its set of resources (quantity and quality) and the objectives held by the farmer and family. For each farmer, and his advisors, the challenge is to work out this optimal system, though, given the nature of farming and primary production, there could well be a number of systems that are near optimal which for all intents and purposes can be called optimal alternatives.

This book is about the methods available to determine optimal systems. Some might be used by farmers themselves, but more likely by farm advisors and consultants, and by farm management researchers interested in, firstly, providing farmers with guidance on optimal systems, and, secondly, providing governments with advice on the impact of farm policy measures. The methods are all about problem solving, as any decision situation implies choice and, therefore, requires a method for deciding which alternative maximizes the objectives. The book is not, however, about carrying out the optimal plans (for a coverage of the skills required in practical farm management, see Nuthall, 2010).

The analytical method used to find an optimal decision must fit the situation, and also relate to the level of increased benefits likely from the improved farming system created. Each method has different costs and features. For example, some decision situations might not involve risk (or more correctly risk is not a major factor, for almost all decisions involve risk) and, consequently, the analytical method used can ignore risk. The model will be less complex than risk-allowing techniques, and therefore cost less to implement.

The range of techniques available is extensive and covers simple budgets, methods used in production economics right through to techniques used in 'operations research' (such as mathematical programming). At one extreme an individual farmer might make decisions based on intuition, and at the other by employing someone to create complex computer-based simulation systems. In most cases the latter will be too costly relative to the benefits over intuition, or to simple budgeting. However, if the results can be applied to a large number of farms then the research cost might be justified. Thus, as noted, the most appropriate analytical method must depend on, firstly, the nature of the problem, and secondly, the value of potential benefits from using the results. Amongst other things, getting it right involves understanding problem solving.

Inquiry, or problem solving, involves:

1. Answering questions and solving problems; and/or
2. Developing more effective procedures for answering questions and solving problems.

The procedures used involve tools, techniques and methods. Tools refer to such things as microscopes, calculators and computers, mathematical symbols, logarithmic tables and so on. Techniques are ways of using the tools. Computer programming, for example, is a technique for using a computer, calculus a technique for finding the value of a variable which maximizes an objective, budgeting for estimating profit, and so on.

Scientific method refers in part to the way techniques are selected, i.e. to the evaluation of alternative methods available to solve a problem. Thus, in this book not only will the alternative methods be explained and discussed, but also considerations given to assessing which method to use. A typical problem, for example, would be whether to use enterprise gross margins or partial budgeting in solving an optimal crop rotation problem. To make these decisions the analyst or researcher must be very familiar with all the methods available, as well as understand the nature of the problem to hand.

Problem research can be divided into two classes:

1. Evaluative; and
2. Developmental.

Evaluative refers to situations in which known courses of action are being evaluated to select the best. For example, cash crop farmers might currently implement a range of crop rotation systems – which is the best? A farm survey might be used to solve this problem. In contrast, where new systems are being developed and tested a developmental research situation exists. For example, the problem might be deciding how best to incorporate soybean production into a mixed cropping system in a new farming area. As known systems do not exist, these would have to be developed.

The steps that must be performed in solving a problem include the following:

1. Formulating the problem, e.g. is it one of the best stock replacement systems, or the best breed of stock, or both.
2. Constructing the model representing the problem, e.g. developing budgets of the alternatives.
3. Testing the model, e.g. comparing budget prediction with actual.
4. Deriving a solution from the model, e.g. comparing budgeted profit of alternatives.

5. Testing and controlling the solution, e.g. trying out the solution on part of the farm.
6. Fully implementing the solution.

The rest of this discussion revolves around steps (**1**) and (**2**) after discussing the objectives in problem solving. The other steps will be covered throughout the rest of the book to a greater or lesser extent.

1.2 Optimal Solutions to Problems

A problem can be described as a situation with the following conditions:

1. An individual or group who has the problem – the decision maker.
2. An outcome that is desired by the decision maker. If there is not a desired outcome, there is no problem as, presumably, s/he already has what s/he wants.
3. At least two unequally efficient courses of action, which have some chance of yielding the desired objective or outcome.
4. A state of doubt in the decision maker's mind as to which is the best course of action.
5. An environment or context of the problem. The environment consists of all factors that can affect the outcome and that are not under the decision maker's control.

Problems can be more complex than the description above. For example, perhaps the decision maker(s) employ people to carry out the decisions; decisions made and implemented may cause a counter action by others; objectives may be complex multidimensional; and so on. Further, the decision answer required may not be a simple 'follow this course of action' result, but rather what is referred to as a strategy. This involves a rule telling the decision maker to follow one of a range of actions depending on the current state of the farm and prices. Thus, a wool producer should, perhaps, shear his sheep in a certain month if wool prices are falling, or at a later date if they are rising. The decision depends on the current state of the environment.

One problem is that analysts tend to superimpose their own goals so the results apply to their objectives. They are not the decision maker, nor the person taking responsibility for the outcome. Thus, it is important to carefully assess a decision maker's objectives and use these in an analysis, or at least use a range of objectives to suit a wider group of people.

It is also true that it is often difficult to find the true optimal solution to primary production problems unless the problem and its environment are very simple. Most realistic problems are quite complex and difficult to model. Further, an analyser is often involved in finding solutions to problems faced by many producers, so the researcher must measure and report errors in such a way that they can be adjusted to suit the circumstances faced by each individual decision maker.

In order to estimate 'errors' the nature of a problem must be accurately specified even though this conceptual framework may not be used. For example, in deciding when lambs should be weaned it is known that this will depend on selling price estimates, the current feed situation, expected feed production and so on, but it may not be considered worthwhile to develop detailed forecasting models of what a particular, for example, lucerne (alfalfa) paddock (field) will produce. Rather, an agronomist could be asked for an estimate based on experience. Thus it is known where an error may be introduced into the system. The objective must be to minimize the expected cost of errors relative to the potential benefits of correct decisions.

1.3 Formulating the Problem

One of the first steps must be to study the environment, or context, in which the problem exists. Producing a general diagram or description of the whole system in which the problem is embedded (a flow chart) can be helpful. The system will consist of desire/s or need/s (objectives), and the potential activities satisfying these. The components are then connected by a flow of information (as represented in Fig. 1.1), which leads to decisions on a choice of system, or in other words, a set of production activities.

When information is received a decision must be made on its use. In some cases it should be discarded. Having described the system it will be clear which information is important and which is irrelevant. No matter what the problem, the same components are present: information inputs, transmission, compilation, decision and action.

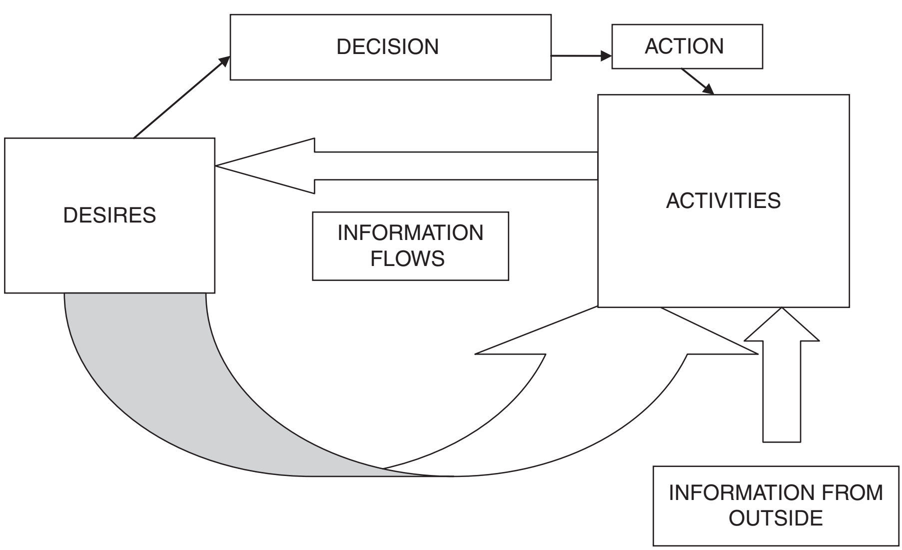


Fig. 1.1. Formulating the problem flow chart.

To enable analysis, a problem must be formulated in rather more detail than presented above. The following components should be considered.

The decision maker's objectives (desires)

Formulating these is difficult, and seldom is the researcher given these in clear detail. If asked, the farmer will probably provide a series of platitudes, e.g. enjoy the farming way of life. Such platitudes have little operational significance. Thus, the researcher will have to extract the objectives himself and in so doing may well provide a useful service. Direct questioning seldom reveals all the relevant objectives, so often a good approach is to present the decision maker with alternative 'solutions' to the problem and note reactions. This often reveals previously unmentioned objectives. Furthermore, objectives often change, so last year's cannot be used in solving this year's problems. Sometimes answers to especially prepared question sets will provide details (see, for example, Nuthall (2009), for a question set).

Alternative courses of action

Identifying the possibilities largely consists of:

- identifying the variables that significantly affect the outcome of the problem, e.g. time of weaning, time of shearing, price of wool, and
- determining which of the variables can be controlled directly, or indirectly, by the decision maker, e.g. price of wool cannot be controlled but time of shearing can.

Variables will be defined in either a quantitative or qualitative way depending on their nature. For example, the decision regarding how much superphosphate fertilizer to apply will be a quantitative decision, but the decision regarding whether to sell the lambs at auction or by weight is a qualitative decision (i.e. yes/no).

In some cases none of the obvious courses of action may seem to solve the problem. In this case the researcher must look for new courses of action – design and search.

More on objectives and their handling in choosing between courses of action

Ideally, it is useful to be able to quantify objectives so that alternative courses of action can be given a value – choice is then easy. However, this may be difficult, so offering a range of efficient solutions to the decision maker will allow choice. For example, in choosing between buying or breeding replacement stock the adviser could estimate the cost of the best way of achieving each method and let the farmer decide on the basis of the cost and other factors such as the risk and work loads.

Models

Models are representations of farm states, objects and events, and are usually simplified representations of the real world (Swinton and Black, 2000). Their advantage is they are easier and less costly to manipulate and experiment with than the real world. It is possible to define three different types of models:

1. Iconic models – small-scale models of the real world, which look like the real thing, e.g. a sand and plaster model of a river system, including water flows, which enables, for example, comparing flood control systems.
2. Analogue models – physical models which resemble, but do not exactly mimic, the real world situation, e.g. a road map, an electrical circuit designed to represent an hydraulic system.
3. Symbolic models – models in which the real world is represented with symbols. This is the type of model commonly used in farm systems analysis, e.g. a simple budget.

Symbolic models

These consist of variable/s representing output (e.g. profit) and input (feed, fertilizer, etc.). Thus we have, for example, a production function:

$$Y = f\,(X_i)$$

Output (Y) is some function of inputs (X_i), where a value for i represents each input (e.g. X_1 might represent the amount of fertilizer per hectare, X_2 = irrigation level, etc.).

Models of problem situations will always take the following form:

$$Y = f\,(X_i,\ W_j)$$

Where Y = measure of the value of the decision/s made (the output);

X_i = the variables under the control of the decision maker – decision variables (e.g. the quantity of fertilizer/ha, the lambing date, etc.);

W_j = the factors (variable or constant) that affect performance but are not controlled by the farmer (e.g. the wool price);

f = the relationship tying the variables together.

In most problems there will be restrictions on the decision variables. There is usually, for example, a limited supply of fertilizer, land and all the other resources. Thus, the problem is constrained.

In constructing decision models other models may be required, though they may not contain any decision variables. For example, in deciding when to wean the lambs (time being the decision variable), selling price is important so a sub-model in the system may be a model predicting the lamb prices even though it is not possible to influence the price.

In constructing models each variable has to be defined so it can be measured, and when the model is being used to predict (e.g. lamb price), samples of variable values must be taken in order to predict what the outcome will be (or observe the whole population). Further, once the model as been constructed, a method of analysis must be designed. Before analysis must come validation. Finally, results must be interpreted and extended to the decision makers who require the information.

In model building considerable time may need to be spent in determining the functional relationships, for these tie the model together and determine the optimal values of the decision variables, e.g. how will altering the lambing date affect the lambing percentage?

1.4 Constructing and Using Models

Models are approximations

Due to the complexity of real life, most models will be approximations of reality. Furthermore, models need to be manageable (capable of being solved), so sometimes simplifications are necessary, bearing in mind the significance of the results must be related to the cost of obtaining them so simplification may be economically justified. The problem is to decide what are reasonable simplifications. In reality, the only way to truly determine the effect of simplifications is to first solve a completely realistic model, then repeat the analysis using simplifications and make comparisons. To make a decision on the detail required you can either:

- ‘sneak up’ on the problem by successively more complex approximations and compare the improved results with the costs; or
- start with complete complexity and gradually simplify until the problem can be solved given the funds available. As this assumes a fully realistic model is available it could be sensible to simply proceed without simplification.

Experienced researchers will be able to decide the best approach without these trial procedures. Past trials lead to a body of knowledge instantly available.

Possible simplification methods

These include:

- Omitting decision variables, e.g. assume all lambs will be given selenium when comparing alternative stock systems.
- Omitting uncontrolled variables, e.g. assume a given lamb price in a budget whereas, in fact, it might be any one of many prices.
- Changing the nature of variables, e.g. ignoring the possibility of hiring casual labour when comparing alternative labour costs.
- Modifying the functional relationships to more easily handled relationships, e.g. using linear relationships instead of more complicated non-linear functions in a production response relationship.
- Changing the constraints to a simpler form, e.g. ignoring the limitations on a cropping system imposed by having two tractors of different size: simply assume the average size when developing a new programme.

In considering these, questions of significance must be considered.

Sequential decision models

Sometimes decision models are constructed for a range of actions, some of which do not have to be implemented for some months (time-based models). When it comes time to make some of the later decisions some of the assumptions may have changed. Thus, models may need to be constructed that enable re-solving just before the decision is made so that all relevant up-to-date information can be used, e.g. don't decide on how much hay to sell until well into the winter.

Deriving solutions from models

In analysing models three possible solution techniques are available. Which one should be used depends on the structure of the models. These are:

- Analytic procedures: where 'one calculation' provides the answer; e.g. use of calculus in finding the unique maximum of a profit function.
- Numerical procedures: where trial and error methods are used to converge on the solution; e.g. a series of budgets, each one being developed on the basis of the conclusions suggested by the previous one; or a linear programming model solution where the algorithm (the solution procedure rules) converges on an optimal system through repeated calculations.
- Complete or near complete enumeration: where all likely alternatives are evaluated so the best can be selected; e.g. a series of budgets which have been calculated concurrently for all the alternatives.

Of course, various combinations are also possible where a problem has subcomponents.

1.5 The Road Ahead

By now it is becoming clear that farm systems analysis is about creating models of alternative systems to allow comparisons and analysis leading to conclusions on improved systems, which, hopefully, are optimal (Petheram and Clark, 1998). This work is largely quantitative in that it produces values of decision variables and outcomes.

However, it is not always appropriate to conduct quantitative research. Such a case is where little information and data are available for solving a problem. Another case is where the problem is simply unquantifiable, e.g. where you wish to compare enjoyment levels experienced by farmers when producing different products. There simply is no scale on which to objectively measure 'enjoyment'. However, in some of these cases inherently non-measureable outcomes can be approximated with various opinion scales.

For situations where little data are available, the problem might be to produce qualitative information that might one day lead to acquiring data that can be used to quantify the problem. For example, when exploring a new development it might be appropriate to ask farmers, extension workers, commercial managers and the like, what their views are on possible costs and returns, conditions and problems. The qualitative conclusions might then be used to conduct experiments and institute recording systems to provide data for future quantitative work.

While most of the chapters that follow relate to quantitative methods, a discussion on qualitative analysis will also be provided. The use of the word 'analysis' might be questioned, but as will be discovered, respondents, and other sources of information, can provide information amenable to scrutiny leading to conclusions on questions that might have been posed. While this is not traditional scientific discovery involving hypothesis testing, it is answering problems. In quantitative work, proposed hypotheses (e.g. it might be proposed that 'a farm system producing wheat and beans is more profitable than one producing wool and meat') are tested, whereas in qualitative work hypotheses might be generated that can be tested later.

In the next few sections what might be called the decision-making environment in primary production is discussed, as it must be understood to enable creating and assessing suitable analytical methods. A strong feature of the environment is its uncertain and risky nature, so a chapter is devoted to covering risk and uncertainty. Models need to allow for this to enable realistic conclusions. A discussion on valuing system outputs is also included, for in real life objectives are multidimensional, i.e. profit is not the only objective. Indeed, many farmers will tell you profit is secondary to 'enjoying farming as a way of life'.

Farm production is a long-term affair with, in many cases, production systems requiring several years to produce saleable products. Clearly forestry is the extreme, but even stock systems can take more than 1 year to produce. An example is beef production in which 2-year-old animals are sold. And it is certainly true that farm development takes several years to reach an equilibrium. If, for example, a block of land is cleared and sowed down into pasture it might be as long as 10 years before the pasture is producing to its potential. This all means that in analysing such problems the timing of costs and returns must be taken into account, for there is an opportunity cost of the committed funds before output occurs. Funds invested could have been put into the bank and returned the going interest rate. Thus, farm investment must return at least this rate to be worthwhile. In analysing situations where time is involved, such factors as the compound interest rate must be allowed for. A chapter covers all the formulae and methods to be used in these time-dependent projects, called cost–benefit analyses. Furthermore, it is not always easy to decide which costs and returns to include in these analyses, so discussion on the choices is provided. For example, income tax is a cost to the farmer and needs to be included. But if a state is considering a group irrigation scheme, tax is not a cost as it is simply a transfer payment. In other words, from the farmer's point of view, a payment from farmer to the state is a real cost, whereas in contrast this payment is just an internal transfer (from farmer to state coffer) and should be ignored for purposes of deciding whether the state should invest.

The subsequent chapter covers farm surveys and their use in comparing farming systems. Every farm can be viewed as an experiment and, consequently, provides information useful in comparing systems and in coming to conclusions over optimal systems. Farm survey technique is about gathering this information and using it to make conclusions. Having collected the data, analyses can use a range of techniques from simple comparisons between groups of farms (e.g. those in three different fertilizer use ranges to compare profit), through to quite sophisticated statistical techniques such as structural equation modelling. Also included is the estimation of production functions, either per technical unit (e.g. wheat output per hectare over a range of input levels), or whole farm production functions where the output might be profit and the inputs could be the expenditure on various inputs. These calculations rely on using regression analysis in most cases (regression analysis involves finding the equation of the line or curve which best fits the data through minimizing the residuals from the estimated curve).

Of course, survey data have other uses as well. An example is the formation of benchmark data, which can then be used to guide farmers on how to improve their farming system through comparing the case farm's data with the benchmarks of the better farms. Another comparison method is the use of efficiency models that determine the farms with the most efficient production functions. This information is then used to rank farms, enabling the poorly ranked ones to compare farming systems used (data envelope analysis). All the methods mentioned are explained in the text. While most of these applications rely on using farm survey data, a special form of data collection is a purposefully set up recording system in contrast to simply using what historic data are available. With modern computers found on most farms, it is not difficult to set up recording schemes which create files that can then be fed into group analysis computer packages.

A basis for much of the analysis mentioned is production economics. There are many texts written on production economics, so the details are not covered in this book. However, it is useful to summarize the conclusions provided by production economics, especially as one of the techniques mentioned later is a direct representation of the decision rules created from production economics. By a decision rule is meant a condition which, if attained, provides optimality. One such rule is to invest in increasing quantities of an input until the cost of the last unit invested just matches the profit increase which results from this last input. This is the 'marginal cost equals the marginal return' rule. Appendix One provides a synopsis of the basics of production economics for students new to the area, and revision for others.

Overall, the techniques mentioned rely on historic records and consequently provide answers about improved systems that can be no better than the best in the population of farmers surveyed. In contrast, it is possible to use a range of analytical techniques to discover systems that are improvements on the existing systems. These rely on having technical information that defines physical input–output relationships, which can then be used to create 'optimal' systems using various standard models. While the 'very best possible' systems might just exist in the farming population, it is more likely that the calculations will come up with better farming systems. However, note that the technical information (e.g. the relationship between animal growth and the intake of protein and energy) can only come from historic studies using experiments and farm observations. There is one exception to this: if all the biochemical relationships are known, it is possible to create mathematical equation models that will predict production relationships. For example, current knowledge enables simulating milk output from a cow's udder, given information about the nutrients being supplied through the cow's blood.

The analytical models outlined in the book include linear programming, dynamic programming and 'free form' systems simulation. Linear programming (LP) selects from the available alternative production systems and activities a combination that maximizes an objective, usually profit. The relationships between input, output and the constraints on production are assumed to be linear. That is, e.g. if 10 ha of wheat produces 150 t of wheat, then 20 ha will produce 300 t. In reality, there are ways to model segmented non-linear relationships if a little imagination is used in defining the problem. This LP methodology has been used for many years, the offshoot of which is that there are readily available standard computer packages for solving LP problems. Thus, once the researcher knows how to input the data, the computer does the rest and comes up with the optimal system. The chapter on LP does, however, explain the solving technique, for understanding this allows understanding special features of the solutions. Linear programming is a form of automatic budgeting. To lead in to studying LP, a previous chapter contains an outline of budgeting in its various forms. Budgeting is the simplest of the analytical approaches and is used by many farmers, but to be useful it does rely on having a skilled and experienced operator.

Dynamic programming (DP) is a more flexible model than LP. It can handle non-linear relationships and risk with conceptual ease, but the downside is the difficulty in solving large problems. The computational burden can be excessive, though with the speed of modern computers tractable problems are increasing in size. As the name implies, DP is particularly suitable for solving time-dependent problems. The model assumes decisions come in stages in contrast to having to make, for example, a whole year's decisions all at once. Thus, a farm decision problem is started by defining the current state of the resources on hand. The first set of decisions gives rise to a new state after, for example, 1 month, depending on the outcome of the non-certain variables (e.g. rainfall). This new state is then exposed to a new set of decisions, leading to an updated set of resource levels (e.g. cash in bank) and so on through to the end of the decision horizon. Solving involves finding the optimal decisions at each stage of the time-dependent processes. Conceptually, the model is the most realistic of all available, but the computational burden does restrict its use. However, a study of DP leads to a very good understanding of realistic decision problems.

While LP and DP have well-defined problem structures, systems simulation (SS) does not have any predefined structure. While all models are simulations of the real world, when talking about SS the general understanding is that SS models are constructed specifically for each problem using computer languages of various kinds. Thus, for example, if you wanted to simulate animal growth to explore optimal feeding regimes, you would search the literature for information on animal maintenance and growth responses to intake of energy and protein. Also relevant would be equations defining intake limits. Equations for maintenance depend on the live weight of the animal, thus:

Maintenance energy (Metabolizable energy ME) = f (live weight in kg).

Similarly for growth:

Growth (kg) = f (ME intake, digestible protein intake).

For intake there will be a relationship:

Max intake = f (live weight, feed concentration).

Given details of these equations, it is possible to predict growth given a certain diet is assumed. Furthermore, as the quality of the food is probably uncertain this can also be recognized. Using the model, via computer programs, experiments can be repeated many times for different feed intake combinations and feed qualities. The results can then be combined with cost and price data to come up with profit ranges from which an optimal decision can be concluded. In effect, SS is a symbolic form of experimenting, which provides data for statistical analysis and a conclusion. With imagination and basic data just about any decision problem can be modelled and experimented with. Of course, the results will only be as good as the data used, and the process is quite expensive as standard computer packages are not available, though there are packages specifically designed with the features typically used in SS making creating and experimenting with SS models easier.

After the chapters explaining these prescriptive techniques, the book has a penultimate chapter discussing a number of analytical models that are commonly used in urban business but which are less important in primary production. One model is used for inventory analysis, where the problem is to determine the optimal level of inventory to hold. For example, how much hay and silage should be held that minimizes the cost of a feed level which satisfies feed demand? In urban business, such as a supermarket, inventory questions are critical and are solved by major and complex computer packages.

Another ‘operations research’ problem, for these problems feature in most operations research texts, is replacement analysis. What is the optimal time to replace productive assets such as a header harvester, or even an animal? With time, productivity and costs change, leading to an optimal replacement time which minimizes the costs of providing the services of the asset.

Two other operations research problems discussed are critical path and queuing analysis, both of which can be critical in urban business but less critical in rural operations. Nevertheless, they are worth explaining to provide a conceptual understanding. Critical path analysis is about working out the task sequence in any operation (e.g. building a major dairy milking parlour) that is critical to determining completion time. Queuing analysis is about providing sufficient resources so an uneconomic queue does not form, e.g. providing harvesting resources to ensure crop loss is not overly expensive.

Along the way in explaining the alternative methods of analysing and creating farming systems, a number of sidetracks appear. For example, the question of how to measure which farms are efficient is discussed, for sometimes analysts get confused over this issue. Similarly, comments on cost accounting are offered, for this is a common approach to deciding on the profit contribution of alternative enterprises. Less of a sidetrack is an earlier chapter on 'representative farms'. Modelling work requires a base from which to work, so the concept of a representative farm is used to ensure the results have the widest application possible.

In all the discussions a farmer and/or analyst is defined using 'he' or 'his'. In most countries most farm managers are male, but there is certainly an increasing number of female managers and, probably, at some time in the future the proportion of each gender will be almost equal. In the interests of ease of reading, 'him', 'his' and 'he' are used, but these terms infer the manager/analyst could be either male or female.

Finally, little attention has been given to defining what is meant by a 'farm system'. It is probably clear that a 'farm system' is a plan or blueprint for operating a primary production unit, be it a farm, horticultural unit, or any other form of primary production. This plan, or system, includes a list of the products produced, the area/quantity of each, a list of the inputs used for each product, and a description of how the inputs should be used both in a timing, quantity and application method. The plan also includes answers to financing questions, and instructions for all other components of the plans, including strategies for handling uncertainties as they occur. In theory, a manager should be able to take up the blueprint and follow the guidelines it provides to produce maximum profit, assuming this is the objective. This book covers the main methods available to an analyst to create these blueprints. Clearly it is not possible to discuss every possible method available, but with a full understanding of the contents of the book an analyst/researcher should have a sound basic knowledge that will lead him or her to seek further refinements where necessary.

References

Nuthall, P.L. (2009) *Farm Business Management. The Human Factor*. CAB International, Wallingford, UK.

Nuthall, P.L. (2010) *Farm Business Management. The Core Skills*. CAB International, Wallingford, UK.

Petheram, R.J. and Clark, R.A. (1998) Farming systems research: relevance to Australia. *Australian Journal of Experimental Agriculture* 38, 101–115.

Swinton, S.M. and Black, J.R. (2000) *Modeling of Agricultural Systems*. Michigan State University, Department of Agriculture, Food and Resource Economics, Staff paper 11581, East Lansing.

2 The Environment Under Which Farming Systems Exist

2.1 Introduction

In creating successful farm systems an analyst must allow for the conditions and background under which any farm operates. This can be called the decision environment. Part of this environment is the set of the farmer's objectives that should be met. This chapter contains a discussion on the components of the overall environment and, consequently, provides an understanding of the factors which must all be allowed for when planning. These factors include the risk and uncertainty that always exists, the resulting management problem structure, the nature of the management input that controls the farm, the influence of the farm household on decisions and, finally, the nature of the objectives and their measurement.

2.2 Risk and Uncertainty

All farms face uncertainty and this affects the type of information and models that are necessary when planning. While in some cases it is legitimate to ignore uncertainty in decision models, it is still necessary to be aware of its impact and, possibly, modify the model to allow for risk in a simple way, such as taking very conservative price estimates.

For descriptive purposes three distinct types of information and decision environments can be defined.

A certainty environment

This is where it is known with certainty the outcome of a particular action. For example, when a farmer borrows money at agreed terms he knows that in time he will have to furnish interest and principal charges, or else the lender will sell the farm to get back the money unless special arrangements are made.

A risk environment

This is a situation where:

- the average outcome over a number of trials of a particular action is known with certainty, i.e. repetition of trials may be in time or space; e.g. wheat yield trials over many years (time) or over many sites of the same fertility (space);
- the outcome in a particular trial cannot be predicted with certainty, but the chance or relative frequency of a particular outcome can be predicted.

For example, if a farmer sows a particular wheat variety in a particular way into a seedbed of a particular fertility and moisture state, and he repeats this trial many times (different paddocks/fields or different years or both), and he knows that the average yield will be 7800 kg/ha, but that in any one trial the yield could be 6500 kg with a 25% chance, or 7200 kg with a 50% chance, or 10,300 kg with a 25% chance, then a risk situation exists. However, the outcome in a particular trial cannot be predicted with certainty. What this means is that given, say, 20 years of trials we would expect 5 years to exhibit a yield of 6500 kg, 10 to exhibit 7200 kg and 5 to exhibit a yield of 10,300 kg. In real life the possible outcomes would be many more than this and would form a continuous stream of possibilities. In this example, the three possible outcomes are used to represent mid-points of ranges.

An uncertainty environment

This is where the possible outcomes might be known from a particular action, but even if they are, the chance of each particular outcome in repeated trials is not known. For example, when a farmer sinks a well he will know what the possible outcomes might be but he does not know the chance of striking a flow of 2000 l/h at a depth of 150 m. Another example might be where a farmer uses a new variety of white clover. As it has never been used on his farm before, the chances of the various possible yields are not known.

In these examples possible outcomes can be considered known, though, strictly, this is a matter of definition to a certain extent. In the white clover example, because the farmer does not know the yield range it could be construed to mean he does not know the possible outcomes. But considered estimates may be quite satisfactory given many years of experience.

The above definitions rely on the term ‘trial’. For the definitions to hold, trials must be repeatable. In farming, this is seldom the case as exact trial conditions are unlikely to be repeated. For example, the soil fertility level is unlikely to be identical in each of ten paddocks. However, the definitions serve a useful reference point.

A typical decision-making environment

Most farmers operate in what is largely an uncertainty environment, though there will be some certainty aspects. While pure risk seldom occurs, many 'uncertainty' problems approach risk situations as it is possible to estimate outcome chances on the basis of past experience of 'similar trials'. For example, in estimating wheat yield chances, past years' records on the particular farm, and on similar farms, may be used to provide a good estimate.

Given risk/uncertainty is the reality, it must be considered when formulating and analysing problems and when defining farmer objectives. Details are introduced in later sections and chapters. Fortunately, due to the relationship between risk and uncertainty, in many cases these two cases can be treated as one.

Types of risk and uncertainty found in primary production

Categorizing together risk and uncertainty environments, and calling this non-certainty, the following distinctive types of non-certainty can be listed.

Price uncertainty of products and input factors

Most products have non-certain prices and examples are common (e.g. beef, wool, lamb, milk solids, wheat, small seeds etc.). However, in some countries a number of products are subject to price stabilization agreements that may remove much of the uncertainty.

Factor or input costs tend to be less uncertain than product prices and exhibit general inflationary increases. However, some input prices fluctuate considerably. These tend to have a fluctuating supply, such as pasture seeds, store and replacement animals.

Some of the reasons for product and factor price uncertainty include:

- weather conditions (supply);
- commodity cycles (supply);
- fluctuations in national and international income and tastes (demand);
- technological change (demand and supply);
- political decisions (demand and supply).

The consultant, researcher and farm manager must be concerned with trying to estimate trends in prices and in estimating the nature of price variability.

Technical and yield uncertainty

This is uncertainty over input–output relationships and, together with price uncertainty, forms the major source of uncertainty in farming. Input–output information refers to crop yields, wool production, lambing percentage, result of spraying a crop for insect control, the input requirement to control a particular stock health problem etc. Quantifying this form of uncertainty usually relies on historical observations of similar situations and the results of field experiments.

Technological advance uncertainty

This refers to the lack of information about the effect new techniques will have on inputs and outputs, and about the yields of new plant varieties, animal breeds etc. Estimates of the effects, input requirements and yields, and the nature of their variability must come from trials and general interpolation. These will be more 'uncertain' than technical uncertainty.

Institutional uncertainty

This is the uncertainty attached to the effect institutions have on farming operations. The term 'institutions' covers organizations from central government through local government and lending institutions to the local soil erosion control board. An example might be the uncertainty over a request to borrow development finance from a bank. Another example, which directly affects all prices, is uncertainty over a government's decision to devalue its currency. Similarly, there is uncertainty about possible new tax laws. This general form of uncertainty is extremely difficult to quantify. Seldom can historical information be used to estimate the chances, though historical experience of how particular governments, lending institutions etc. operate will suggest the nature of the uncertainty.

Financial uncertainty

This refers to the uncertainty associated with the many financial matters facing a farmer, such as the interest rate on a mortgage, or the interest received on an investment. Financial uncertainty is affected by what happens round the globe, by governmental decisions etc. An important influence is the exchange rate between a country's trading partners.

Quantifying uncertainty

To incorporate uncertainty into an analysis it must be quantified. The logical approach is to use probabilities, i.e. the chance of an event occurring based on a 0 to 1 scale (e.g. the chance of a 6 on the toss of a die is 0.167) or relative frequency information. The estimation of such probabilities can be difficult and is considered in more detail later.

2.3 The Nature of the Management Problem

Management must decide on the products and inputs to produce and use, and ensure the decisions are efficiently implemented. In many cases the manager will also be involved in physically carrying out the decisions. This section, however, is not directly concerned with these aspects, but with how the non-certainty environment dictates the structure and form the decisions must take.

In a certainty environment the manager only has to make a series of decisions at the start of a particular planning period – no further thought is required. The decisions made are carried out at specified times and the results that were predicted occur without question. However, certainty environments do *not* exist, so the well known management functions of planning, execution and control and/or re-planning must be followed through.

In a non-certain environment plans are still made, but in their construction full account must be taken of the unpredictable outcomes. This has two implications:

- returns may be less, or greater, than expected so that consumption may have to be altered to fit the circumstances and/or adjustments made to plans so that consumption will be maintained;
- plans for future periods may no longer apply as they have become infeasible or sub-optimal.

Future 'period' lengths can range from daily to periods of several years. Which length is relevant depends on the nature of the plans made. If plans are, for example, for labour utilization, daily periods may be appropriate or, alternatively, if the plans are for borrowing development finance, 6-monthly periods may be appropriate.

Given a particular planning period has just passed, plans made initially for the next period may no longer be feasible. For example, hay production may have been much lower than expected so it is no longer feasible (possible) to carry the number of stock originally envisaged through the winter (though perhaps the plan could be made feasible by buying feed, but it may not be optimal).

Furthermore, the original plans may now be sub-optimal. For example, a farmer may have planned on carrying lambs over the winter to sell as hoggets (yearlings), but at the start of winter hogget price forecasts may suggest this action would be sub-optimal even though it may be feasible from a feed point of view. It may pay this farmer to sell off the lambs now and use the feed in some other way.

The implication is that when making the original plans uncertainties must be taken into account. If the farmer follows a course of action and the outcome does not turn out as expected, what alternative courses are available? These alternatives form part of the original decision and must be included in the original analysis, or at least considered. For example, given the hogget fattening example, the return on this alternative is not simply the return from hoggets, but the return from the hoggets, together with any other course of action which may be associated with this decision if conditions are not as expected.

Diagrammatically, the decision complex might look like Fig. 2.1.

In making the original decision to retain the lambs the other possibilities that may occur must be included in the analysis.

This decision-making problem can be described in another way. When plans are made they are made for the current *state* of the farm. By the 'current state' is meant, for example, the amount of working capital available, the condition of the various paddocks and the crops/pastures that are in them, the machinery on hand and its working condition, stock numbers etc. Given this current state, the plans made are designed to 'take' the farm into a different state at some time in the future. The 'different' state may not in fact be very

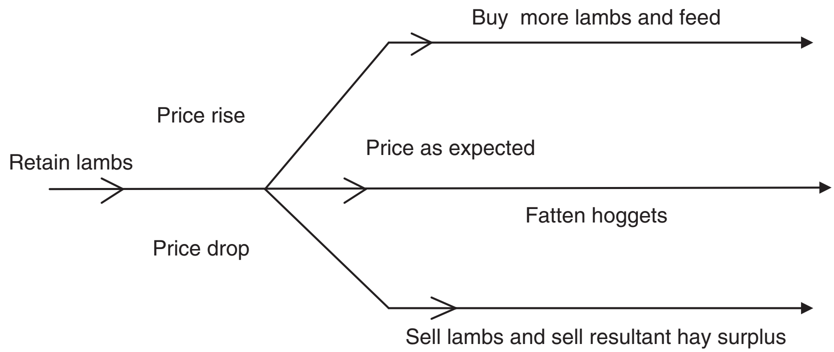


Fig. 2.1. Choices depend on earlier outcomes, in this case prices.

different from the current state in non-developmental situations. Subsequent plans then rely on the fact that this future 'different' state will occur and this state is the starting point for the next period. These periods may be quite small, for example, plans made to grow a wheat crop rely on the fact that the seedbed will be prepared by a certain time, i.e. the field will be in a certain state. If it is not, the original plans cannot be implemented exactly as planned.

The state of a system has been defined as its physical characteristics, but previous examples also show that the current state includes price and cost estimates. In a nutshell, the 'state' of a farm is a value for all the variables that go to make up a farm and the economic environment around it.

When evaluating alternative systems, the manager must take into account that planned states may not occur. Further, as prices, costs and conditions change the management is faced with a continuous re-planning problem, assuming that adequate records are kept so that the manager knows when expected states do not occur. This re-planning phase of management is often called *control*, but it is probably more than this. Control suggests making short-term adjustments as conditions change, but at any particular time a manager must be thinking of positive plans for the whole farm for all future periods and this may require major re-planning as different states eventuate through time.

However, in making plans it must be emphasized that the only action decisions necessary are those that have to be implemented in the current time period. All other decisions are irrelevant in an immediate action sense, as with changing states and conditions they may no longer be optimal once the period arrives. This does not mean that future actions can be ignored, but if a future decision is completely disconnected from a current decision there is no point in considering it now. For example, there is no point in deciding the type of fence to be used in a subdivision in 5 years' time as this decision will not affect what should be done now. In general, if a current decision will not affect the state of the farm at some future time, the decision can be made without reference to this future period.

Because future plans cannot be mapped out in detail, the idea of what is called a *strategy* can be introduced. A strategy is a 'group' of courses of action, one of which is optimal for each particular state the farm system might be in ('state' here refers to physical features as well as price expectations). Thus, in the hogget example, the actions might be (Fig. 2.1):

- If prices are as expected, finish off the hoggets for sale, i.e. 'fatten' them.
- If prices fall, sell lambs and sell surplus feed.
- If prices rise, buy more lambs to fatten.

Thus, a complete farm plan for 6 months could consist of a series of strategies, one for each part of the farm system, together with a description of the states under which each course of action should be implemented. This is vastly different from the idea of having a single farm plan.

The idea of strategies can be further broadened to encompass what is referred to as a *decision rule*. A decision rule is a statement informing the manager which particular course of action should be followed given certain circumstances, i.e. a rule indicating which particular action to follow. Thus, in the hogget example, the decision rule would be:

> if prices are within $\$X \pm 0.5$, implement strategy one, if prices are $> \$X + 0.05$, implement strategy two etc.

The existence of non-certainty makes the decision-making problem somewhat complex. A consultant could quite easily spend all his time evaluating and planning for a single farm, but this would clearly be an unprofitable use of planning resources. The potential improvement would not warrant the expense. In practice, each farm manager operates more or less in the manner suggested even though he is often not consciously aware of this. The process becomes intuitive, with active managers constantly reviewing decisions in their mind and considering future strategies.

2.4 The Nature of the Management Input

While it has been noted a manager must work with strategies, this section will broaden this discussion and generally consider the functions of management, managerial ability and the importance of estimating ability.

In managing a farm, the manager must carry out the following functions:

1. Formulate the goals of the farm-family complex. This involves constructing an objective function through which choices can be made between alternative systems.

Conceptually, an objective function can be regarded as a relationship indicating the net satisfaction (utility) obtained as a result of how the farm is run ('*f*' refers to 'a function of'):

> Net satisfaction = f (no. of sheep carried, hectares of wheat grown, level of debt, hours of leisure time, etc., cash take home pay).

The critical question is determining the details of the relationship. How does cash take home pay contribute to satisfaction? What is the satisfaction gained from an hour of leisure time etc.? Some of these problems will be discussed later. It is important to note that every farm-household complex will put different values on each of the sources of satisfaction, so what is correct for one case will not be correct for another. Further, objectives change with time, so that what is correct today may not be correct next period.

2. Recognize and define problems and opportunities. If change is to occur, this function is clearly crucial.

Research suggests this function is one of the most difficult. Farmers, and others, get used to a particular system and provided they are not forced to change through extremely poor economic circumstances, tend to maintain traditional patterns. Often how well they perform depends on their contact with advisers, better farmers, appropriate reading, TV programmes and other conduits of ideas.

3. Obtain information and observe the relevant facts. Given an opportunity has been recognized then, for analysis to proceed, data must be obtained.

Whether this function will be carried out satisfactorily depends in part on the farmer's training and experience. In order to obtain the required data, the farmer must have some knowledge of how to analyse the problem.

4. Specify and analyse the alternatives.

5. Make decisions from the results of the analysis. These functions (**1–4**) are part of planning and control (or re-planning) in the simple planning, execution and control description of management functions.

6. Taking actions as a result of decisions made. This is the execution referred to above. Depending on what is meant by planning, action-taking may involve some decision making, e.g. should the wheat in paddock *X* be harvested today or in 2 days' time?

7. Bearing the responsibility of outcomes. It is obvious that farmers must perform this function but how well they adjust to undesirable outcomes, and favourable ones too, is important in managerial efficiency.

8. Evaluate outcomes and re-planning. This must involve keeping adequate records and assessing the 'degree of lack of success' and eventual re-planning for future periods. This re-planning involves functions (**1–4**).

It is the role of farm advisers or consultants to assist the farmer or manager in carrying out the various functions of management.

2.5 Managerial Ability

How well a manager carries out these functions defines his managerial ability. All farmers will differ in their ability, both in general and for specific functions. One farmer might be good at managing stock operations but poor at making cultivation and crop decisions, and so on.

An adviser or researcher must assess ability as it affects the expected returns from the alternative systems. Particular abilities must be made use of. For example, while in general wheat production may be more profitable than lamb production, a particular manager's capabilities might reverse this ordering. An analyst needs to quantify ability as differences may be the reason for different responses to given inputs. Financial lending institutions are also going to be interested in ability.

Methods of measuring ability are still problematic. Most advisers must rely on qualitative assessments in which they observe a farmer's success relative to other farmers. Attempts have been made to measure ability by noting attributes that are regarded as being correlated with ability. Examples include intelligence rating, years and type of education, years of experience as a manager, etc. These can be combined to give an ability rating. For an extensive discussion on managerial ability, see Nuthall (2009).

Particular points that must be considered when considering ability include the following:

- Good ability is not necessarily associated with high profits, as this may not be the farmer's objective.
- Measurable attributes that are positively and significantly correlated with success for a particular objective function may not be highly correlated with success for a different objective function. For example, a high intelligence rating may be necessary for a farmer to be successful in maximizing cash return. In contrast, intelligence may not be highly correlated with success where the objective is to simply 'enjoy farming as a way of life'.

2.6 The Farm–Household Complex

Background

Throughout the world most farms are owned and operated by a proprietor. The extent of the ownership may vary in that some farmers will not own all the resources used, e.g. land may be leased. The form of ownership may contain trust, partnership or private company arrangements. However, in general most managers receive an appreciable portion of the returns from the farm and have appreciable control over the use of the resources. In addition he usually lives on the farm with his family, so that the farm and the family have an intimate relationship. This means that the whole family (including the manager) receive satisfaction from the farm rather than the farmer simply bringing home a weekly pay packet to a location separated from the farm. As a result the farm–household complex frequently has important implications when planning.

Contrasting this situation are the public company arrangements. The owners are separated from the business both in space and in management, though they have the right to elect the people who will control the business. The only satisfaction the owners receive is share dividends, so the aim of a public company's management must be to maximize profits long term.

The general nature of the relationship

Farming production and profit are only intermediate ends, to a certain extent. As far as a public company is concerned, profit is the end point of the process. An owner–operator farmer, however, operates to maximize family satisfaction, and profit is only one contributor to this. Cash is simply a link in the chain leading to family satisfaction. The farm can be described as the production entity and the household the consumption entity. Consumption in this case contains many aspects, ranging from monetary consumption, ‘way of life’ consumption, ‘fishing from the farm dam’ consumption and a wide range of other ‘products’.

This farm–household complex complicates decision making, in that the objective function or goals may well be somewhat different compared with the public company case, or the case of a farmer without a family. It is this need to consider the household that complicates the assessment of the goals, though in some cases the farmer will formulate the family goals on his own, or at least define what he thinks are the family goals.

Each individual in the household will obtain varying degrees of satisfaction from different outputs from the farm. The problem is to determine which individual’s requirements have first priority and what are the trade-offs between them. As quantitatively measuring satisfaction that is comparable between individuals is generally considered impossible, general estimates must be used. Even if satisfaction could be measured it would probably require a considerable time input and the increased satisfaction from the improved decisions are unlikely to warrant the cost. Measuring satisfaction, or utility, is discussed later. It will be shown that utility can be measured in an ordering (ordinal) sense in contrast to a counting sense (cardinal).

2.7 Conflict Between Farm and Household

Production and consumption require, basically, the same resources so that the farm (production entity) and the household (consumption entity) are in direct conflict for resources. The conflict largely revolves around capital and labour.

Capital

Where funds are non-limiting there is no problem. But as this situation seldom exists, decisions must be made on the division of available funds to farm and house and family. Farms can be categorized into situations where:

- the majority of surplus funds are being directed to farm development to provide greater consumption at a later date (be this actual monetary returns or satisfaction from being associated with a ‘developed and prosperous farm’);
- sufficient funds are being diverted into the farm to maintain the real income, while the remainder is being used for consumption;
- insufficient funds are being diverted into the farm to maintain the real income, while consumption is receiving a major share. This situation will eventually lead to zero consumption.

The degree of conflict between house and farm will depend on:

- in which of the above categories the complex finds itself;
- the availability of funds;
- the extent of the list of commodities and services the household regards as luxuries as against necessities.

Funds for necessities tend to be allocated without question. It is between luxuries and the farm that conflict arises. Different households will have different ideas on necessities and luxuries, which will be affected by the current and potential income from the farm. In times of economic stress the lists may need to be altered for survival. With time, the households' ideas tend to change, particularly after 'prosperous' periods in which 'luxuries' have been regularly acquired. This is reflected in the difficulties that farmers have in trying to reduce consumption to provide funds for development.

Where farm development is occurring, the household has decided it is worth reducing consumption now for greater consumption at a later date. Just how much will be put off depends on the satisfaction obtained now compared with future consumption. In general, people prefer to consume today rather than at some future point in time. This is referred to as *time preference* and this idea will form the basis of later discussions on the analysis of dynamic problems in which time is an important variable.

The conflict has a life cycle with changes due to:

- the tastes and ideas of the family changing;
- the structure of the family unit changing. This means that the family's demands will change;
- the changing availability of real income.

A typical life cycle might be:

1. Young farmer, single or newly married. Capital very limiting but consumption demands quite low so little conflict exists. Farm is being developed.
2. Farmer with family. The farm is still being actively developed but the consumption demands have increased considerably in order to support a larger family. There may be requirements for boarding school fees, and similar such as health costs. Capital is probably not so limiting in an absolute sense. However, given the high consumption demands, the degree of conflict is considerable.
3. Children have started to leave school. Development is progressing slowly. Consumption demands are not increasing so that the pay-off from previous development means the conflict is decreasing.
4. Farmer is approaching retirement. Development has probably ceased and direct consumption demands are lower so that conflict is minimal. However, the farmer may be making retirement provisions and if the farm is not to be sold there may be considerable conflict between off-farm investment and consumption.

Asset ownership and capital conflict

The household may seek asset ownership and low indebtedness for the sake of the satisfaction that this may provide in itself, and also the risk reduction may be important. Thus, capital may be used to freehold a leasehold property and/or pay off debt, even though this may be contrary to farm profit maximization. This may provide further conflict.

Labour

The same basic principles apply to the labour situation as for the capital conflict. The farm requires the farmer's time for management and physical tasks while the household similarly requires the farmer's time for holidays, looking after children etc. Thus, there is a conflict between work and leisure. This conflict is greater in a farm situation relative to urban jobs, as the household has immediate access to the operational time, and the farmer is not bound by office conventions and the demands of employees to the same extent as a manager of, for example, a public company.

A non-conflict aspect

The intimate relationship between farm and household means farmers can prudently include the family in some kind of multiple ownership system, thus enabling profit sharing and the resultant decrease in overall family income taxation (and thus an increase in family take-home cash).

2.8 Objectives and Decision Making

Introduction

Decisions are not possible without defined objectives that enable ranking alternative farm systems. The farm–household complex has implications in deciding on the objectives, but understanding this does not help in their quantification or in actually making choices. This section contains a discussion on quantifying objectives and their use in decision making. The section will by no means treat the subject exhaustively.

Theoretical background

In many cases farmers and their advisers choose between alternatives simply on the basis of monetary profit. However, most farmers have many other objectives, as has already been commented on. Conceptually, it is possible to talk of utility as an all-encompassing measure of satisfaction and use this to make the decisions. Utility is defined as the satisfaction obtained by a farmer and household as a result of a particular course of action. A unit of utility is an 'utile'. Thus, conceptually, satisfaction obtained from the various outputs from a defined farming system can be measured in utiles and combined to form an overall measure of satisfaction. For example, a particular action might provide:

$$\begin{aligned} \text{20 hours of leisure time} &= \text{3 utiles} \\ + \$350 &= \underline{\text{2 utiles}} \\ &= \underline{\text{5 utiles}} \end{aligned}$$

A choice can now be made between this and other alternatives as an overall value is put on each.

Diminishing marginal utility

Because $350, for example, provides 2 utiles this does not necessarily mean $700 will provide 4 utiles. In general, additional units of a product (e.g. cash or hours of leisure) will provide a diminishing level of additional satisfaction, so we have diminishing marginal utility as shown in Fig. 2.2.

Marginal utility is defined as:

$$\mathrm{MU} = \frac{U}{P}$$

where U = change in utility for a given increase in the satisfaction-producing product (e.g. $), and P = change in the quantity of the satisfaction-producing product (e.g. $).

Thus, for example, if utility *increases* by 5 utiles when net income *increases* from $30,000 to $30,750, the marginal utility is:

$$\mathrm{MU} = \frac{5}{750}$$

Thus, MU = 0.0067.

Individuality

Every farm–household complex will derive different quantities of satisfaction from various products so that an optimal system for one farmer may not be optimal for another farmer. A further complication is that the rate at which utility per additional unit of product declines will vary from farmer to farmer as shown in Fig. 2.3.

Fig. 2.2. Total and marginal utility.

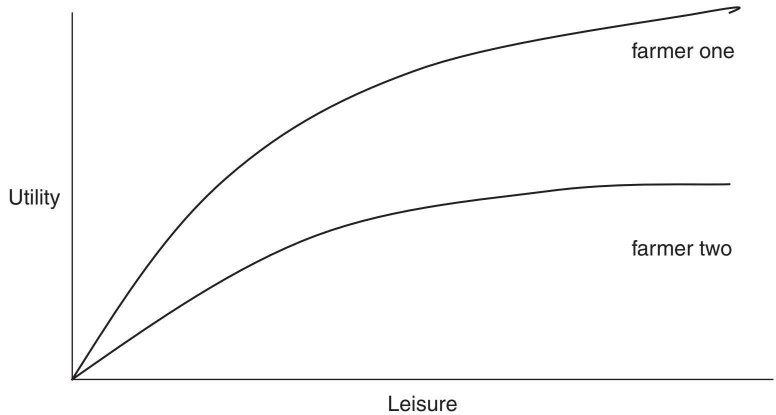


Fig. 2.3. Individuality of farmer's utility functions.

This graph suggests that different individual's utility functions can be quantifiably compared. It will be shown later that this is not the case. The idea expressed, however, is acceptable.

2.9 Optimal Decisions

Given a mathematical relationship of utiles obtained from receiving various levels of each particular satisfaction-producing product, alternative farming systems can be evaluated and the optimal system determined. For example, the utility functions for cash and leisure might be:

Monetary utility $U = 16x - 0.13x^2$

where x = cash income in \$1000 units,

Leisure utility $U = 55y - 0.65x^2$

where y= hours of leisure in 100 h units.

Thus, a particular farm system which produces (i) \$20,000 and (ii) 500 h of leisure would be:

$$\begin{aligned} U &= ((20 \times 16) - (0.13 \times 400)) + ((5 \times 55) - (0.65 \times 25)) \\ &= 526.75 \text{ utiles} \end{aligned}$$

Note that in this example the utility functions exhibit diminishing marginal utility as the squared terms have negative coefficients.

In reality, the situation will probably be more complex. There is probably an interaction between cash income and leisure, so some of both products will provide more utility than each taken separately. Further, most farmers would have minimum requirements on the various satisfaction-producing products so that a large level of cash income may provide zero satisfaction without any leisure combined with it. This is a special form of the interaction effect.

2.10 The General Utility Maximizing Model

These ideas can be generalized to produce a general theory of decision making, which you are probably familiar with from economics courses.

A farmer has a given set of resources that dictates the possible farming systems. Each possibility produces a given combination of the various utility-producing products. Where only a simple two-product case is considered, these combinations can be graphed in the form of a production possibility curve (Fig. 2.4).

To choose the optimal point that has associated with it a particular farming system, the concept of an iso-utility curve can be introduced. Such a 'curve' joins points on the graph which provide equal satisfaction or utility. Thus, a whole series of curves can be drawn (Fig. 2.5), each one representing a different level of utility.

The shape of each iso-utility curve reflects the relationship between the utility-producing products and is reflected in what is called the marginal rate of substitution of one product for the other while maintaining the same level of utility. A flatter curve, for example, reflects that you need a considerable amount of leisure to substitute for cash income, and vice versa. The shape varies with the distance from the origin. The further away, the more likely the curve will be steeper for cash becomes less valuable as you acquire more.

The optimal point (Fig. 2.6) is given by superimposing the utility curves on the production possibility curve and finding the point that is feasible (i.e. touches the production possibility curve) and produces the greatest utility (the curve furthest from the origin but still touches the possibility curve).

Where more than two products provide utility, the graph becomes multidimensional, but the same principle applies.

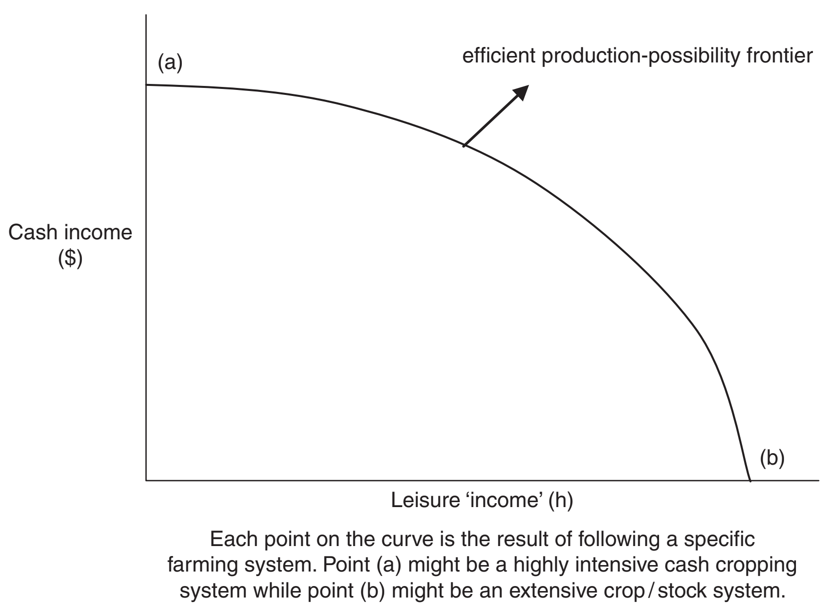


Fig. 2.4. Production possibility curve.

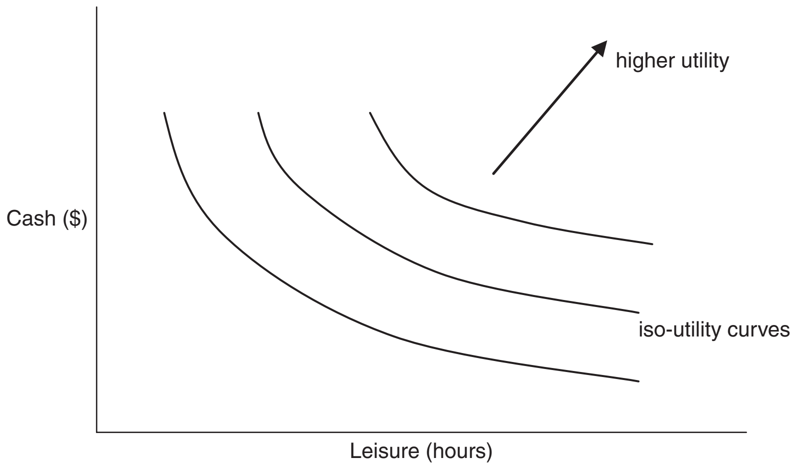


Fig. 2.5. Iso-utility curves representing higher and higher utility.

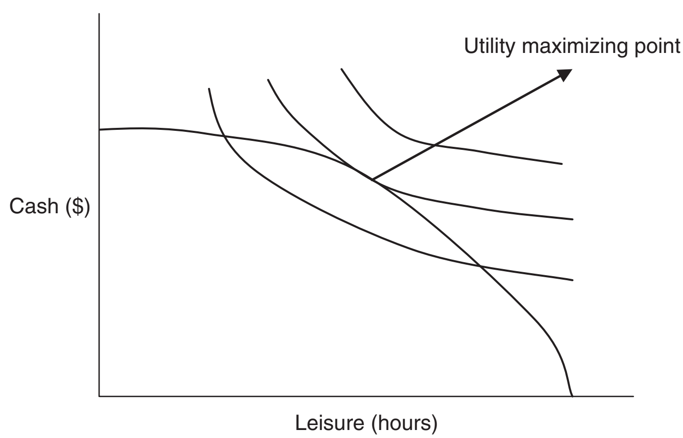


Fig. 2.6. Utility maximizing graph.

To actually determine the optimal point, all possible alternative farming systems need to be evaluated to provide the production possibility curve, then a series of iso-utility curves would be superimposed on the graph and a decision made. Alternatively, the utility functions (equations) could be used to estimate the utility from each alternative farming system to see which provides the greatest utility.

The examples used suggest cash income and leisure are important sources of satisfaction. Most farmers will, of course, be interested in many other satisfaction-producing products. One important 'product' is the level of *outcome variability* that occurs over a number of trials, or years, of a particular system. Most farmers prefer a system with as little variability as possible. It will be shown later there is an intimate relationship between the shape of a farmer's utility function and his attitude to income variability; and this attitude to variability leads to a way of measuring a monetary utility function.

2.11 Practicality of Using the Theory

The practicality of solving problems using utility will emerge in later discussion. Briefly, the broad principles introduced are correct, but the cost of using them, including determining utility functions for each farmer, does not generally warrant their use. Further, this assumes that a many factor utility function can be measured, and currently there is considerable doubt about this. This is not the case, however, with monetary utility (discussed later). Nevertheless, every analyst should take these principles into account when comparing systems, even if this is only in a subjective manner. An adviser, for example, might devise a system for a farmer which provides a minimum of leisure time as the farmer has few other interests besides 'farming'. A researcher interested in general results might present a wide range of possible farming systems, each of which is efficient in producing different levels of possible objectives. A farmer can then make a subjective choice between them.

2.12 The Road Ahead

The remaining sections in the book which consider objectives, decision making and optimal systems, concentrate on how to make decisions where either time is an important variable or where income variability is important. However, from time to time reference will be made to general utility. This approach is taken because of the difficulties in quantifying general utility. Future research may change this situation, but even if it does there is still the question of whether spending time on a detailed analysis of utility will be worth the increased satisfaction. On very large farms this might not be the case, or where the research has general interest to many. But whatever the case, it must be re-emphasized that the principles must at least be taken into account in a subjective manner.

References and Further Reading

Harwood, J., Heifner, R., Coble, K., Perry, J. and Somwaru, A. (1999) *Managing risk in farming: concepts, research and analysis*. USDA Economic Service Agricultural Economics Reports 34081.

Lee, J.E. (1965) Allocating farm resources between farm and non-farm uses. *Journal of Farm Economics* 47(1), 83.

Lin, W., Dean, G.W. and More, C.V. (1974) An empirical test of utility vs profit maximization in agricultural production. *American Journal of Agricultural Economics* 56(3), 497–508.

Nuthall, P.L. (2009) Modelling the origins of managerial ability in agricultural production. *Australian Journal of Agricultural and Resource Economics* 53, 413–436.

Nuthall, P.L. (2009) *Farm Business Management. The Human Factor*. CAB International, Wallingford, UK.

3 Decisions Under Non-certainty – Probability, Methods and Models

3.1 Introduction

Farmers often prefer one system over another solely due to the degree of risk associated with each system. It is important, therefore, to understand both how the degree of risk might affect objectives and how the degree of risk of alternative systems can be quantified.

Chapter 3 contains a discussion on all these relationships. As noted earlier, the term non-certainty is used to represent both risk and uncertainty, so one of the topics discussed brings risk and uncertainty together for decision making. True uncertainty is difficult to cope with, for it is largely non-quantifiable. But prior to these discussions, the first sections of the chapter cover quantifying risk through probabilities and how they can be manipulated. This is followed by a discussion on random variables and their distribution – any variable which can take on a range of values with defined probabilities is called a 'random variable'. Examples include the crop yield experienced in any one year, or the level of meat production from animals. The end point of an understanding of probability and random variables is their use in constructing farm systems with different levels of risk, and choice based on these risk levels.

The simplest method of representing risky choice is a *decision tree* in which possible outcomes are presented as a tree-like structure. This leads to quantifying the range of outcomes and their probabilities. The next step is to devise methods of choosing between the alternative systems. The first uses 'certainty equivalents', which are single values reflecting a farmer's assessment of a risky alternative. More complex approaches are also explained. An example is a choice based on the maximum average profit subject to a minimum in the worst of years. Further examples are also provided.

As part of the discussion, the idea of the 'cost of non-certainty' is introduced. Effectively this is the loss in profit resulting from not being able to predict the future. Understanding this concept helps better to appreciate risky decisions.

Finally, what is called 'monetary utility' is introduced. A farmer's attitude to risk is embedded in the value he places on increasing quantities of monetary income, so if this can be defined it is then possible to use this function to make decisions under non-certainty in complex models. As a postscript, the chapter finishes with a discussion on simple and short-cut practical approaches to comparing systems based on their risk levels.

3.2 Probability

Risk-based choice requires a method of quantifying chance effects through what are called *probabilities*. The discussion that follows considers their estimation leading to their use for risky decision making.

Objective probability

Given a random variable, i.e. a variable whose value is not known before the event, but its possible values and the chance of each is known (e.g. rainfall, profit), the value of the variable in any particular trial (e.g. rainfall on a particular day, profit in a particular year) can take on a number of values. Given, say 100 trials so that we have 100 observations of the random variable, the probability that a particular value will occur in future trials is given by:

Probability = Q/N

where Q = the number of times the particular value occurred and
N = the total number of trials or observations on the event.

For example, given 100 observations of the event 'wheat harvested/ha', 28 of these observations might have been 8200 kg/ha. Thus the probability of the particular event 8200 kg/ha is:

$$P = \frac{28}{100} = 0.28$$

Given this definition, this 'model' of probability ensures that the sum of the probabilities of all possible outcomes is equal to unity.

That is:

$$\sum_i p_i = 1.0$$

where there are i possible values that the random variable can take, and the Greek symbol epsilon means the sum of all the values p_i. Thus, if an outcome has a probability of 1, it is certain to occur.

If probabilities are estimated from a small sample of observations, it is possible that if more observations are taken the probability estimates will change. Thus, the requirement that the number of observations approaches infinity should be added. However, this creates problems as there will never be an infinite number of observations on trials that are identical. A wheat yield observation taken 10 years ago is unlikely to be equivalent to an observation made today as conditions will have changed (e.g. different variety, different cultivation techniques, different disease problems etc.). The definition, however, serves as a basis for the idea of probabilities. This type of probability is referred to as *objective probability* and is based on a *frequency* concept with the relative frequency determining the probability of an outcome.

Objective probabilities can also be defined using a *logical* concept. For example, we would expect a fair coin, when tossed many times, to land 'heads' 50% of the trials. This conclusion is based on logical reasoning. Only two outcomes are possible and there is no reason to suspect one will occur more often than the other.

Subjective probability

Rainfall is probably one of the few random variables for which objective probabilities can be estimated using relative frequency information. The reason is that conditions do not change greatly from trial to trial and there is a large number of observations available. For most random variables (e.g. lambing percentage, wool weight, growth rate, price of 13 kg lambs, sorghum price/t, cost of white clover seed etc.) purely objective probabilities cannot be estimated. This means that if non-certainty is to be incorporated into systems analysis some other means of calculating probabilities must be used, though it must be noted that even with limited historical information statistically 'most likely' probability estimates can be made, and in such estimates allowances can be made for changing conditions.

Another method of obtaining probabilities stems from the fact that farmers do make decisions in an uncertain world, so that it is very likely that they have made, in their own minds, some kind of probability or chance estimates of the possible outcomes. To be useful these estimates should be based on the limited amount of historical information available, indications which past experience suggests, assessments of likely future conditions, and so on. These estimates are based on subjective estimates and so are referred to as *subjective probabilities*. More correctly these probability estimates can be defined as the 'degree of belief or strength of conviction an individual has for the various possible outcomes'.

Probably the most significant argument for using subjective probabilities, even if objective probabilities are available, is the fact that it is the decision maker who bears the responsibility of any decisions made and the resultant outcomes, so he should incorporate his degrees of belief in the analysis. This does not mean, of course, that an individual needs reject the use of objective probabilities. In estimating probabilities he may accept available objective assessments as his own subjective estimates.

3.3 Risk and Uncertainty

The above discussion indicates pure risk environments seldom occur where it is assumed objective probabilities must be available for a situation to be defined as a 'risk situation'. However, if it is accepted that subjective probabilities are a logical basis for decision making, then the uncertainty environment previously defined no longer exists. Reasonable subjective probabilities can be placed on practically all random variables. Thus, the decision environment can be reduced to certainty and non-certainty. Non-certainty environments are all situations in which certainty does not exist, but probability estimates, be they objective or subjective, do.

3.4 The Farmer, Advisers/Consultants and Decision Making

Many farmers will not be able to estimate probabilities without the assistance of advisers or consultants. However, where subjective probabilities are used these should largely be those of the farmer as he takes the responsibility, but probably created with the help of an adviser. On the other hand, calculations containing risk assessments would not, in general, be calculated by farmers due to possible misconceptions. Thus, while farmers would find an understanding of risk principles useful, it is the analysts that need a full understanding of the theories and calculation methods.

3.5 Manipulating Probabilities

Introduction

Given a particular farming system possible profit levels can be estimated, each of which will have an attached subjective probability. For example, the manager of a mixed cropping farm might, with an analyst's help, estimate that using a particular crop rotation and stock policy will produce the following possible cash return outcomes (note that these are summary estimates – in reality, the likely income levels can range, say, anywhere within \$25,000 to \$65,000. As it is confusing to list *all* possibilities, a practical approach is to consider figures that summarize possible ranges):

\$28,000 probability 0.1
\$39,000 probability 0.2
\$50,000 probability 0.4
\$61,000 probability 0.3

However, it must be noted these simple summary estimates may be incorrect, for the final profit outcome is made up of many sub-outcomes such as the wheat yield, lamb growth rate, wool price etc. As it is difficult to combine all this complex information in your mind and produce correct summary probability estimates it is important, where possible, to build up the final outcomes and probability estimates from the basic factors. The following sections contain a discussion on the rules of probability manipulation so that these final outcome probability estimates can be calculated logically.

Probability manipulation rules

Mutually exclusive events

1. A set of mutually exclusive events is defined as a group of events in which only one event can occur in a particular trial. For example, possible lambing percentages are mutually exclusive as in any one year only one lambing percentage will eventuate. The group of events consisting of possible lambing percentages and death rates is not a mutually exclusive set as in any particular year one lambing percentage *and* one death rate will occur.

2. The sum of the probabilities of each event in a set of mutually exclusive and exhaustive events must be unity. By exhaustive is meant all possible outcomes are included in the set of events. For example, the following list of lambing percentages, assuming it is exhaustive, conforms to probability logic:

Lambing percentage	Probability
85	0.02
90	0.08
92.6	0.09
97.8	0.15
100	0.60
120.1	0.06
	1.00

3. The probability that any one single event from a group of events, being members of a set of mutually exclusive and exhaustive events, will occur is the sum of their probabilities. For example, the probability that one of the lambing percentages 85%, 90%, 97.8% occurring in a single trial is 0.02 + 0.08 + 0.15 = 0.25.

Independent events

1. Events can be either independent or dependent. An independent event is defined as an event whose occurrence is not affected by the occurrence of other events, whereas, for dependent events the occurrence of one is affected by the occurrence of another. Putting this another way, two random variables are independent if the possible outcomes of one are not affected by the actual value taken on by the other. For example, the random variables lamb price and wool clip/head are most likely independent variables, while rainfall and wheat yield are dependent variables.

2. When two (or more) events are independent, the probability of both events (or more) occurring together in a particular trial is equal to the product of the probabilities of the individual events. That is:

$$P(AB) = P(A) \times P(B)$$

where $P(A)$ = probability of event A

$P(B)$ = probability of event B
$P(AB)$ = probability of both A and B occurring in a single trial.
$P(AB)$ is termed the *joint probability* of the two events ($P(ABC\ldots)$ for more events).

For example, the probability of lamb price being \$4/kg = 0.6 ($P(A)$)
probability of wool/head being 5 kg = 0.25 ($P(B)$)
thus, $P(AB) = 0.6 \times 0.25 = 0.15$

3. With independent events, the probability of event B occurring, given event A has occurred, is equal to the probability of B occurring. That is:

$$P(B/A) = P(B)$$

where P(B/A) refers to the probability of the event preceding the slash occurring given the event/s following the slash have occurred.

$P(B/A)$ is termed the *conditional probability*, as it defines the probability of an event given a certain condition has occurred.

Dependent events

The conditional probability of event B, given that event A has occurred, is equal to the joint probability of A and B divided by the probability of event A, i.e.:

$$P(B/A) = P(AB)/P(A)$$

(Note $P(AB)$ will always be $\leq P(A)$) or

$$P(AB) = P(B/A) \times P(A)$$

That is, the joint probability of two dependent events is the product of the conditional probability of one and the probability of the other event.

For example, a farmer's records, or perhaps his subjective estimates, show that the probability of getting 28 cm of rain (event A) during the wheat growing season *and* a wheat yield of 7800 kg/ha (event B) to be 0.18. If this was an objective probability it would be estimated by observing the number of times these two events occurred together and dividing this number by the total number of observations. Further, the probability of getting 28 cm of rain is 0.21. Thus:

$$P(B/A) = \frac{0.18}{0.21} = 0.86$$

3.6 Using the Probability Rules

Method

To estimate the probabilities attached to the possible end-point outcomes of a farming system, the system has first to be broken down into its component random variables. The relationship between the variables must then be assessed so the rules can be used in estimating the probabilities of possible end-point outcomes. For example, the end-point outcomes might be the profit contribution of a wool-producing wether (castrated male sheep). This profit will depend on many factors, the more important of which might be the random variables wool cut/head, quality of wool, price received/kg of wool, animal health costs, the death rate, shearing costs and the relationship between fleece weight to other components. Some of these variables are clearly related. One example would be wool quality and price/kg. These relationships would have to be determined and similarly the probabilities of all the possible values each random variable can take. Examples of these calculations will be given in a later section. The simplest way to handle the problem is to draw a flow diagram as shown in Fig. 3.1.

This shows the possible end-point outcomes and, given the relevant probability information, working back through the below tree from each end point or outcome will enable its probability to be estimated.

This simple example indicates that if a whole farm system is to be divided into all its events there is a considerable amount of calculation involved, which will depend on how many outcomes for each variable are considered. In reality, most of the variables can take on an infinite number of values. For other than detailed research it is usual to consider only a limited number of possibilities and to break the overall system into as small a number of summary random variables as possible. Just what level of detail is required will depend on the problem and in some cases it might even be prudent to ignore the fact that non-certainty exists. The answers to some of these questions will become evident in later sections. Whatever the case, it is useful to be aware of how to estimate correct probabilities. This understanding may enable estimating summary probabilities on a subjective basis.

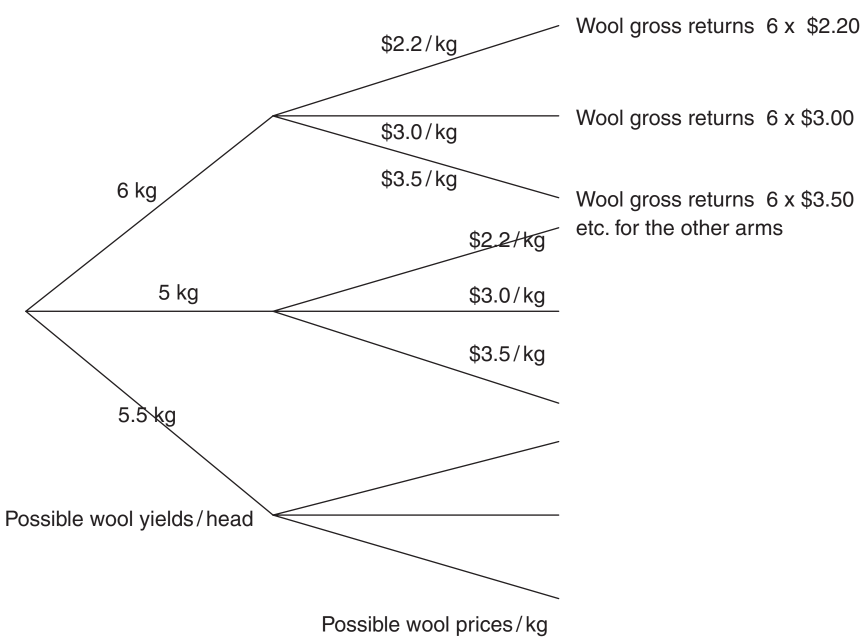


Fig. 3.1. Tree diagram of event combinations (wool gross income per animal).

Revising probability estimates

Estimates should be revised when new information becomes available, for example the results of experimental trials. Probability theory provides logical rules on how to revise estimates given new information (Bayes' theorem). However, such rules have greater application to objective probability situations. Where subjective probability is used, given new information, the subjective estimates would be revised provided the new estimates still conform to probability logic (i.e. $\Sigma p_i = 1.0$).

3.7 Continuous Outcome Possibilities

It has been noted that in reality there is often an infinite number of possible values a random variable can take. For example, wheat yield/ha can be 7600.1567 kg, 7600.1568 kg and so on. The probability values attached to such outcomes can be expressed in the form of a probability function or graph. An example is shown in Fig. 3.2.

Functions for the constituent variables can be used to produce end-point variable (e.g. profit) distributions, but such an approach can be mathematically difficult. For most purposes it will be sufficiently accurate to consider a number of discrete outcomes for which probabilities are estimated.

This is achieved by dividing the curve into a number of segments and estimating the area under each segment. The area gives the probability of each discrete outcome. An example is provided in Fig. 3.3.

3.8 Describing a Random Variable

The distribution of a random variable is characterized by a number of parameters. These parameters depend on the shape of the particular distribution. Two important parameters are the average, or mean, value of the variable, and the variance. The mean is usually referred to as the *expected value*, as when

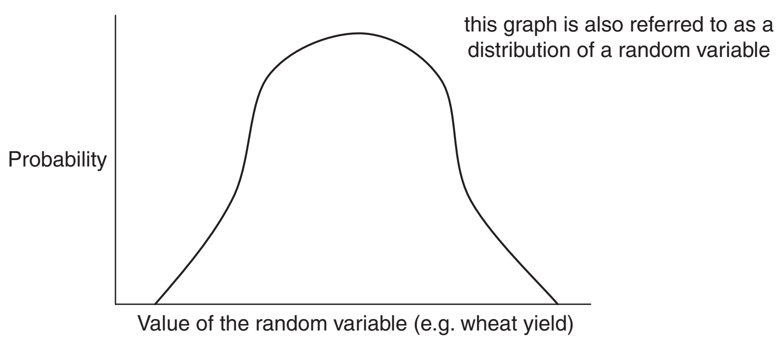


Fig. 3.2. A typical continuous probability distribution.

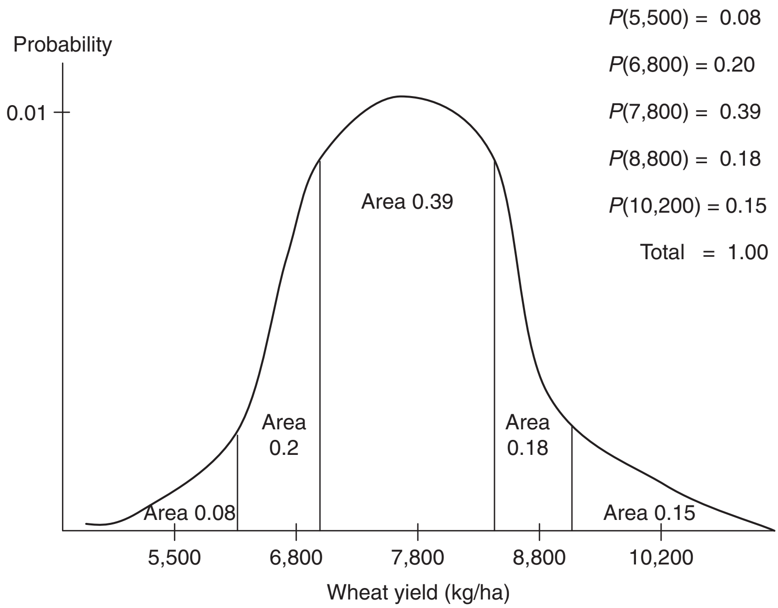


Fig. 3.3. Summarizing a continuous probability distribution with discrete segments.

dealing with forecasts clearly the 'mean' is usually the most likely value expected. Where it is assumed only discrete outcomes can occur the expected value of a random variable is given by:

$$E(V) = \sum_{i=1}^{n} p_i V_i$$

where p_i = probability of the ith value of the variable occurring
V_i = the ith value of the variable
n = the total number of values or outcomes considered.

For example, possible values and probabilities of wool cut/head might be:

Kg/head	Probability
3.5	0.10
4.0	0.25
4.5	0.50
5.0	0.15

Then,

$$E(\text{kg/head}) = (0.1 \times 3.5) + (0.25 \times 4.0) + (0.5 \times 4.5) + (0.15 \times 5.0) = 4.35$$

The general formula for the *variance* is given by:

$$V(V) = \sum_{i=1}^{n} p_i (V_i - E(V))^2$$

The variance for the above variable would therefore be:

$$V(\text{kg/head}) = 0.1(3.5 - 4.35)^2 + 0.25(4.0 - 4.35)^2 + 0.5(4.5 - 4.35)^2 + 0.15(5.0 - 4.35)^2 = 0.178$$

The standard deviation of a variable taking on only discrete values is:

$$S(V) = \sqrt{\sum p_i (V_i - E(V))^2}$$

The expected value locates the distribution on the graph, and the variance (and standard deviation) defines the degree of spread of the distribution. These two parameters are the first and second moment of a distribution. Further moments define other characteristics of a distribution.

3.9 Decision Making Under Non-certainty

Introduction

This section will demonstrate how decisions might be made for various forms of monetary based objectives. It will become clear how other measurable objectives could similarly be considered. Decision making under certainty will not be discussed, as choice under this unrealistic situation is obvious, assuming the objective is known. Choice simply involves choosing the course of action, or system, that provides the greatest, certain, objective value.

The problem under non-certainty arises as output is a random variable. A particular farming system cannot be characterized by a single summary figure, such as expected profit. To describe output the random variable's distribution must be used, or alternatively the parameters that characterize the distribution used. Farmers are not indifferent to alternative profit distributions even though the expected value might be the same. For example, two farming systems might produce the profit distributions shown in Fig. 3.4.

Most farmers would prefer system B as there is less variability, i.e. less *risk*. For practical purposes the most important distribution parameters are probably the expected value and the variance, though in some cases other distribution moments may be important. Farmers may not be indifferent to two systems with the same expected outcome and variance but with a different skewness (skewness characterizes the proportion of possible outcomes falling above or below the expected value, as shown in Fig. 3.5).

Evaluating alternatives in a non-certainty environment

Before examining actual decision making using possible criteria it is necessary to develop a framework for estimating outcomes and their probabilities. One framework uses the following procedure.

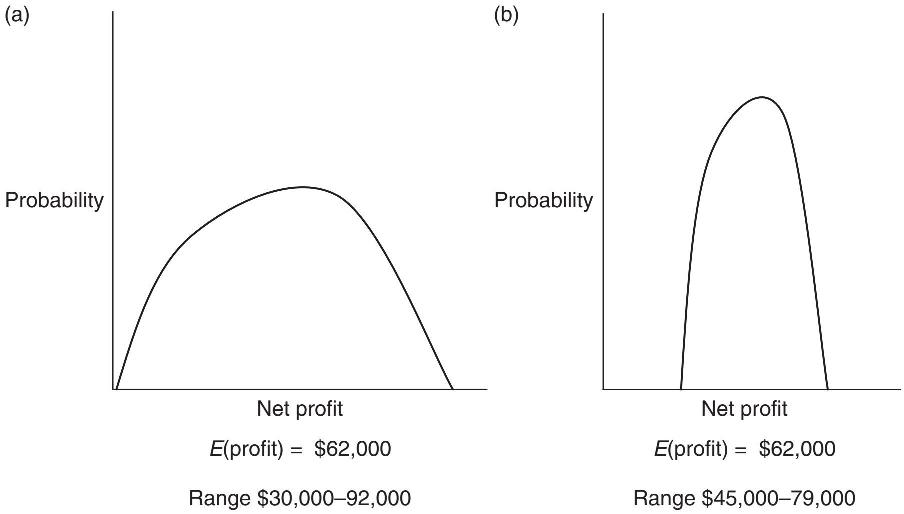


Fig. 3.4. Variation in profit distribution shapes.

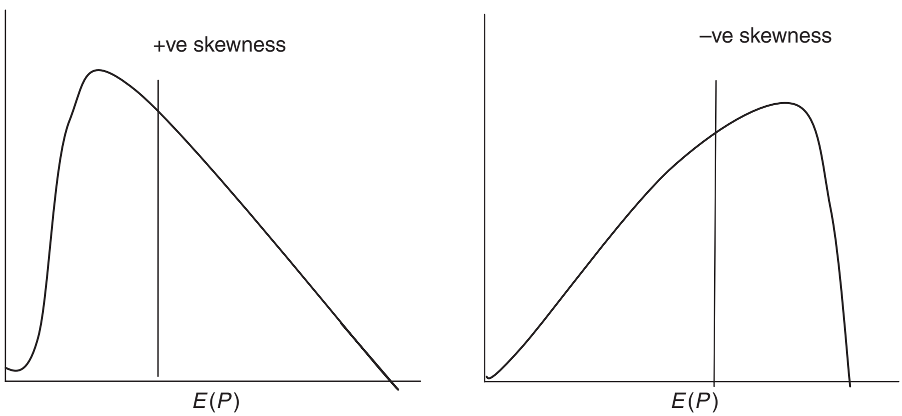


Fig. 3.5. Distributions showing different types of skewness.

Construct a 'cash' flow decision diagram or tree

This diagram follows through a series of actions showing where decisions must be made, the possible outcomes for each decision, and the relevant costs. In particular, the decision tree or diagram shows:

- acts – points in the system where a decision must be made between alternative actions;
- events – the possible physical outcomes resulting from the various acts;
- outlays – the cost involved in each act;
- returns – the monetary return resulting from each act and event;

- payoffs or net returns – the monetary inflow less costs for each act and event (or set of acts and events);
- probabilities – the probabilities attached to each event and, thus, using probability rules, the probabilities attached to final outcomes.

The purpose of a diagram is to logically set out the decision problem and to assist in: (i) locating where decisions must be made; (ii) estimating the possible final outcomes; and (iii) their probabilities.

The best way to describe this process is to consider a simple example. A farmer is thinking about applying nitrogen fertilizer to a wheat crop. The decision tree, on a per hectare basis, might take the form shown in Fig. 3.6.

The diagram contains a number of *forks*, hence the term 'decision tree'. These forks will be either:

- Act forks – points in the process where a decision is made. The decision directs the process along a particular branch. In Fig. 3.6 point A is an act fork (the only one); or
- Event forks – points where non-controllable factors split the results of an act into possible outcomes. In Fig. 3.6 these are points B, C and D.

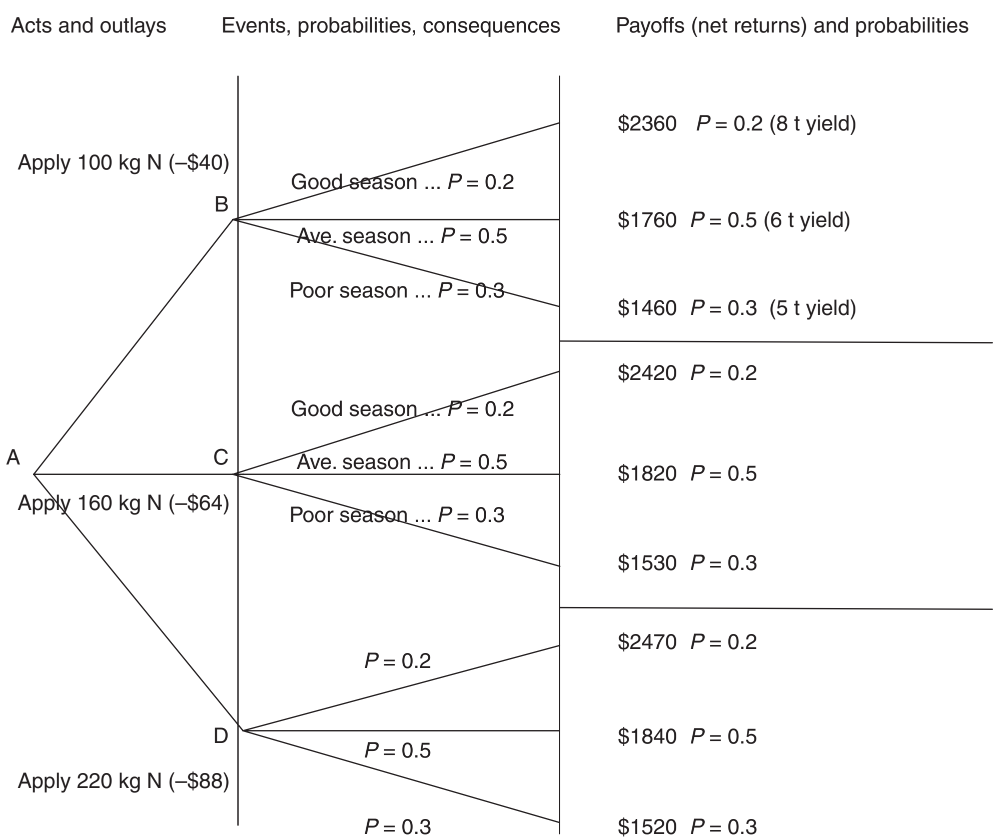


Fig. 3.6. Decision tree for applying nitrogen on a wheat crop.

The example given is quite simple. Firstly, it contains only one decision point or act fork. There would normally be several of these in practical farming systems (Fig. 3.7). Second, it contains only one event fork per act branch. It would not be unusual to have several event forks following an act fork. For example, in the wheat–nitrogen example, the outcomes should have been dependent on yield and price received per kg, as shown in Fig. 3.8.

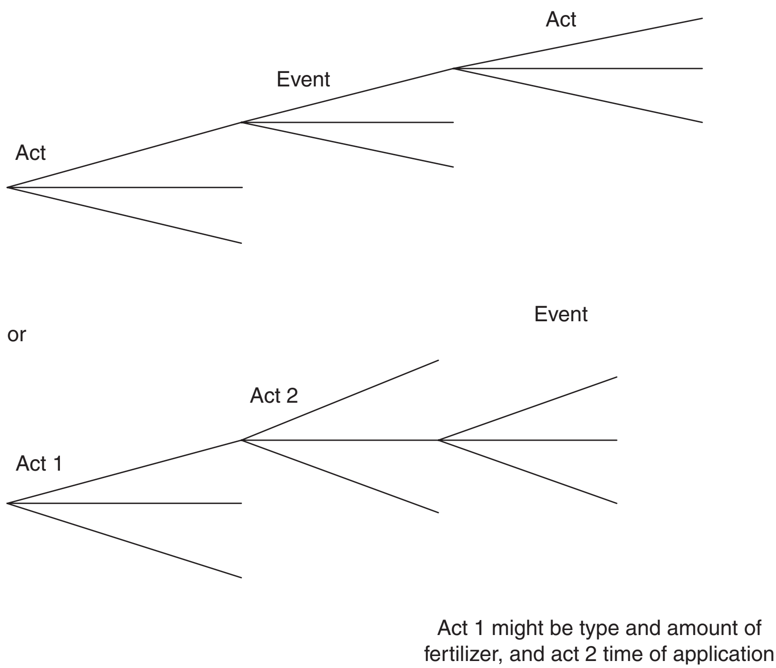


Fig. 3.7. Acts, events and forks in a decision tree.

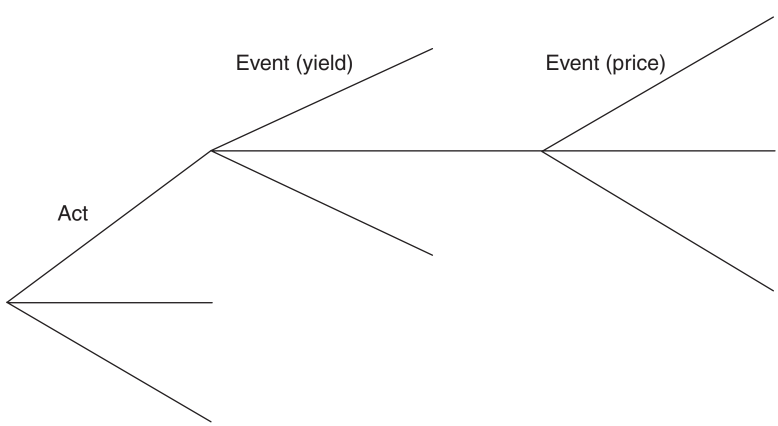


Fig. 3.8. Acts and multiple events in a decision tree.

The decision tree idea can also be used to estimate final outcomes and attached probabilities for a farming system. In this case there will be no act forks. Using a decision tree diagram simply facilitates the identification of end-point outcomes and the calculation of probabilities.

Construct a 'payoff matrix' from the cash flow decision diagram

This is not always necessary, but where the decision tree is complex and therefore requires the use of several pages, it is useful to summarize the end-point outcomes and probabilities in the form of a single table to facilitate decision making. For the example the payoff matrix is:

	Actions		
Events and probabilities	100 kg N	160 kg N	220 kg N
Good season (0.2)	$2360	$2420	$2470
Ave. season (0.5)	$1760	$1820	$1840
Poor season (0.3)	$1460	$1530	$1520

Where there are several act and event forks the payoff matrix might have the form shown in Table 3.1, where each column in the table represents the final outcomes possible from following a series of actions and each row represents the outcomes from the act combinations for a particular set of events occurring. A spreadsheet might well be used to create such tables, including doing the sums.

Table 3.1. A table showing many events and acts.

		Act fork 1							
		Act 1		Act 2		Act 3		Act 4	
Events and probabilities		Act fork 2							
Event fork 1	Event fork 2	Act 1	Act 2	Act 1	Act 2	Act 1	Act 2	Act 1	Act 2
Good season (0.6)	Good price (0.2)	$2360	$2420	etc.					
Good season (0.6)	Ave. price (0.8)								
Ave. season (0.2)	Good price (0.2)								
Ave. season (0.2)	Ave. price (0.8)								

Having quantified alternative courses of action, decision making analyses can now be considered.

Decision making for a range of objectives

THE CERTAINTY EQUIVALENT APPROACH Where the farmer's objective is made up of a non-definable combination of the profit distribution parameters, the *certainty equivalent* approach can be used. The certainty equivalent for an action is defined as:

> the sum of money, available with certainty, that would make the farmer indifferent to a choice between the action (which has a range of outcomes with attached probabilities) and the certain sum of money.

Further, even where the farmer's objective is definable it may be extremely complicated to incorporate, so the only practicable approach in an economic and calculational sense may be to use certainty equivalents.

The certainty equivalent (CE) approach involves asking the farmer to estimate his CE for each alternative course of action (as in the payoff matrix). A choice is then made by selecting the alternative with the greatest CE. For a farmer to be able to estimate accurate CEs he clearly must have reasonable understanding of chance and probabilities.

For example, in the nitrogen/wheat example (see Fig. 3.6) the procedure might be:

Consider action 1 (100 kg N)

Ask the farmer what sum of money would give him the same satisfaction, assuming there is no doubt about the exact quantity that would be received (i.e. received with certainty), as that provided by choosing action 1. The possible outcomes from action 1 were:

\$2360	with a chance of	0.2
\$1760	with a chance of	0.5
\$1460	with a chance of	0.3
		1.00

In many cases a farmer might not fully understand what is involved. The alternative is to arbitrarily suggest a possible sum to be received with certainty and ask whether he would prefer, assuming the choice did in fact exist, the certain sum or the risky action. If he definitely prefers one or the other, increase or decrease the hypothetical certain sum. Continue this procedure until he is not sure whether he would prefer the certain sum or action 1. Once this point is reached, the level of the 'certain sum' can be regarded as the certainty equivalent of action 1 and should reflect the overall value the farmer attaches to this action. For example:

Q: Do you prefer \$2000 with certainty **or** action 1 (in which the outcome is not certain)?

A: Definitely prefer \$2000.

Q: Do you prefer $1600 **or** action 1?
A: Definitely prefer action 1.
Q: Do you prefer $1850 **or** action 1?
A: Both about the same.

As the farmer is indifferent between $1850 and action 1 the certainty equivalent is $1850.

Similarly, the farmer's certainty equivalents of the other actions might be:

Act	1	(100 kg N)	$1850
Act	2	(160 kg N)	$1900
Act	3	(220 kg N)	$1890

Thus, the optimal action is to use 160 kg N (Act 2), but only marginally so.

The farmer may find it easy to estimate his CEs where only a few outcomes are considered for each course of action, but where there are many it may be more difficult. For example, what is your certainty equivalent for the following range of outcomes?

$ payoff per ewe	Probability
–10.0	0.05
20.0	0.12
25.0	0.10
30.0	0.20
40.0	0.04
55.0	0.40
80.0	0.03
90.0	0.06

However, in practice many farmers and advisers probably intuitively use this approach without formalizing the alternatives in a payoff matrix.

The approach has the advantage of being conceptually simple, and where the significance of the decisions over variability is not great, the technique is particularly useful. Further, when the CE is estimated it is likely a farmer takes into account more than monetary considerations, so other satisfaction-producing products will be taken into account.

In some cases, however, it will be easier, given the payoff matrix, to simply ask the farmer which of the alternatives he prefers. The assumption is the farmer will intuitively assess the alternatives on the basis of informal CEs.

CHOICE ON THE BASIS OF EXPECTED PROFIT Where the farmer is indifferent to the shape of the profit distributions, his monetary objective is the maximization of expected profit ($E(P)$)**.** Choice is made by using the payoff matrix to estimate the expected profit for each alternative and selecting the greatest $E(P)$. In the nitrogen/wheat example, the alternatives and the expected return for each from the matrix table are:

(1) 100 kg N; $E(P) = (0.2 \times 2360) + (0.5 \times 1760) + (0.3 \times 1460)$
$= \$1790.0$

(2) 160 kg N; $E(P) = (0.2 \times 2420) + (0.5 \times 1820) + (0.3 \times 1530)$
$= \$1853.0$

(3) 220 kg N; $E(P) = (0.2 \times 2470) + (0.5 \times 1840) + (0.3 \times 1520)$
$= \$1870.0$

Alternative (**3**) has the greatest expected return.

When they carry out budgeting, many consultants and advisers assume farmers want to maximize expected profit. In estimating the budget parameters (death rate, yields, calving percentage, costs, prices etc.) they use expected values and proceed to calculate a single valued expectation. Provided there are no interactions between the variables, and also provided the plan or system evaluated in the budget can be implemented no matter what conditions eventuate, the result is acceptable. That is, provided the plan will be feasible under all situations. If interactions do occur the expected profit estimate will be incorrect and should be estimated using a decision tree in which all branches are evaluated. Strictly this is always likely to be the case, so it is a matter of the degree of distortion.

For example, consider the case where additional winter feed, compared with that assumed, may have to be purchased to maintain feasibility (perhaps stock numbers are greater than expected and/or hay production less than expected). This possible extra expense will not be included in a budget based on single valued expectations. The problem is related to an earlier discussion about the need to consider that expected *states* may not eventuate.

CHOICE ON THE BASIS OF EXPECTED PROFIT MAXIMIZATION SUBJECT TO A RANGE OF POSSIBLE CONSTRAINTS A farmer's objective may primarily be the maximization of expected profit with other special features of the profit distribution playing a lesser role. The two most likely requirements will be: (i) profit must not fall below a certain level under all conditions; or (ii) profit must not fall below a specified level with more than a given probability.

In order to make decisions for these types of objectives a payoff matrix is calculated for all the courses of action, and the alternatives that do not satisfy the special requirements are deleted from further consideration. A decision is then made by selecting the alternative from those remaining with the greatest expected profit.

As an example, consider a problem in which the following alternative farming systems are possible:

System 1		System 2		System 3	
Payoffs	Prob.	Payoffs	Prob.	Payoffs	Prob.
\$2000	0.2	\$2500	0.3	\$1500	0.1
2500	0.2	2800	0.4	2500	0.2
3000	0.5	3200	0.2	3500	0.5
3400	0.1	4000	0.1	4500	0.2
$E(P) = \$2740$		$E(P) = \$2910$		$E(P) = \$3300$	

Case 1: Profit must never fall below $2,000. Eliminate system 3, select system 2.
Case 2: Profit must never fall below $2,100 with a probability greater than 0.15. Eliminate system 1, select system 3.

CONCLUSION A range of possible choice systems have been suggested, although many more exist. Which should be used depends on the farmer's objective and the likely increase in satisfaction resulting from using a correct analytical procedure compared with the cost and time in its implementation. In some cases none of these procedures may be appropriate, so further possibilities are considered in later sections. As most farmers have slightly different objectives it is not possible to cover them all. The section on monetary utility to be covered later, does, however, have considerable theoretical generality. Finally, it must be strongly emphasized that due to the extra work involved in introducing non-certainty into decision making, this should only be done for major decisions, e.g. the choice of stocking rate. In other situations, risk should be considered subjectively.

POST-CONCLUSION – THE COST OF NON-CERTAINTY As the future cannot be predicted, farmers (and advisers) will make incorrect decisions. After an event has occurred this may become evident, but hindsight is of little use except for gaining experience. The correct decisions prior to an event will not necessarily be correct for the conditions that eventuate. That is, with non-certainty what is best, given the estimates of future conditions and their probabilities, often turns out to be less than optimal. On the other hand, if the optimal farming system is repeated over many years, the average profit will be optimal provided the characteristics of the situation do not change.

An estimate can be made of the cost of having to operate in a non-certainty environment. This is calculated by determining the profit that would be made assuming perfect prediction and then obtaining the difference between the *expected perfect prediction profit* and the expected profit for the case where perfect prediction is not assumed. As an example, consider the following case:

(1) Expected profit from optimal system = $6200
without perfect prediction
(2) Profit from optimal system for event set 1 = $5000
Profit from optimal system for event set 2 = $6800
Profit from optimal system for event set 3 = $8500

These outcomes are worked out assuming the optimal system for each event set is used. These optimal systems will probably not be the same as (**1**) – they are optimal for the particular set of conditions or events. They are worked out by taking the variable values in each event set and estimating the best system, assuming you know that these values will occur with certainty.

(3) Probability of event set 1 occurring = 0.25
Probability of event set 2 occurring = 0.45
Probability of event set 3 occurring = 0.30
Cost of non-certainty:

(i) Expected profit assuming can predict conditions:
$(5000 \times 0.25) + (6800 \times 0.45) + (8500 \times 0.3) = 6860$

(ii) Comparison gives cost of non-certainty as \$6860 − 6200 = \$660

In drought-prone areas many farmers have low incomes from trying to maintain stock numbers when 'drought' conditions occur. But if a farmer could predict when a drought will occur he could use optimal systems for the low food production. The cost of non-certainty in these cases is usually very high due to all the extra purchased feed or animal sale costs.

3.10 Monetary Utility Functions – their Estimation and Use

Introduction

The question of how to determine a farmer's objective has so far not been considered in any detail. A number of approaches, all relying on observation and questioning procedures, are possible. An adviser, for example, might simply conclude from general discussion and observation that a particular farmer's objective is the 'maximization of expected returns subject to gifting $\$x$ to his children per year', or something similar. These kinds of observations will always be important. This section, however, is concerned with developing a formal method for estimating a *monetary utility function* for use in decision making under risk.

The method relies on the direct relationship between the shape of a farmer's utility function and the profit distribution parameters which should be used in making decisions. This knowledge helps decide which of the processes discussed in the last section is appropriate for any particular farmer. While in theory the certainty equivalent and monetary utility approaches should lead to the same decisions, a clear understanding of the reasons for considering risk is important and helps resolve any inconsistencies.

Monetary utility functions and risk

The general theory

Given a monetary utility function and equivalent graph, decisions can be made by reading off the utility created by the profit from each alternative farming system. For example, given the following function and two possible farming systems A and B as shown in Fig. 3.9, the choice would be system A as it provides the greatest utility (and profit in this case). The same procedure allows decisions for other outputs such as leisure. This would produce a multidimensional graph provided a function (equation) for each 'product' can be determined.

Note that an analyst might comment that a farming system which produces *greater* monetary returns than alternatives will be preferred. Thus, why not simply base decisions on monetary returns and ignore utility? The fallacy in this approach, however, is that a particular system does not produce a *certain* outcome enabling a simple comparison as suggested. Utility can also embody more than just monetary returns.

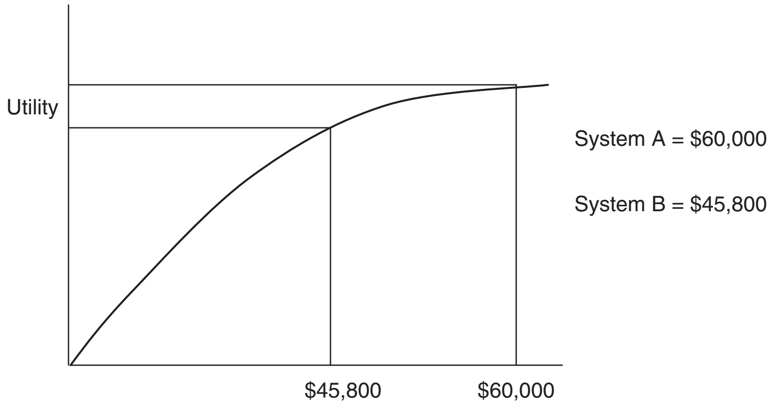


Fig. 3.9. Profit and utility.

Where output is a random variable it is necessary to express output as a single figure to enable comparisons, although this may not always be an acceptable statement for some farmers (this question will be discussed later under *lexicographic utility*). This single figure can be created using a monetary utility function. Given the possible monetary outcomes, together with their probabilities, the *expected utility* of the alternative can be estimated from the graph or function. This provides a *single summary figure* enabling comparisons. The expected utility is estimated by obtaining from the graph or function the utility for each possible monetary gain and by summing the product of the utility levels and the associated probabilities. That is:

$$E(U) = \sum p_i U_i$$

An analyst might also question why it would not be easier to use *expected profit* rather than expected utility. The answer is straightforward. The expected profit parameter does not take into account that the farmer's utility function is usually non-linear. That is, high gains in a profit distribution should not be weighted to the same extent as the lower gains where the utility function exhibits diminishing marginal utility. (The probabilities will also weight the possibilities.) The expected profit criterion weights all gains from the distribution equally so that, for example, $10,000 has one-tenth of the value of $100,000. If the farmer's utility function exhibits diminishing returns this is not the case. Consider Fig. 3.10.

It is clear expected profit does not equal expected utility except under the special condition of a linear utility function. That is:

$$E(U) \neq E(P)$$

for non-linear utility functions, even where the scales for money and utility are identical (i.e. $1 = 1 utile as is the case for $10,000 in the above example).

In the above example, if a particular farming system had two possible outcomes, $10,000 and $100,000 with probabilities 0.4 and 0.6 respectively, the expected profit and utility figures are:

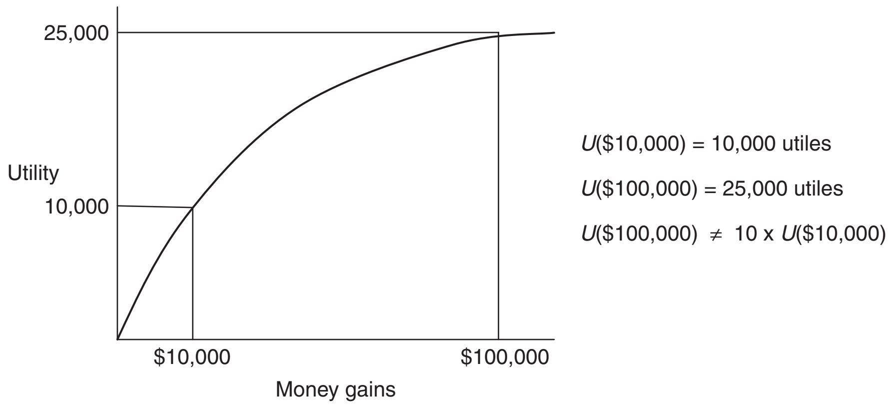


Fig. 3.10. Diminishing marginal utility.

$$E(P) = (0.4 \times 10{,}000) + (0.6 \times 100{,}000) = 64{,}000$$
$$E(U) = (0.4 \times 10{,}000) + (0.6 \times 25{,}000) = 19{,}000$$

There is a slight problem in the above analysis. For the conclusion of $E(U) \neq E(P)$ to hold, monetary gains and utility must be measured on a comparable scale, but in reality this is difficult as utility cannot be measured in absolute terms. The basic conclusion, however, holds despite this complication. Where expected utility is used to select an optimal system it is most likely to be different from the one which maximizes expected profit. As noted, the exception is where the utility function is approximately linear and/or the alternatives have very similar risk levels.

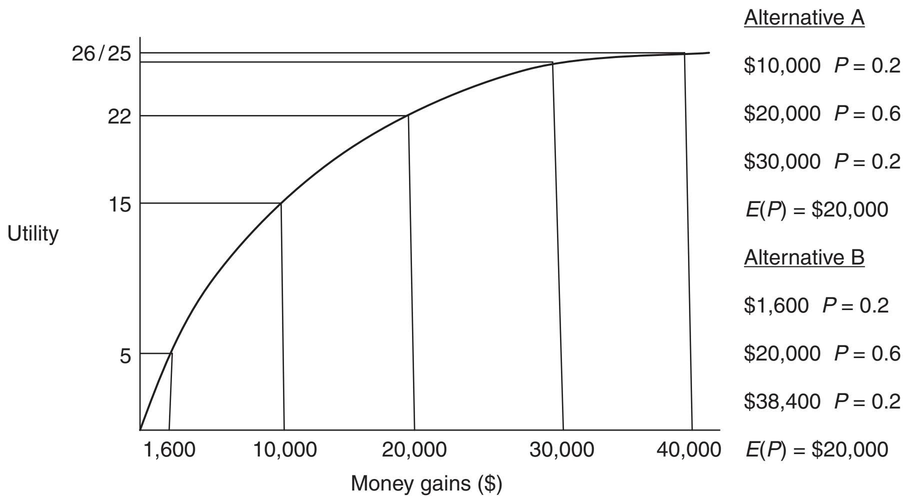
The relationship between risk and utility can now be further explored. A risky alternative farming system with a given expected profit will, where diminishing marginal utility exists, have a lower expected utility than a less risky alternative with the same expected profit. The term 'risky' is used here to describe the variability of possible outcomes.

In the risky alternative there will be some very high and low outcomes. Due to the diminishing marginal utility, the value given to the high profit outcomes will not compensate for the low profit outcomes compared with the less risky alternative. For example, given the function and alternatives in Fig. 3.11, the less risky alternative has greater expected utility.

The idea of a certainty equivalent discussed previously can now be explained logically with the use of this example. The certainty equivalent for alternative A, provided the farmer can estimate it correctly, should be the sure monetary gain equivalent to a utility of 21.2 utiles. Reading from the graph this is approximately $18,000 whereas for alternative B it is approximately $16,000. Thus, certainty equivalents should reflect the farmers' utility functions so that the alternatives will be ordered in terms of preferences.

Two examples will reinforce the arguments presented.

1. Which of the following alternatives would you prefer?
(i) $100,000 gift, tax free, with certainty.



Alternative A $E(U) = (0.2 \times 15) + (0.6 \times 22) + (0.2 \times 25) = 21.2$

Alternative B $E(U) = (0.2 \times 5) + (0.6 \times 22) + (0.2 \times 26) = 19.4$

Note carefully that $E(U(\$20,000)) = 22$ which is *not* 21.2 *or* 19.4

Fig. 3.11. The difference between expected profit and expected utility.

(ii) On the toss of a fair coin, a zero return if heads occurs, *or* a tax free gift of \$201,000 if tails occurs.

Most individuals would prefer alternative (**i**), though if their objective is the maximization of expected returns they should prefer alternative (**ii**) as E(**i**) = \$100,000, whereas E(**ii**) = \$100,500. This example indicates that the riskiness of alternatives is important in decision making (and in the sums involved).

2. Which of the following alternatives would you prefer?
- **(i)** \$105 tax free gift with certainty.
- **(ii)** \$400 tax free gift if, given two tosses of a fair coin, two heads occur, *or* zero return if this condition is not met.

Most decision makers would prefer alternative (**ii**) to (**i**) despite E(**i**) > E(**ii**). (E(**i**) = 105, E(**ii**) = $(0.25 \times 400) + (0.75 \times 0) = 100$). Now consider the same problem but where the payoffs are multiplied by 1000. In this case alternative (**i**) will probably be preferred. This suggests that different levels of monetary gains provide proportionately different satisfaction. If this did not hold, the preferred alternatives would not alter as the sums involved changed.

These concepts of monetary utility and decision making have been approached from a general reasoning point of view. All these ideas can in fact be rigorously proved.

Types of utility functions

In general, three distinctive types of utility functions can be isolated for descriptive purposes. These utility functions relate to an individual's attitude to risky outcomes.

NO PREFERENCE OR AVERSION TO RISK For this case the utility function will be linear as shown in Fig. 3.12.

As high gains have proportionately the same value as low gains, the riskiness of alternatives will be unimportant. That is, choice made on the basis of expected profit will provide the same answer as choice made using expected utility.

RISK AVERSION ATTITUDE This is the most common attitude. Farmers with this attitude exhibit diminishing marginal utility as shown in Fig. 3.13.

The reason for the risk aversion tendencies is that the high gains in a risky alternative do not provide sufficient utility to compensate for the low gains relative to a less risky alternative. The farmer is not prepared to gamble heavily for the slight chance of a high gain, i.e. the utility from the possible high gain is not sufficient to warrant the gamble.

RISK PREFERENCE ATTITUDE This attitude is uncommon. Farmers with a preference for risky alternatives exhibit a function of the form shown in Fig. 3.14.

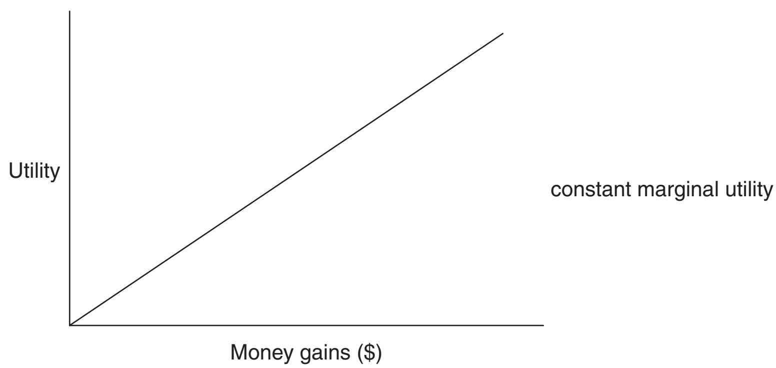


Fig. 3.12. The utility function and risk indifference.

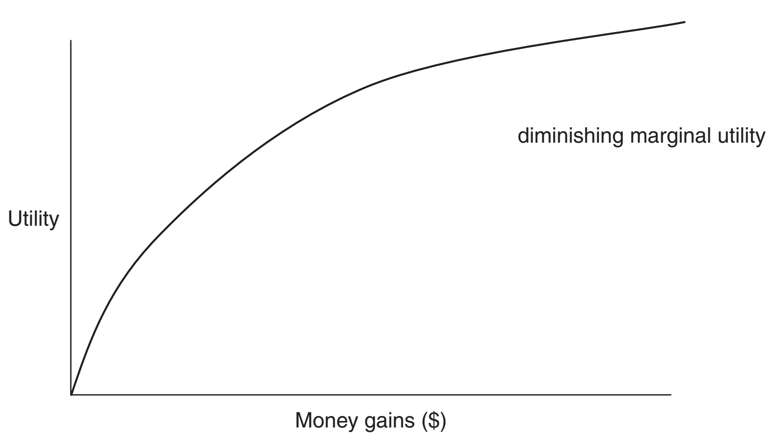


Fig. 3.13. Diminishing marginal utility and risk aversion.

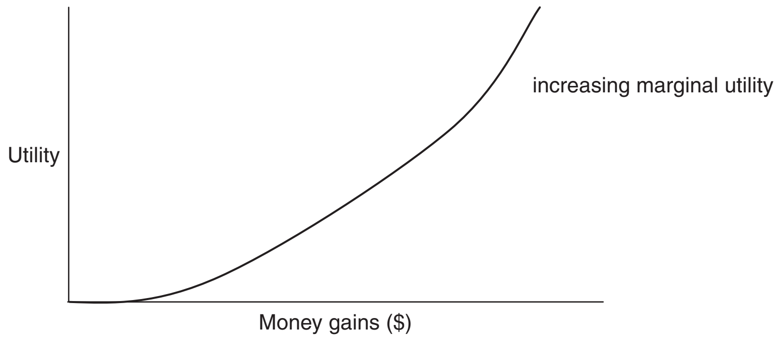


Fig. 3.14. Increasing marginal utility and risk preference.

Farmers prefer risky alternatives as the possible high gains provide much greater utility proportionately than the low gains. Thus if an alternative exists in which there could be an extremely high gain they prefer it to surer, but lower value, alternatives.

Thus, clearly the degree of risk aversion a farmer will exhibit depends on the rate at which marginal utility declines, or increases, as the case may be. (It will be demonstrated later that for functions that are nearly linear the objective of maximum expected profit can be used.) But also note a farmer's monetary utility function may be complex and change from increasing to decreasing marginal utility with increasing income levels. Similarly, the asset base of the farmer may influence the shape of the function. A farmer with large net assets is more likely to have increasing marginal utility, for losses have less impact.

The general utility function

Research has shown that a common utility function has the form shown in Fig. 3.15.

This curve exhibits: (i) increasing marginal *disutility* for money losses (section A); (ii) increasing marginal utility for gains up to a certain point (section B); and (iii) decreasing marginal utility for gains thereafter (section C). This suggests risk preference up to a certain point and risk aversion beyond this point. The changing point is commonly termed the *aspiration level*. Below this, decision makers are prepared to gamble to achieve a certain minimum standard of living. This is to be expected as individuals with a low level of income can make considerable gains in satisfaction given a favourable outcome. The aspiration level for different individuals will vary. For most farmers, however, an analyst will be dealing with the higher section of the curve, as most outcomes for whole farm systems will fall above the aspiration point.

An individual's utility function is unique, as everyone has a different background, different experiences and a different psychological makeup. One particularly important factor is the decision maker's current asset situation. With few assets some farmers are likely to be risk preferers, and similarly if the net assets are very high, for losses are less important.

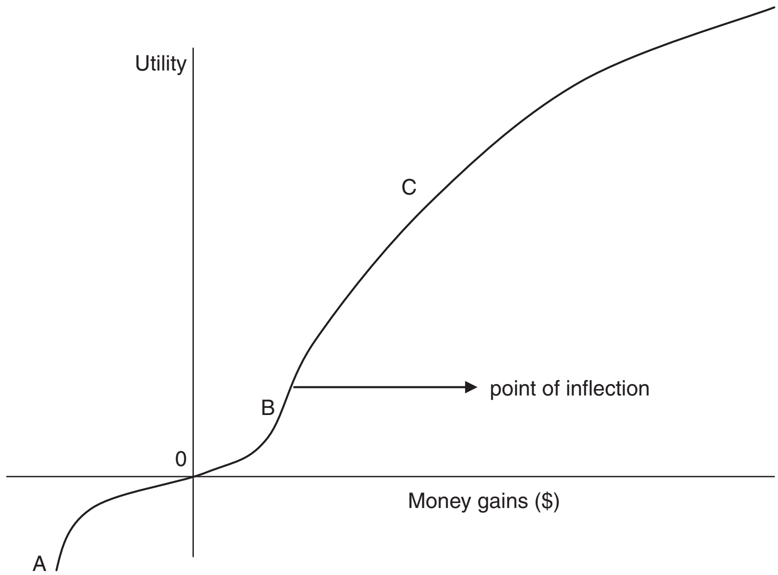


Fig. 3.15. The full range of a typical utility function.

The shape of a utility function is also likely to change with time as asset situations, experience types and levels, and fixed commitments are all likely to change.

Individual's utility functions cannot be directly compared as there is no way of *absolutely* measuring the worth of a product or output. Given the utility functions for two farmers, they might show that the utility of $1000 to farmer A is 50 utiles while for farmer B it is 160 utiles. This does not mean, however, that farmer B derives more satisfaction from $1000 than farmer A. There is no way of measuring this as there is no standard measure for a unit of satisfaction as there is, for example, the length of different cars in feet, metres etc. Thus, a utility function is a *personal concept* enabling alternatives to be ranked in order of preference for that particular individual. This means utility functions cannot be combined to form group functions. This discussion will be extended when the construction of utility curves is considered below.

Decision making where the objective is maximum expected utility

To choose between alternatives a payoff matrix, or tree, must be created, giving the final monetary outcomes and associated probabilities for each action. Using the utility graph, or to give greater accuracy, the equation of the utility function, the expected utility for each action is calculated by converting each monetary outcome into utility values and summing the product of the utility outcomes and their probability. Thus, given two alternatives with the following monetary outcomes and probabilities, the expected utility of each will be:

Alternative A: \$$X_1$, X_2, X_3 … X_n with probabilities p_1, p_2, p_3 … p_n
Alternative B: \$$Y_1$, Y_2, Y_3 … Y_n with probabilities q_1, q_2, q_3 … q_n

Let $u(X_1)$ and $u(Y_1)$ be the utility of monetary outcomes X_1 and Y_1 and similarly for the other possibilities. Then:

$$E(u(A)) = p_1u(X_1) + p_2u(X_2) + \dots + p_nu(X_n)$$
$$E(u(B)) = q_1u(Y_1) + q_2u(Y_2) + \dots + q_nu(Y_n)$$

If $E(u(A)) > E(u(B))$, choose alternative A or vice versa.

In carrying out these calculations, do not fall into the error of assuming that the expected utility of an alternative is equal to the utility of the expected monetary return. That is:

$$E(u(E(\$\text{ return}))) \neq E\text{ (utility)}$$

This will only hold where the utility function is linear.

When using utility for decision making *only whole systems can be compared*. The reason is that where the utility function is curvilinear, total system utility is not equal to the utility of a unit of the farming system multiplied up to give the whole system. For example, on a particular 50 ha block a crop rotation/system might return \$800/ha. Given the utility function shown in Fig. 3.16 the utility of (\$800 × 50) is not equal to the utility of \$800 multiplied by 50 ($U(800) \times 50$).

This point has particular relevance when comparing risky and non-risky alternatives, as when 'per unit' returns are multiplied up the shape of the utility curve will probably ensure that the ranking of the two alternatives will change. For example, consider two alternatives which provide the following returns per hectare:

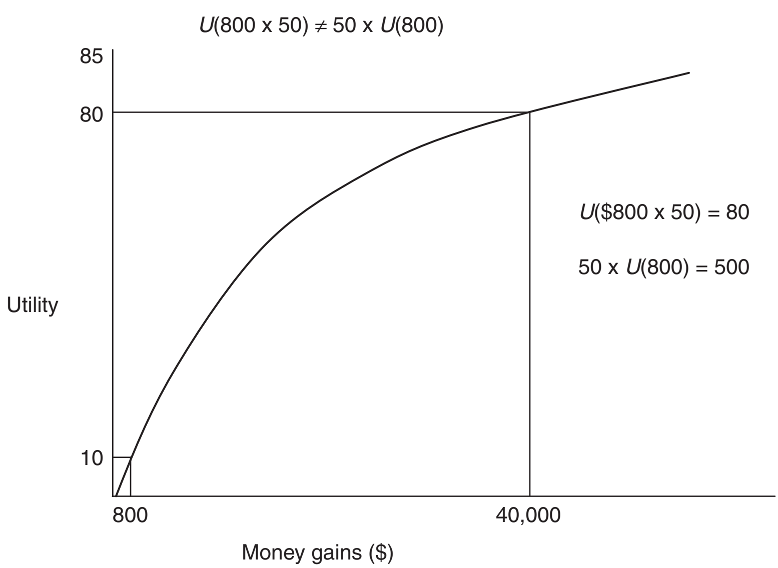


Fig. 3.16. Utility from a whole farm system relative to per hectare utility.

Alternative A: \$750 with $p = 0.4$, \$900 with $p = 0.6$
Alternative B: \$100 with $p = 0.2$, \$900 with $p = 0.5$, \$1233 with $p = 0.3$

Both alternatives have the same E(Profit) = \$840.

(a) On a per hectare basis, expected utilities for the above function are:

A: $E(u) = 0.4(9) + 0.6(11) = 10.2$
B: $E(u) = 0.2(1.45) + 0.5(11) + 0.3(15.2) = 10.35$

(b) On a 50 ha basis, expected utilities are:

A: $E(u) = 0.4(u(37{,}500)) + 0.6(u(45{,}000)) = 82.8$
B: $E(u) = 0.2(u(5{,}000)) = 0.5(u(45{,}000)) + 0.3(u(61{,}650)) = 72.2$

Thus, the optimal choice changes where the whole system is considered, as it should.

3.11 Estimation of Monetary Utility Functions

Introduction

Considerable research has gone into determining whether a farmer's utility function can be estimated successfully, and in determining the best method for estimating the function. The main method relies on the relationship between a farmer's attitude to risky situations and the form of his utility function. The procedure involves:

1. Asking the farmer a series of questions to determine his certainty equivalent for a series of simple 50–50 gambles, and
2. Using his answers to relate his preference for each gamble to an arbitrary utility scale.

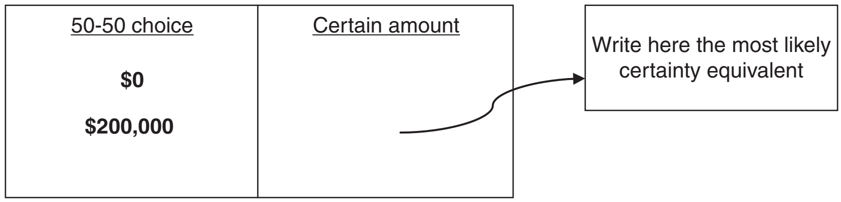
In considering the details, only a utility function for monetary gains will be considered. With slight modifications the system can be extended to cover monetary losses (see Further Reading).

Determining the function

The detailed steps are:

1. Determine the certainty equivalent of a 50–50 gamble involving the lowest and highest money values for which the utility function is to be estimated.

For example, the range of interest might be \$0–200,000. Thus, the farmer would be asked what sum of money, received with certainty, would make him *indifferent* to this certain sum *or* the gamble of \$0 with a probability of 0.5 or \$200,000 with a probability of 0.5. The 50–50 gamble is used as it is the easiest chance situation to understand. A successful method of finding the certainty equivalent is to prepare a card in the following format and ask which alternative is preferred. If the certainty equivalent is preferred, decrease it and re-ask the question and vice versa. This process is repeated until indifference is



indicated. Research has indicated a series of questions will provide a more accurate answer compared with directly asking the farmer to nominate a certainty equivalent.

2. Assign arbitrary utility levels to the highest and lowest gains considered.

For example, assign a zero utility to the money gain of zero (i.e. $u(0) = 0$) and assign a utility figure of, say, 100 utiles to the highest gain (i.e. $u(200{,}000) = 100$). These figures can be any level provided u(lowest gain) < u(highest gain). As these values are arbitrarily assigned, different individual's utilities cannot be compared. It is impossible to overcome this problem as there is no absolute measure of satisfaction. If money losses are also considered it would be logical to set $u(0) = 0$ so that the utility of losses would be negative figures, i.e. disutility.

The utility of the certainty equivalent in the highest–lowest gamble case can now be estimated. This will be equal to the expected utility from the gamble as the farmer is indifferent between the gamble and the certainty equivalent. That is:

$$U(\text{certainty equivalent}) = 0.5(u(\text{lowest gain})) + 0.5(u(\text{highest gain}))$$

For example, assume the certainty equivalent is \$90,000. Therefore:

$$U(\$90{,}000) = 0.5(0) + 0.5(100) = 50 \text{ utiles}$$

Thus, the utility of one monetary value (\$90,000) lying between \$0 and \$200,000 has now been estimated.

3. Proceed to ask a series of similar questions that are designed to evaluate the utility of various money values between the ranges considered.

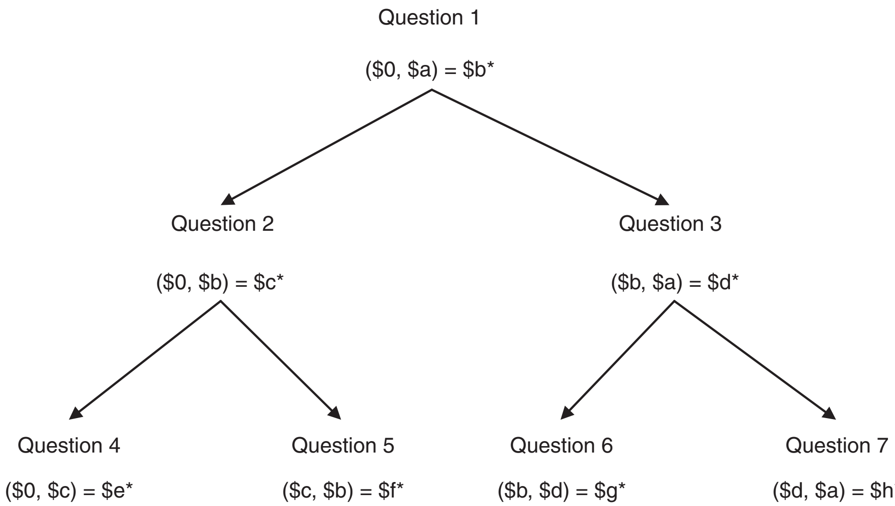
In general, it has been found that it is usually only necessary to ask a total of approximately seven questions. These questions should follow the sequence set out in Fig. 3.17.

Each question involves a 50–50 gamble between two monetary sums for which the utility levels are known, so that the utility of the certainty equivalent provided by the farmer can be estimated using:

$$u(\text{certainty equivalent}) = 0.5\,u(\$x) + 0.5\,u(\$y)$$

where $\$x$ and $\$y$ are monetary gains for which the utility level is known.

Once the utility of a new monetary gain is determined, a new question is formulated using this monetary gain in the gamble together with another value for which the utility level is already known.



Where: $a = highest monetary gain considered; ( ) represents a 50-50 gamble for the two values within the brackets; * the certainty equivalents to be determined.

Fig. 3.17. Schematic of the question procedure for estimating 50–50 gambles and certainty equivalents.

For example, a farmer might provide the following answers:

Q1, the certainty equivalent to (0, 200,000) = $90,000
Q2, the certainty equivalent to (0, 90,000) = $48,000
Q3, the certainty equivalent to (90,000, 200,000) = $125,000
Q4, the certainty equivalent to (0, 48,000) = $19,000
Q5, the certainty equivalent to (48,000, 90,000) = $59,000
Q6, the certainty equivalent to (90,000, 125,000) = $105,000
Q7, the certainty equivalent to (125,000, 200,000) = $147,000

Thus, the utility levels of the various monetary sums are:

$u(90{,}000) = 0.5\ (0) + 0.5\ (100) = 50.0$ utiles
$u(48{,}000) = 0.5\ (0) + 0.5\ (50) = 25.0$ utiles
$u(125{,}000) = 0.5\ (50) + 0.5\ (100) = 75.0$ utiles
$u(19{,}000) = 0.5\ (0) + 0.5\ (25) = 12.5$ utiles
$u(59{,}000) = 0.5\ (25) + 0.5\ (50) = 37.5$ utiles
$u(105{,}000) = 0.5\ (50) + 0.5\ (75) = 62.5$ utiles
$u(147{,}000) = 0.5\ (75) + 0.5\ (100) = 87.5$ utiles

Using this information the utility function can be graphed and used to make decisions, or the observations used to determine an equation of the function.

The main doubt about the correctness of utility functions estimated in this way is whether farmers can answer the hypothetical 50–50 gamble questions to reflect how they would make real-world decisions. If the questions can be formulated using real farm problems the credibility of the answers will be enhanced. For example, the 50–50 gamble could be the possible outcomes from growing a specific crop, or some other realistic gamble appropriate to the monetary levels in the particular gamble question.

3.12 An Appraisal of the Use of Utility Functions

Background

There is no doubt that many farmers actually make decisions using the principles outlined for maximum expected utility. Farmers do not, however, formalize their decisions in this way. While maximizing utility may well create a better farming system than intuitive approaches, the real question is whether using these techniques will give sufficient improvement to warrant the analytical costs. The alternative possibility is not limited to using a single valued parameter system, but to the whole range of semi-correct techniques, some of which have been discussed in earlier sections. These rely on subjective judgment and involve considerably less work. An example is putting a subjective limit on the number of hectares of an extremely variable crop (e.g. potatoes) that can be grown, as this recognizes that the farmer is a risk averter (it may also recognize a restriction is necessary for economic survival). On the other hand, if the analyst is considering optimal systems for a range of situations, a formal analysis may be worthwhile given the many beneficiaries. In such cases a range of utility functions should be used, i.e. carry out a sensitivity analysis.

No definitive statements can be made as each case must be considered on its own merits. Whatever the case, the discussion provides a logical framework to assess subjective constraints and simplified objectives. It is usually more efficient to operate from an understanding of a problem than using a set of rules suggested by somebody else.

The individuality of utility functions poses a problem. Even if there existed a group of similar farms and farmers, general recommendations can not be made. It is only possible to consider a range of typical functions and develop optimal systems for each.

The factors important in whether utility theory will make a difference are:

(i) the shape of the farmer's utility function,
(ii) the shape of the profit distributions from alternative farming systems.

If the farmer has a linear objective function, utility can be ignored as maximizing expected utility will give the same answer as maximizing expected profit. Similarly, where the farmer is only a slight risk averter, or preferer, the same conclusion will provide near optimal decisions. Many farmers will fall into these categories.

Where *alternative* farming systems provide similar profit distributions, and/or these exhibit low variability, the same conclusion can be reached as expected profit will rank similarly to expected utility.

Points (**i**) and (**ii**) above must be considered together, as even if alternative systems have widely differing profit distributions but the farmer's utility function is linear, decisions can be made on the basis of expected profit. Similarly, if a farmer is an extreme risk averter but the feasible alternatives are only slightly variable, making decisions on the basis of expected profit will provide near optimal solutions.

As an example, consider the following situation consisting of three alternatives between which two different farmers must choose:

(a) \$5,000, $p = 0.4$; \$7,500, $p = 0.6$
(b) \$2,500, $p = 0.2$; \$7,500, $p = 0.4$; \$10,000, $p = 0.4$
(c) \$5,000, $p = 0.3$; \$6,250, $p = 0.7$

To assess these alternatives, consider the utility functions shown in Fig. 3.18.

Expected profits:

$E(P_a) = \$6{,}500;\ E(P_b) = \$7{,}500;\ E(P_c) = \$5{,}875$

Expected utility: Farmer 1, a risk averter

$E(u(\text{a})) = 0.4(54) + 0.6(66) = 61.2$
$E(u(\text{b})) = 0.2(37) + 0.4(66) + 0.4(68) = 61.0$
$E(u(\text{c})) = 0.3(54) + 0.7(60) = 58.2$

Thus, farmer 1 should select alternative **(a)** even though this does not provide maximum $E(P)$.

However, if this extreme risk averter had only alternatives **(a)** and **(c)** available, he would select alternative **(a)**, which is the alternative with the greatest expected profit. Thus, the utility function has not assisted in decision making.

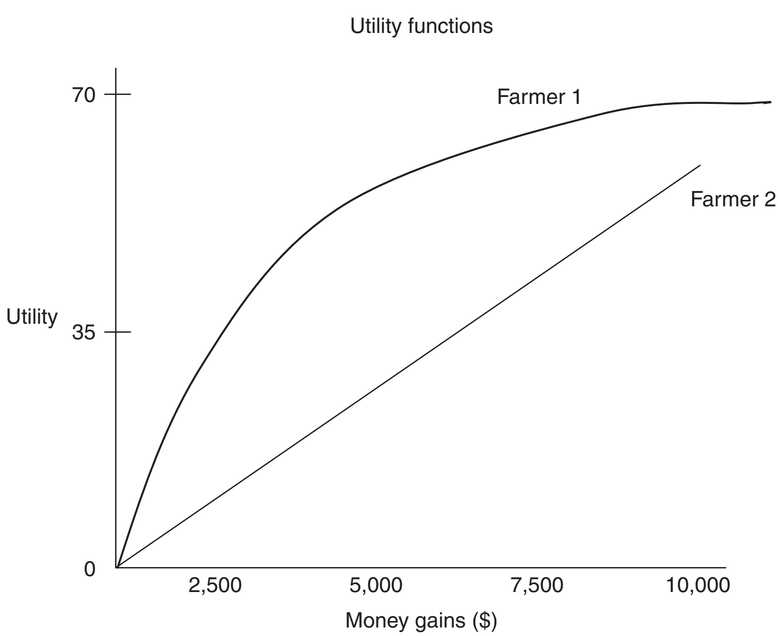


Fig. 3.18. Impact of different risk attitudes on decisions.

The reason is that the shape of the two distributions for alternative **(a)** and **(c)** are similar.

Expected utility: Farmer 2, virtually indifferent to risk

$E(u(\mathrm{a})) = 0.4(22) + 0.6(40) = 32.8$
$E(u(\mathrm{b})) = 0.2(10) + 0.4(44) + 0.4(59) = 41.6$
$E(u(\mathrm{c})) = 0.3(22) + 0.7(30) = 27.6$

Thus, for farmer 2, the alternatives giving maximum expected utility and profit are one and the same. Decision making using a utility function can be ignored in this case.

When using utility, other outputs besides monetary gains must still be included in the analysis in a subjective manner. Given various alternatives, the leisure hours produced, the complexity of management, the preferences for various product types and so on must be assessed.

A practical approach

If a farmer is an extreme risk averter, or preferer, then the use of expected utility may give a different optimal farming system compared with the use of expected profit. However, in most cases it will be time consuming to estimate expected utility. A practical approach is to use some of the techniques suggested in the previous discussions. The most useful is probably the maximization of expected profit subject to various side conditions such as minimum income or low variability requirements. An understanding of the use of expected utility enables the subjective estimate of important side conditions.

For individual farmer advice, careful observation may indicate whether he is an extreme risk averter or preferer. The principles indicate a series of practical questions and careful observations will usually provide the answer. For example, asking the farmer how many hectares of a risky crop he is prepared to grow (e.g. potatoes) will provide clues, as would asking him whether he would prefer to invest surplus funds in Government bonds (sure return) or in, e.g. importing a new breed of stock. Similarly, observing what he has done with surplus funds in the past will indicate his attitude to risky ventures.

3.13 The Passive Approach to Risk Attitudes and Decision Making

The use of utility functions can be called an *active approach*, as the utility function is determined and used to select an optimal system. The function is *actively* used in the process. An alternative is to sidestep the question of quantifying the objective or utility function and to let the farmer choose between possible alternatives. This involves constructing a set of farming systems, each having a different profit distribution, and then presenting these to the farmer with the request that he selects the most preferred alternative. This is a *passive approach* as it is left to the farmer to make the decision.

This approach can be placed in an *efficient frontier* framework. An efficient frontier is the set of alternative systems producing the maximum output for various ratios, or proportions, of the inputs given a fixed set of resources to work with. A familiar example comes from production economics. A two product production possibility curve represents the maximum quantity, in various proportions, of the two products that can be produced with given resources. As output is at a maximum, the graph is an efficient frontier, as shown in Fig. 3.19.

As each point on the curve (and each point below the curve) represents a particular output, it represents a particular farming system or combination of resources.

Returning to the risk case, what is called an *Expected Profit–Variance* (EV) curve can be developed. This has the general form shown in Fig. 3.20.

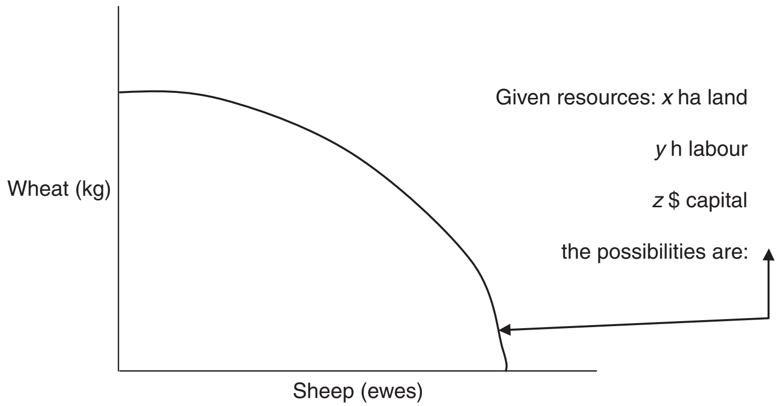


Fig. 3.19. Efficient production possibility curve.

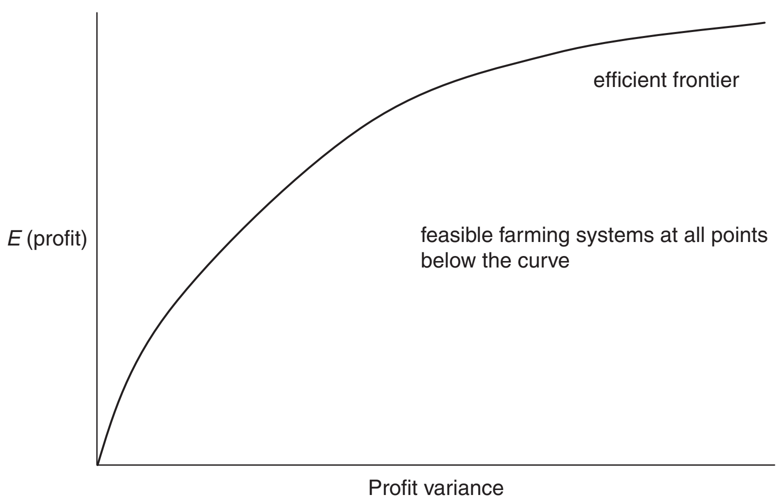


Fig. 3.20. Efficient frontier for combinations of *E*(profit) and profit variance.

The curve represents EV combinations that are *efficient*. Given a farming system with a particular expected income and variance, it is not possible to construct a better system having less variance and the same expected income *or* greater expected income with the same variance. There will, of course, be possible farming systems that have a lower expected income and a higher variance, but these are not efficient. Each point on the curve is representative of a particular, EV efficient, farming system.

The passive approach proceeds by determining a number of EV-efficient systems and then lets the farmer make his own choice.

This approach suggests that only the expected profit and the profit variance distribution parameters are important. While this is often the case, there is nothing to prevent multidimensional efficient frontiers from being constructed where each axis represents a particular distribution parameter. This process, however, can become complex.

Which parameters of the profit distribution are important depends on the shape of a farmer's utility function. It can be shown that if the farmer's utility function can be represented with a quadratic function, then *the only two* distribution parameters of importance are the expected profit and profit variance. As quadratic functions can satisfactorily approximate most farmers utility functions, the EV approach has considerable value.

Which approach should be used? *Active or Passive*? This will depend on whether accurate utility functions can be estimated, or in practical terms whether you can estimate the shape of a farmer's function from general questioning and observation. If not, there is no choice but to use the passive approach. Where reasonable functions can be estimated, the choice must depend on the work involved (in some cases neither approach should be used) and the potential value of the results.

Finally, consider the relationship between the maximization of expected utility and the EV efficient frontier curve. The farming system giving maximum expected utility can be estimated from the EV curve. To do this, curves representing points of equal utility are superimposed on the EV graph and the point of tangency of the highest iso-utility curve and the EV curve is the optimal combination, as shown in Fig. 3.21.

The optimal farming system is the one giving the EV combination at the point of tangency. This relationship, however, only holds if the utility function from which the iso-utility curves were estimated has a quadratic form. To determine points of equal utility the function would be set equal to a constant and solved for various combinations of $E(P)$ and $V(P)$ giving this utility level.

The iso-utility curves given in the above described graph are for a risk averter. A risk preferer would have the curves shown in Fig. 3.22.

The work involved in estimating EV curves for all farmers is, of course, extensive. However, where it is considered that risk should be taken into account it is often only necessary to analyse a few representative case farms. The results can then be subjectively interpolated to individual farms.

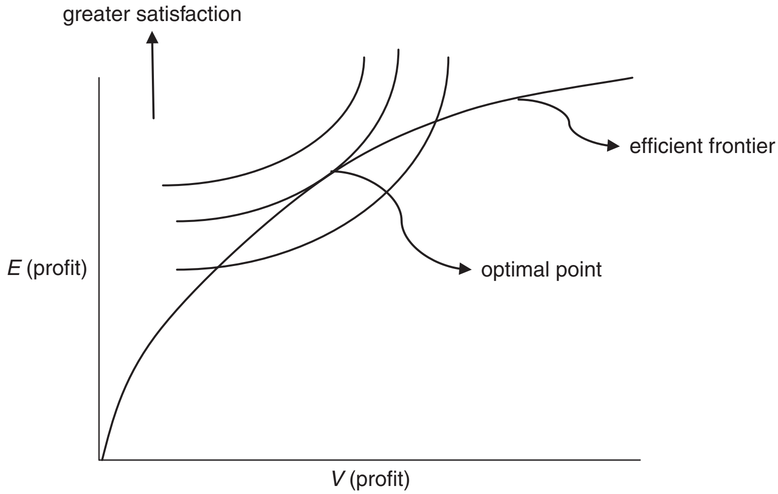


Fig. 3.21. The optimal *E*(profit)/profit variance point (farming system).

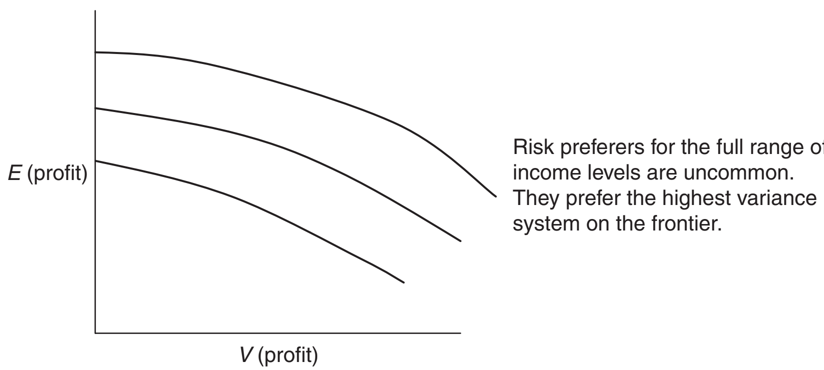


Fig. 3.22. Iso-utility curves for a risk preferer.

3.14 Game Theory – an Approach to Uncertainty

While the earlier discussion has assumed uncertainty can be converted to risk using subjective probabilities, some analysts believe *uncertainty* can be directly allowed for when developing improved systems through a body of theory referred to as *game theory*. The idea is that a decision maker is playing a game against an opposition and both are actively trying to get the better of the other. In farming, the 'opposition' is frequently the weather, or perhaps a distant market. The theory assumes the decision maker has an idea of possible outcomes, but not their probabilities and must choose a strategy which achieves his objectives. As noted, the discussion in the previous sections has assumed a decision maker can make a reasonable estimate of a probability even if objective data is not available. In game theory this assumption is dropped, and as there may be cases in agriculture where there is simply no evidence on which to base probability estimates, it is valuable to consider the basic concepts used in game theory.

The analyst must create a payoff matrix giving the return for each alternative relative to the approach the 'opposition' may take. Choice of action is then based on the action which achieves the objectives, assuming each outcome is equally likely. An example is choosing a strategy to ensure income never drops below some base level.

Imagine a farmer is faced with deciding on the nitrogen fertilizer to apply to a maize crop. The outcome depends on the quantity and speed of rainfall as well as the temperatures. The farmer might conclude the possible profit outcomes are:

	Cash surplus/ha		
Nitrogen level (kg/ha)	Good weather	Average weather	Poor weather
50	645	318	132
100	710	415	160
150	843	540	140
200	980	620	80

If the farmer reckons he must have at least \$150/ha to cover fixed costs it is clear the two higher rates of N fertilizer should not be selected, and nor should the lowest rate. Thus there is only one choice, which in this case is 100 kg N, for if 'poor weather' occurs this is the only option guaranteeing at least \$150. Other selection approaches which might be more appropriate are listed below. In this example the 'opposition' (weather) is benign in that it does not actively try to minimize its loss.

Pure game theory, on the other hand, works on the basis that the 'players' are actively opposing each other. An example might be a grain merchant trying to make deals with a farmer, or with a farmers' cooperative. The grain merchant wants to obtain supplies as cheaply as possible, whereas the farmer/s want to get the highest price possible. Contracts can take many forms, so both the farmers and the grain purchasers have a number of strategies they can follow. For example, the purchaser might want a percentage of the grain delivered before a fixed date and offers a price accordingly. Many other strategies exist. In some cases the optimal system for each may coincide so there is little to argue about. In other cases this will not be the case so a mixed strategy might be best – perhaps the farmer sells half his crop using one policy, and the remainder using another.

If the nitrogen/weather payoff table is imagined as alternative strategies in this marketing example with columns representing the costs to the purchaser, and the rows the returns to the farmer per tonne of grain, the game theory approach would, first, see if any strategies dominated. For example, from the purchasers point of view the last column would be preferred no matter which strategy the farmer selected (choice is between the rows). But in the farmer's case, no one row dominates the others, though the last strategy does dominate for all but the last column. In these cases it is not always possible to find a simple match of what is best. In this example the purchaser would always select the last column strategy, and if the farmer can see this he should select the second strategy with a return of $160. Thus both strategies are clear and provide what is known as the *value of the game* .... in this case, $160. To find a simple solution like this the analyst can, say, circle the column maximums and put a square round row minimums. Where the two match up provides the value of the game, i.e. $160. This common point for both competitors is called the 'saddle point' of the game. Also note in this example the farmer would never choose the first strategy, for the payoffs are less than row two which dominates.

In games without a saddle point an optimal strategy will be to play a mix of the strategies, as noted above. Determining the mix is a little more complicated for many strategy cases and is probably best solved using linear programming, which is the subject of a later chapter. Where an analyst believes he is faced with a competitive game in which probabilities are not known, a text on game theory should be consulted.

Much more relevant to most farm decision making is the case where the farmer is facing a non-active competitor such as the weather, or a major market where one person is unable to influence the outcomes and the 'market' is not actively opposing the farmer. The example of the use of N on a crop is a good example. In these cases a number of choice criteria can be used, any one of which might be appropriate for a particular farmer.

The commonly referred to criteria are called *The Hurwicz*, *The Wald*, *The Savage* and *The Laplace*. Consider the Hurwicz decision criteria. This assumes nature is usually favourable to the decision maker and comes up with the best 'state of nature'. The procedure is to select the best outcome payoff for each row, and then select the strategy which has the highest of these. In the example, the figures in the first column are the maximums for each row. So, under this criteria the farmer would choose the maximum of these which, of course, is to apply 200 kg N. Effectively, the selection is based on the maximum of the maximum so the strategy is called the *maximax*. Clearly this approach is only for the most optimistic of farmers.

The Wald criteria is the complete opposite and involves selecting the maximum of the minimums. Thus the minimum is noted for each row, and the strategy which has the highest minimum is selected. In the example this approach results in selecting 100 kg N. This is the *maximin* criteria, which is suitable for very conservative farmers and ensures the minimum income is kept to a maximum possible.

The Savage criteria, on the other hand, takes quite a different approach and works with 'regret'. Instead of a payoff matrix, Savage suggests a 'regret' matrix is constructed made up of data expressing the regret a farmer might express given a particular strategy's outcome. This matrix might come from subtracting the monetary payoffs from a figure with which the farmer would be very happy. For example, say this was $1000/ha. By taking the N payoff matrix and subtracting each from 1000 the regret matrix is created. Various methods using the minimums of the rows can then be used to decide. Thus regret can be minimized by selecting the *minimax* strategy. Such a farmer is clearly somewhat negative about life, believing it will deal him the worst situation possible.

The Laplace criteria is rather more even handed. This assumes the probability of each state of weather (state of nature) is equal and then proceeds to calculate the expected outcome for each row with the farmer selecting the strategy which has the greatest expected value. Thus, for the N decision each strategy has an expected cash surplus of:

$$E(50\,\text{kg}) = (645 \times 0.333) + (318 \times 0.333) + (132 \times 0.333) = 364.63$$
$$E(100\,\text{kg}) = (710 \times 0.333) + (415 \times 0.333) + (160 \times 0.333) = 427.91$$
$$E(150\,\text{kg}) = (843 \times 0.333) + (540 \times 0.333) + (140 \times 0.333) = 507.16$$
$$E(200\,\text{kg}) = (980 \times 0.333) + (620 \times 0.333) + (80 \times 0.333) = 559.44$$

As might be expected from a casual inspection, the 200 kg strategy gives the highest expected cash surplus payoff. The extra costs of more N are more than compensated by the increased yields except in the 'poor' weather condition.

In this last criterion example, in essence subjective probabilities have been used. It is only that the evidence is insufficient to assume other than equal probabilities. In real life an analyst has to decide whether it is logical to use what information is available to create subjective probabilities, or revert to one of the 'game theory' approaches.

3.15 Concluding Comments

Non-certainty prevails in primary production. An analyst developing optimal, or at least 'improved', production systems must take it into account in some way or other. This chapter has covered methods of quantifying non-certainty, and of choosing between alternatives based on the variability of each system. Whether they should be used in any analysis must depend on the degree of riskiness inherent in the production system, the farmers' predominant attitude to risk, and the risk information available about the alternatives. Also relevant is the degree of system improvement likely to be relative to the costs of analysis, which can be extensive in complex risk analyses.

If a formal analysis is not considered worthwhile then a range of approximations might be instituted. These should be used with a full understanding of risk and uncertainty, for otherwise the possible biases will not be understood. For example, one approach might be to discount yields and prices on the basis of perceived riskiness. Thus, for example, a particularly variable crop might have its average yield downgraded by, say 25%, relative to a crop which is very reliable despite weather variations. These approaches can be called 'single valued', and as such may lead to infeasible and certainly sub-optimal systems. For example, calculating how many animals a particular farm might support using single values may mean in poor conditions there is inadequate feed supplies to maintain the animals, and vice versa. The proposed system just will not work under some conditions. In this example perhaps more feed must be purchased, or animals sold. In either case, the profit outcome will be much less than projected.

In the end it is a judgement call the analyst must make. It should be based on experience and the best information available. A good understanding of the discussion on the cost of uncertainty can be helpful in making this decision.

Further topics using the concepts introduced in this chapter, and extending the discussion, will be introduced from time to time in subsequent chapters. For example, formulae for estimating profit variance for a combination of crops will be defined later.

Overall, it is not possible to divorce non-certainty from analytical thought in farm systems analysis.

Further Reading

Anderson, J., Dillon, J. and Hardaker, B. (1972) *Agricultural Decision Analysis*. Iowa State University Press, Ames, Iowa.

Baumol, W.J. (1965) Neuman – Morgenstern Cardinal Utility. In: *Economic Theory and Operations Analysis*. Englewood Cliffs, New Jersey.

Dillon, J. (1971) An Expository review of Bernoullian decision theory in agriculture: is utility futility. *Review of Marketing and Agricultural Economics* 39(1), 3–80.

Hardaker, B., Huirne, R., Anderson, J. and Lien, G. (2004) *Coping With Risk in Agriculture*, 2nd edn. CAB International, Wallingford, UK.

Roe, T. (1996) Application of game theory in agricultural economics: discussion. *American Journal of Agricultural Economics* 78(3), 761–763.

4 Cost–Benefit Analysis – Recognizing Input–Output Timing

4.1 Introduction

In analysing farming systems it is important to recognize that the timing of input use and the resultant output sales is seldom the same, particularly where a farm is being improved in some way. This affects the economic viability of a project. Furthermore, production often 'uses' time such as in orchard, forestry, or even milk and beef enterprises. It can be shown alternative systems that are identical except for the timing of inputs and outputs have a different value or utility. Thus the timing aspects need to be assessed and valued.

The sections in this chapter will examine how these timing differences should be taken into account. Throughout this discussion non-certainty will largely be ignored. This does not mean it is not important in dynamic problems (time dependent), but it is easier to understand the concepts if non-certainty is initially left out of the discussion. Comments on handling non-certainty will be provided later, so it will then be possible to adjust an understanding of the points raised.

The chapter will proceed by first considering why time, in its own right, has value (in an interest rate sense it clearly has value for depositing money in interest bearing loans is possible), and using these concepts show how time-based calculations can be used (such as compounding interest) to compare investments. Furthermore, a discussion on *investment analysis* relative to *cost–benefit* analysis is included as the methods used in their calculations need to be different; the discussion will clearly define what is meant by each. Essentially, investment analysis refers to projects conducted by individuals, whereas cost–benefit analysis is about state-initiated projects. The criteria used in choosing the best investments will be explained, and methods of their calculation discussed. Finally, conditions which increase the criteria values are explored so that an analyst knows how to design projects to maximize the outcomes.

4.2 The Value of Time

Most individuals prefer to consume today rather than in the future, though there is a limit to this preference. In order to live in the future some present consumption has to be deferred to allow future consumption. For example, you do not borrow to consume today to the extent that all future earnings will be used in paying back the borrowed funds.

If there was not a preference for current consumption investors would not require interest on money lent. They would be indifferent to whether they used the money today, tomorrow or at some other time and would not demand compensation for giving up the right to use the funds. In reality there are sufficient lenders in the market requiring compensation to create a positive interest rate (this rate can be had by the investor with little or no time preference as a bonus). Given this positive interest rate, it is clear most individuals exhibit this positive *time preference* for consumption, i.e. $100 received today will not provide the same satisfaction as $100 received in a year's time. The current 'point in time' is the relevant time for an analysis, as it is the time at which decisions are being made. Similarly, spending $100 on an input today has greater disutility than spending $100 in one year's time. These comments do, however, ignore inflation, which also influences behaviour in that people need to receive a positive interest just to maintain purchasing power. Tax must also be considered. For the moment the discussion assumes an inflation-free and tax-adjusted situation. Allowances for inflation and tax are discussed later in the chapter.

The arguments presented (i.e. that timing in itself is important) also emerge when farmers' investment decisions are examined. When investing in a development project, for example the development of pastures, there is an expectation that the farmer will receive a greater return from this investment than the cost involved. The farmer requires compensation for giving up consumption today. The extent of the additional return necessary before investment will take place is an indication of the farmer's *rate of time preference*. In practice, the situation is probably somewhat more complicated than this as, for example, there are other satisfaction-producing commodities arising from an investment besides the monetary returns. The process of farm development itself may provide satisfaction.

4.3 Optimal Investment Decisions

The nature of time-based investment decisions can be shown graphically and also algebraically (there is one-to-one correspondence between a graph and algebraic explanations/equations). Given a set of resources at current time a farmer can consume all today, or invest all for future consumption, or operate somewhere between these extremes. This gives the opportunity curve shown in Fig. 4.1.

Each point on the curve is representative of a particular farming system. The farmer must decide which of these will provide the greatest satisfaction. The time-preference arguments presented suggest that iso-utility, or indifference, curves can be superimposed on this opportunity curve. Effectively, there are points on the graph representing various combinations of consumption in period t_1 and in period t_2 that provide equal satisfaction (see Fig. 4.2).

These curves are not straight lines with a negative 45° slope, as greater consumption in t_2 than in t_1 is necessary to provide the same utility (given time preference). As income increases the curves have a greater slope, as increased consumption in t_2 is necessary to give the same satisfaction as a given consumption in t_1 compared with lower income situations.

The point giving the greatest utility (there will be a farming system associated with this point) is where the highest utility curve that can be reached is a tangent to the opportunity curve (Fig. 4.3).

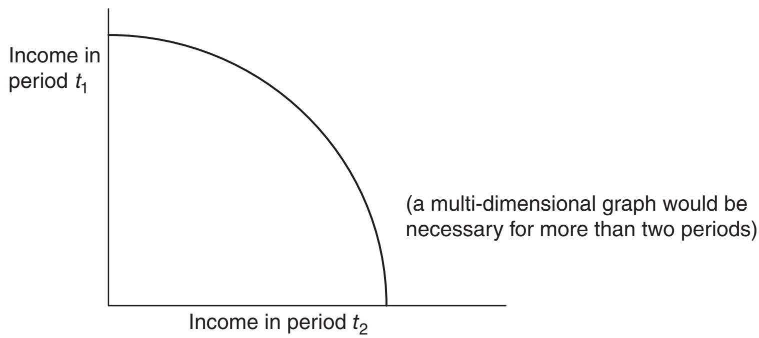


Fig. 4.1. Income possibility graph covering two time periods.

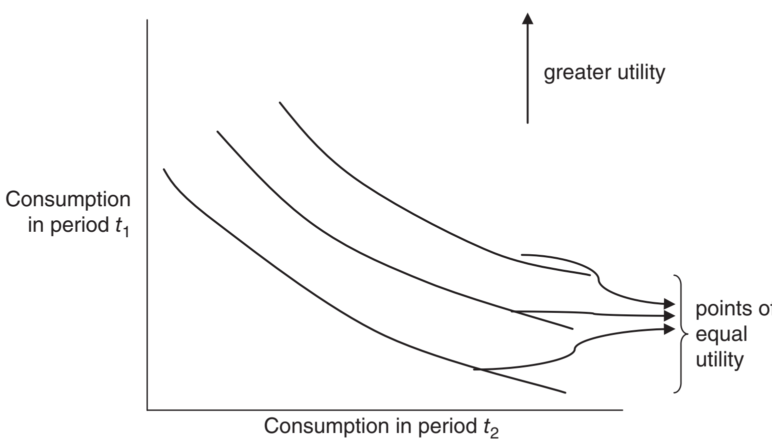


Fig. 4.2. Iso-utility curves for income over different time periods.

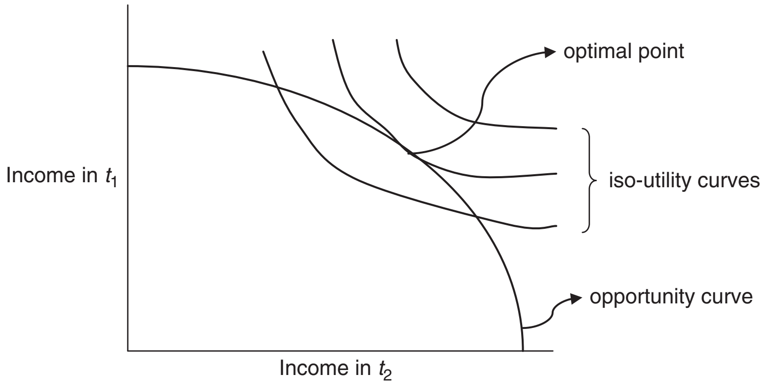


Fig. 4.3. Point of maximum utility for investment between two time periods.

4.4 Discounting

The arguments show that when alternative farming systems have inputs and outputs occurring over a number of time periods they must not simply be valued, or assessed, on the basis of their monetary face value. The value of expenditure and sales in the future must be *discounted* to account for time preference so comparisons can be made directly. The greater the time period between now and the future, the greater must be the *discounting* in order to make comparisons. For example, \$100 received in 1 year's time needs to be discounted by a factor (or adjusted by a factor) to be comparable to sums received now. This factor must be a reflection of the rate of time preference. For a particular farmer this might be, for example, 0.9, so that \$100 received in 1 year's time is equivalent to \$(100 × 0.9) received today. The slope of the iso-utility curves indicate the level of the discount factor. The steeper the iso-utility curve, the lower is the discount factor as less future consumption is necessary to provide the same utility as a given current consumption level compared with a flatter iso-utility curve. This is shown in Fig. 4.4.

The use of discounting means that multidimensional opportunity and iso-utility graphs, as demonstrated, do not have to be used in comparing systems. This would be an impossible task given a large number of time periods. The same result is achieved by estimating the costs and returns for each farming system and discounting them according to their timing. Adding each discounted figure provides a single summary profit figure, which can then be compared with alternative systems.

This discounting approach is demonstrated through the graph shown in Fig. 4.5. Farming system one is represented by point A (q in t_1 and r in t_2) so that using a discount factor d, this system is equivalent to point $q+dr$ on the income t_1 axis. Similarly, system two (given by B) is equivalent to point $s+dt$ on the income t_1 axis. Thus, system one is preferred.

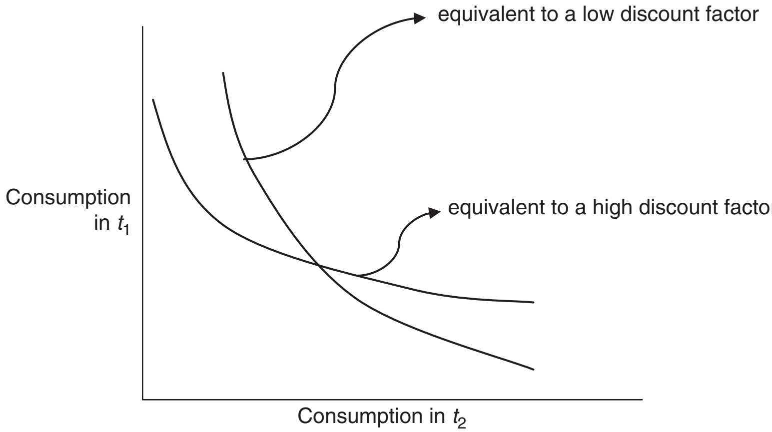


Fig. 4.4. Iso-utility curves reflecting different discount rates (factors).

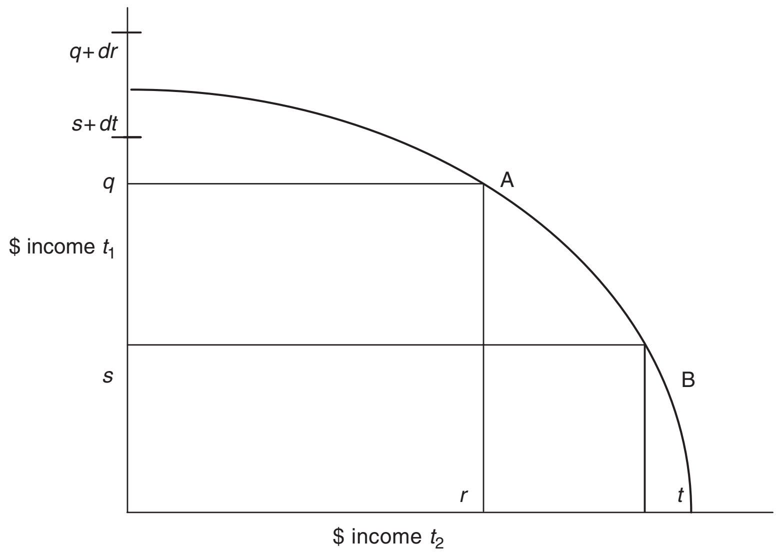


Fig. 4.5. Comparison of multi-output systems using discounting.

The discount factor can also be represented graphically. The iso-utility curve gives points of indifference so that the current equivalent to a future quantity can be read off the graph and the discount factor estimated as shown in Fig. 4.6.

Consumption q in t_2 is equivalent to consumption r in t_1 in terms of utility. Thus, the farmer's (whose iso-utility curve has the shape shown in Fig. 4.6) discount factor will be:

$$r = q \times d$$

i.e. the equation which relates a future sum to the present value. Thus:

$$d = r/q$$

defines the *discount factor d* expressed as a fraction.

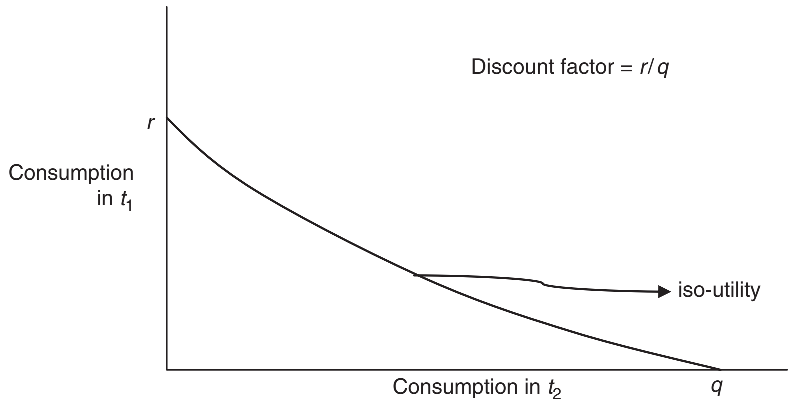


Fig. 4.6. Iso-utility and the discount factor.

The critical question is, of course, deciding on the shape of the iso-utility curve. It should be noted that the curve's shape varies according to the level of utility, thus the discount factor varies according to the level of income. In practical work it is usually assumed that a constant discount factor can be used. A value proportional to the current rate of interest is often used under the assumption that the market reflects the social discount rate as the average rate of time preference.

4.5 Related Arguments for Differentiating Between Input and Output Timing

The fact that when funds are borrowed interest must be paid and when funds are invested in a project there is an opportunity cost, which is at least equal to the interest that could be earned if the funds were invested in a fixed interest deposit, means sums of money received at different times cannot be directly compared. Time has value. The existence of interest charges and returns stems, of course, in part from time preference considerations.

Consider the opportunity cost argument. If a farmer invests \$100 today and receives \$200 in 2 years' time, this investment is not equivalent to one in which he invests \$100 in 1 year's time and receives \$200 in 2 years' time. The \$100 could be invested in the market for 1 year before it is required so that the total return is \$200 plus interest on the \$100 for 1 year. Thus the real cost of the first investment is not \$100 but \$100 plus the interest for a year which was foregone by investing in this particular alternative.

Clearly, the farmer must not treat as equal the \$100 costs as they occur at different times. For a comparison the opportunity cost must be taken into account, so the real cost of the second investment is \$100 less the interest received. That is, \$100 × a discount factor (assuming the farmer would in fact invest the \$100 for the year over which it would otherwise be idle). If the farmer could invest the \$100 at 6% he would only need \$94 (approximately) now to have \$100 at the start of next year, i.e. \$94 today is equivalent to \$100 in a year so that the discount factor is:

$$d = 94/100 = 0.94$$

Similarly, if the equal investment costs for each investment occurred at the same time but the equal returns occurred at different times, they would not be equivalent investments. The return from the investment paying off, say, a year earlier than the alternative could be re-invested for a year and thus give a greater total return. Output timing must be accounted for.

Now consider the cost of borrowing argument. If a farmer must borrow to develop, or invest, this cost must be taken into account when comparing alternative investments. If one system requires an input today compared with another which returns the same cash output but does not require the input until a year has elapsed, this second alternative is clearly the best as the total cost of borrowed funds necessary to purchase the inputs will be less as borrowing is delayed for a year. Thus, input timing must be accounted for. Similarly, where funds are borrowed the alternative giving the sooner pay off will be preferred as the principal borrowed can be paid off sooner giving a lower total interest charge.

Even if a farmer does not exhibit a time preference for consumption, the argument presented in this section indicates that when alternative systems are compared, input and output timing must be accounted for.

Furthermore, besides time preference there is also a supply and demand argument on why positive interest rates exist. Funds for investment are always in limited supply so farmers, and others, wanting to borrow must bid up in the market, giving rise to the current market rate which ensures supply equals demand. The rate is also influenced by government policies requiring banks to hold certain reserves and affecting the supply of cash. So, which ever way you look at the investment problem, the time of inputs and outputs must be allowed for. Furthermore, discounting creates a single summary figure for each investment, enabling a comparison. It is also possible to compound all cash flows to a common future time for comparisons. Either approach provides the same ordering of alternative investments.

The supply–demand argument is referred to as the *liquidity preference* theory. Some people like to hold funds that can be easily liquidated thus meeting cash demands. They will hold bank and short-term investments to allow this, and thus create a supply of funds for others to use. In addition, of course, some people just simply wish to invest surplus funds. The supply of money relative to the demand (for a range of activities such as working cash, building a new milking parlour etc.) then creates a positive interest rate.

4.6 Cost–Benefit and Investment Analyses

Introduction

This need to recognize input and output timing has given rise to *cost–benefit* analysis (some would say ‘benefit–cost analysis’, but this is putting the cart before the horse). Such analyses are concerned with the evaluation of alternative community investment projects, which involve many time periods. The projects analysed range from hydro-electric power schemes to farm development projects. As the name implies, the costs are compared with the benefits, with particular emphasis being placed on the timing of costs and benefits, so that the feasible alternatives can be ranked, or simply set against the going interest rate, which is always available as an alternative. In general, the term 'cost–benefit' analysis is used to refer to public projects (such a hydro-electric scheme, an irrigation scheme for a region etc.), whereas the term *investment* analysis is used when the alternatives available to a *private* individual are being analysed (such as a farmer investing in a private irrigation scheme). The analytical principles used in both are basically the same. The differences lie in the nature of the costs and benefits, which must be considered. While the discussion that follows is largely concerned with investment analysis, initially the types of costs and benefits that must be considered in cost–benefit analysis will be outlined to provide perspective.

Costs in cost–benefit analysis

Costs can be divided into tangible and intangible costs.

- Tangible costs are defined as costs that can be measured in monetary terms. For example, the cost of erecting power lines from a hydro-electric scheme.
- Intangible costs cannot be directly measured in monetary terms. For example, the dis-attraction of overhead power lines to the general landscape.

Tangible costs can be divided into two groups.

- Primary costs. Defined as those directly incurred by the project, e.g. the cost of building a dam.
- Secondary costs. Defined as those indirectly incurred by the project, e.g. when a dam is built labour may be taken from other industries, thus giving rise to a loss of production. Such costs are difficult to assess.

Benefits in cost–benefit analysis

Benefits can similarly be divided into tangible and intangible benefits. An intangible benefit of a dam might be the availability of a water surface for aquatic sports. Tangible benefits can be divided into primary and secondary benefits. An example of a secondary benefit could be the advantages conferred on the fertilizer industry when a large block of land is developed for farming.

Cost–benefit analysis and investment analysis again

The preceding discussion makes the distinction between cost–benefit analysis and investment analysis clear. Cost–benefit analysis must consider *all* costs and benefits, as the projects are being evaluated from the *public point of view*.

That is, intangibles and secondaries are part of the costs and returns affecting the public and must, therefore, be considered.

In investment analysis, however, *only* the primary costs and benefits must be considered, as these are the only factors affecting the *private investor*. Private investors, however, must consider an additional cost. This is *income tax* and any other direct taxes reducing the farmer's profit. Taxation is a cost to an individual but not to the nation, as from the public point of view it is a simple transfer payment.

The difference in costs and benefits, depending on the point of view taken, gives rise to a possible argument for subsidies. A large scale investment project, such as the development of an area for farming by a number of farmers, may be profitable to the nation as a whole but not profitable to the individual farmers. This could mean, dependent on the alternatives available, that the Government should subsidize these farmers so that development becomes profitable to them as individuals. Without the subsidy the farmers would not proceed, but of course the subsidy level must still allow the nation to make a profit. However, subsidies are often given for other reasons, e.g. placating a prominent protest group.

One of the major problems in cost–benefit analysis is valuing intangible costs and returns. Researchers have come up with a range of possibilities. These include the following:

- Travel cost method. To find the value of a benefit (e.g. the recreational fishing value of a river) the beneficiaries can be asked to indicate how far they travel to experience the benefit under the assumption that those travelling the furthest put a value on the resource at least equal to the travel cost. They clearly value the resource to at least this value, and perhaps more.
- Contingent valuation. A sample of users are asked how much they would be prepared to pay to use/experience the benefit. Of course, their answers may not reflect what would happen in reality, especially for people never having experienced the situation being valued.
- Hedonic pricing. This compares the market price of objects, some of which contain the benefit and others not. The difference is assumed to be the value. For example, compare the value of houses on a lake side with those nearby to estimate the value of the view and amenity associated with the lake.
- Defensive expenditure method. Ask a receiver of a benefit just how much he would be prepared to spend to protect the benefit. For example, how much would a house owner near a proposed motorway be prepared to spend on fencing and noise insulation to remove the potential nuisance. Thus the intangible cost of the motorway can be estimated. Similarly, house owners next to existing motorways could be asked how much they have spent on mitigating the noise.
- Value of proxy (similar) goods. Ask a potential user how much they would be prepared to pay for a similar product. For example, how much would they be prepared to pay for a water well when trying to value the benefit of a public irrigation system.

A search of the literature will provide many examples of the use of these techniques, and others, in valuing and costing intangibles. See, for example, Sinden (1994) for a synopsis of the approach.

4.7 Formulae Used in Investment Analysis and Related Work

Introduction

Discounting must be used to compare cash flows at different times. Alternatively costs and returns can be *compounded* to some common future time instead of being discounted. The discussion which follows develops these and related interest rate formulae. Initially, however, general comments are offered about the criteria that might be used in ranking alternative investment projects and a conclusion made on the most appropriate.

Some possible criteria for ranking alternative projects

The desired features of an efficient criterion

In general, the criteria used to choose between alternatives must:

- take into account the differences in timing of input costs and output returns, as has been shown;
- take into account the level, or profitability, of each investment, i.e. take into account the amount of *profit surplus* earned by the project over its entire life span.

By using criteria which satisfy these requirements the farmer can chose alternatives that maximize profits (and other quantifiable objectives).

Possible criteria

Project evaluators have used many different criteria. Three commonly used criteria have been:

- the payback period;
- the average percentage return on capital invested;
- the net present value or worth.

Using the requirements specified it will be shown that *the net present value or worth* is the appropriate decision-making criterion.

To arrive at this conclusion, firstly consider the payback period criterion. This is the number of time periods (years) necessary for the summed net returns to equal the initial capital investment. The best alternative would be chosen on the basis of the shortest payback period. This criterion does not satisfy the requirements as it only partially considers the total profit surplus. Profit made after the payback period has expired is ignored. Similarly, it only partially considers timing, though it favours alternatives with a quick return, as would farmers with a high rate of time preference.

The average percentage return on capital is the average yearly net return expressed as a percentage of the capital invested. The preferred alternative has the greatest percentage return. It is clear that this criterion is also unsatisfactory as, while it accounts for the total profit surplus, it does not take into account any differences in input and output timing.

The *net present value or worth* of a project is the sum of the discounted benefits or returns less the sum of the discounted costs, where each value is discounted by a factor dependent on how many years from the current time it occurs. The table below shows an example of the net present value of an investment.

	Beginning of						
	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6	Discounted sum
Costs ($)	700	2300	800	200	0	0	3775
Return	0	0	1000	2000	2500	0	4600
Discount factor	1	0.95	0.9	0.85	0.8	–	

Net present value = 4600 – 3775 = $825

This example makes it clear that this criterion takes into account both the profit surplus and the timing of the costs and returns and must, therefore, be accepted as the preferred criterion compared with the other two given as examples. In practice there are a number of other criteria that can be used that are based on the same principle of taking into account the timing of inputs and outputs. These will be discussed in later sections. One example is *net future worth*, which is identical to the net present worth except that costs and returns are compounded to a future time so direct comparisons can be made. (This suggests that any point in time can be taken as the comparative point provided all costs and returns occurring prior to this point are compounded and those occurring after the point are discounted.) The essential concept is adjusting all cash flows so they are comparable. They can then be added and subtracted to provide a single net benefit figure, which is easily compared with the alternative projects.

Investment analysis formulae

The future value or worth of a monetary sum

The future value of a sum is given by:

$$S = P(1 + i)^n$$

where S = the future value;

P = the sum invested at time t (where t = current point in time);

i = the rate of interest received on the investment expressed as a fraction per time period (usually 1 year), *or* the opportunity cost of the investment expressed as a fraction per time period, *or* a figure indicative of the rate of time preference expressed as a fraction per time period. The value used will depend on the reason why the calculation is being carried out;

n = the number of time periods (usually years) into the future for which the S value is required.

PROOF

at t	value of sum invested	$= P$
at t+1	value of sum invested	$= P + Pi = P(1+i)$
at t+2	value of sum invested	$= P(1+i) + P(1+i)i$
		$= P + Pi + Pi + Pi^2$
		$= P(1+i)^2$
at t+3	value of sum invested	$= P(1+i)^3$
at t+n	value of sum invested	$= P(1+i)^n$

The term $(1+i)^n$ is called the *compound amount factor* and is equivalent to the discount factor that has previously been discussed for the discounting process.

As an example of compounding, consider the problem of choosing between two investments both involving the same capital investment. Alternative one returns \$10,000 at the end of year three and the other returns \$20,000 at the end of year fifteen. In order to compare the two, compound the \$10,000 to an equivalent sum in year fifteen. Assuming an interest rate of 5%, the future value is:

$$S = 10{,}000\ (1.05)^{12} = \$17{,}959$$

Thus, the second alternative is preferred.

The present value or worth of a monetary sum

The present value is given by:

$$P = S\frac{1}{(1+i)^n}$$

where the terms are as defined above.

PROOF As shown above:

$$S = P(1+i)^n$$

$$\therefore\ P = \frac{S}{(1+i)^n} = S\frac{1}{(1+i)^n}$$

The factor $\frac{1}{(1+i^n)}$ is referred to as the *discount factor*.

For example, using the example investments given above, comparisons could be made by discounting the returns to equivalent current point in time values:

$$\text{Alternative (1)}\ \ P = 10{,}000\frac{1}{(1.05)^3} = \$9{,}070$$

$$\text{Alternative (2) } P = 20{,}000 \frac{1}{(0.05)^{15}} = \$9{,}620$$

The future value of a uniform series of monetary sums

If an investment yields a constant return per period (or requires a constant \$ input) for a defined time, its future value could be estimated by summing the future value of each period's return. This, however, is not necessary as given a uniform series the future value can be determined from:

$$S = a\left[\frac{(1+i)^n - 1}{i}\right]$$

where a = the constant \$ return or cost per period.

For example, the future value at the end of year twenty of a \$5,000 per year stream of incomes is:

$$S = 5000\left[\frac{(1.05)^{20} - 1}{0.05}\right] = \$180{,}330$$

PROOF

$$S = a_1(1 + i)^{n-1} + a_2(1 + i)^{n-2} + \ldots + a_n \tag{1}$$

where $a_1 = a_2 = \ldots = a_n$ and the cash flows occur at the end of each period. Reverse equation (1):

$$\therefore S = a + a(1 + i) + (a + i)^2 + \ldots + a(1 + i)^{n-1} \tag{2}$$

Multiply equation (2) by $(1 + i)$

$$\therefore S(1 + i) = a(1 + i) + a(1 + i)^2 + \ldots\ldots + a(1 + i)^n \tag{3}$$

Subtract equation (2) from (3)

$$\therefore S + S_i - S = a(1 + i)^n - a$$
$$\therefore S_i = a\,[(1 + i)^n - 1]$$

$$S = a\left[\frac{(1+i)^n - 1}{i}\right]$$

The term $\left[\frac{(1+i)^n - 1}{i}\right]$ is referred to as the *uniform series compound amount factor*.

Note that the time at which the cash flow occurred within the period was clearly defined. If the flow occurs at a different point the formula would be different. This comment applies to all the formulae developed.

The present value of a uniform series of monetary sums

Instead of summing the discounted values of each \$ sum the following formula can be used to obtain the present value of a uniform series:

$$P = a\left[\frac{(1+i)^n - 1}{i(1+i)^n}\right]$$

where the term in the brackets is referred to as the uniform series present worth or value factor.

PROOF

$$P = \frac{S}{(1+i)^n} = a\left[\frac{(1+i)^n - 1}{i(1+i)^n}\right]$$

$$\therefore\ P = \frac{a\left[\dfrac{(1+i)^n - 1}{i}\right]}{(1+i)^n}$$

(i.e. the future value of a uniform series is simply discounted back to the current time).

For example, the present worth of $5,000/year at 5% for 20 years is:

$$P = 5{,}000\left[\frac{(1.05)^{20} - 1}{0.05(1.05)^{20}}\right] = \frac{180{,}330}{(1.05)^{20}} = \$68{,}525$$

These uniform series are sometimes referred to as annuities where the time period is a year.

The present value of a uniform series of monetary sums which continue into infinity

If the present value of a uniform series which is expected to continue for a 'large' number of periods has to be calculated, a good approximation (how close the approximate value is to the true value depends on the number of periods involved) is given by the *capitalization* formula. This is:

$$P = \frac{a}{i}$$

This formula is used in *productive valuations*. That is, the expected average yearly return for a 'developed' farm is estimated and, assuming this will continue for many years, the monetary economic worth of the property is estimated from:

$$\text{Capitalized worth or productive value} = \frac{\text{yearly net return}}{i}$$

PROOF It was shown that the present value of a uniform series for a given number of periods is:

$$P = a\left[\frac{(1+i)^n - 1}{i(1+i)^n}\right] = \frac{a}{i}\left[\frac{(1+i)^n}{(1+i)^n} - \frac{1}{(1+i)^n}\right]$$

$$P = \frac{a}{i}\left[1 - \frac{1}{(1+i)^n}\right]$$

As n tends to infinity, $(1 + i)^n$ tends to infinity

$\therefore \quad \frac{1}{(1+i)^n}$ tends to an infinitely small amount.

$\therefore \quad \left[1 - \frac{1}{(1+i)^n}\right]$ tends to 1

$\therefore \quad P$ tends to $\frac{a}{i}$ as n tends to infinity.

The capital recovery or amortization factor

This factor is used to determine the *uniform* series $ amount that must be paid each period in order to extinguish (or amortize) a sum borrowed together with paying interest on the loan. In other words it is used to determine the instalments for an equal instalment table mortgage. This factor is not used directly in investment analysis but is included to complete the discussion.

The equal instalment is given by:

$$R = P\left[\frac{i(1+i)^n}{(1+i)^n - 1}\right]$$

where

R = the instalment;

P = the sum borrowed.

The term in brackets is referred to as the amortization factor.

PROOF It was shown the present worth of a uniform series is given by:

$$P = R\left[\frac{(1+i)^n - 1}{i(1+i)^n}\right]$$

where

R = the uniform series $ amount.

Thus, it is required to determine that R which gives a present worth equal to the sum borrowed.

$$\therefore \quad R = \frac{P}{\left[\frac{(1+i)^n - 1}{i(1+i)^n}\right]} = P\,\frac{1}{\left[\frac{(1+i)^n - 1}{i(1+i)^n}\right]}$$

$$= P\left[\frac{i(1+i)^n}{(1+i)^n - 1}\right]$$

For example, if $20,000 is borrowed for 15 years at 5%, what is the annual instalment necessary to amortize the loan?

$$R = 20{,}000\left[\frac{0.5(1.05)^{15}}{(1.05)^{15} - 1}\right] = 20{,}000 \times 0.0963 = \$1{,}926$$

The sinking fund factor

This factor is used to determine the sum that must be invested each period at compound interest so it will accumulate to provide a given sum at a given time. Again, this factor is not used directly in investment analysis work, but is included because of its general interest. For example, what is the sum a farmer must set aside each year at 6% interest in order to have sufficient funds to replace a tractor in 6 years time?

$$a = S\left[\frac{i}{(1+i)^n - 1}\right]$$

where a = equal annual investment;
S = sum required at year n.

PROOF It was shown that the future value of a uniform series is:

$$S = a\left[\frac{(1+i)^n - 1}{i}\right]$$

Thus, it is required to obtain an expression for a given S:

$$a = \frac{S}{\left[\frac{(1+i)^n - 1}{i}\right]} = S\left[\frac{i}{(1+i)^n - 1}\right]$$

The term in brackets is known as the sinking fund factor.

Tables of factors

In carrying out these calculations considerable time is necessary to evaluate the various factors. In order to minimize the time, various organizations have published tables giving the *factors* referred to for varying interest rate levels and time periods. For example, when carrying out discounting calculations, a table of the following form can be used to obtain the factor by which each value must be multiplied (see section 4.9 for a more complete table).

Table of the Discount Factor $\left[\frac{1}{(1+i)^n}\right]$

	Interest rate			
Years	4%	5%	6%	7%
5	0.8219	0.7835	0.7473	0.7130
6	0.7903	0.7462	0.7050	0.6663
7	0.7599	0.7107	0.6651	0.6227
8	0.7307	0.6768	0.6274	0.5820
9	0.7026	0.6446	0.5919	0.5439
10	0.6756	0.6139	0.5584	0.5083

Thus, for example, the present worth of the following \$ sums are:

(i) \$2500 at 5% received in 8 years = 2500 × 0.6768 = \$1692
(ii) \$8460 at 7% received in 10 years = 8460 × 0.5083 = \$4300

Also note most spreadsheets have functions for calculating many of the factors.

4.8 The Internal Rate of Return

It was noted the criterion *average percentage return on capital* incorrectly ranks alternatives and incorrectly measures profitability as it ignores the importance of input and output timing. However, there is a measure, known as the internal rate of return, which is the correct counterpart of the average percentage return on capital. This measure recognizes timing through the use of discounting techniques.

The internal rate of return is:

> given an investment in which P is invested at time t and the return is S at time $t+n$, the internal rate of return for this investment is *the interest rate that equates the present worth of* S *with* P.

The internal rate of return for this simple single-input, single-output case is given by:

$$r = n\sqrt{\frac{S}{P}} - 1$$

where:

the positive root is the IRR (internal rate of return) i.e. = r
r = IRR

For example, if \$5,000 was invested today and \$10,000 received in 10 years time,

$$r = 10\sqrt{\frac{10{,}000}{5{,}000}} - 1 = \ 0.072 = 7.2\%$$

In effect, this means that if \$5,000 was invested today at 7.2% compound interest it would accrue to \$10,000 in 10 years' time.

PROOF It was shown that the present worth of S is given by:

$$P = \frac{S}{(1 + \ i)^n}$$

Thus, it is necessary to determine the value of i that will give a present worth of S equalling the initial investment. To avoid confusion the term r is used to designate the IRR rather than i, which is used to designate an interest rate.

Solving for r:

$$\frac{S}{P} = (1 + r)^n$$

$$\therefore (1+r) = n\sqrt{\frac{S}{P}}$$

$$\therefore \ r = n\sqrt{\frac{S}{P}} - 1$$

This formula is for the single input-single output case. Where the investment consists of multiple inputs and outputs a different technique must be used as r cannot be isolated in complex equations. This problem will be discussed later.

4.9 Discount Factor Tables

Listed below is an example of a table of factors. In this case it is the discount factors.

Period	Discount Rate (*i*)									
n	4%	5%	6%	7%	8%	12%	16%	20%	25%	30%
1	.9615	.9524	.9434	.9346	.9259	.8929	.8621	.8333	.8000	.7692
2	.9246	.9070	.8900	.8734	.8573	.7972	.7432	.6944	.6400	.5917
3	.8890	.8638	.8396	.8163	.7938	.7118	.6407	.5787	.5120	.4552
4	.8548	.8227	.7921	.7629	.7350	.6335	.5523	.4823	.4096	.3501
5	.8219	.7835	.7473	.7130	.6806	.5674	.4761	.4019	.3277	.2693
6	.7903	.7426	.7050	.6663	.6302	.5066	.4104	.3349	.2621	.2072
7	.7599	.7107	.6651	.6227	.5835	.4523	.3538	.2791	.2097	.1594
8	.7307	.6768	.6274	.5820	.5403	.4039	.3050	.2326	.1678	.1226
9	.7026	.6446	.5919	.5439	.5002	.3606	.2630	.1938	.1342	.0943
10	.6756	.6139	.5584	.5083	.4632	.3220	.2267	.1615	.1074	.0725
11	.6496	.5847	.5268	.4751	.4289	.2875	.1954	.1346	.0859	.0558
12	.6246	.5568	.4970	.4440	.3971	.2567	.1685	.1122	.0687	.0429
13	.6006	.5303	.4688	.4150	.3677	.2292	.1452	.0935	.0550	.0330
14	.5775	.5051	.4423	.3878	.3405	.2046	.1252	.0779	.0440	.0254
15	.5553	.4810	.4173	.3624	.3152	.1827	.1079	.0649	.0352	.0195
16	.5339	.4581	.3936	.3387	.2919	.1631	.0930	.0541	.0281	.0150
17	.5134	.4363	.3714	.3166	.2703	.1456	.0802	.0451	.0225	.0116
18	.4936	.4155	.3503	.2959	.2502	.1300	.0691	.0376	.0180	.0089
19	.4746	.3957	.3305	.2765	.2317	.1161	.0596	.0313	.0144	.0068
20	.4564	.3769	.3118	.2584	.2145	.1037	.0514	.0261	.0115	.0053

Continued

Period	Discount Rate (*i*)									
n	4%	5%	6%	7%	8%	12%	16%	20%	25%	30%
21	.4388	.3589	.2942	.2415	.1987	.0926	.0443	.0217	.0092	.0040
22	.4220	.3418	.2775	.2257	.1839	.0826	.0382	.0181	.0074	.0031
23	.4057	.3256	.2618	.2109	.1703	.0738	.0329	.0151	.0059	.0024
24	.3901	.3101	.2470	.1971	.1577	.0659	.0284	.0126	.0047	.0018
25	.3751	.2953	.2330	.1842	.1460	.0588	.0245	.0105	.0038	.0014
26	.3607	.2812	.2198	.1722	.1352	.0525	.0211	.0087	.0030	.0011
27	.3468	.2678	.2074	.1609	.1252	.0469	.0182	.0073	.0024	.0008
28	.3335	.2551	.1956	.1504	.1159	.0419	.0157	.0061	.0019	.0006
29	.3207	.2429	.1846	.1406	.1073	.0374	.0135	.0051	.0015	.0005
30	.3083	.2314	.1741	.1314	.0994	.0334	.0116	.0042	.0012	.0004
31	.2965	.2204	.1643	.1228	.0920	.0298	.0100	.0035	.0010	.0003
32	.2851	.2099	.1550	.1147	.0852	.0266	.0087	.0029	.0008	.0002
33	.2741	.1999	.1462	.1072	.0789	.0238	.0075	.0024	.0006	.0002
34	.2636	.1904	.1379	.1002	.0730	.0212	.0064	.0020	.0005	.0001
35	.2534	.1813	.1301	.0937	.0676	.0189	.0055	.0017	.0004	.0001
36	.2437	.1727	.1227	.0875	.0626	.1069	.0048	.0014	.0003	.0001
37	.2343	.1644	.1158	.0818	.0580	.0151	.0041	.0012	.0003	.0001
38	.2253	.1566	.1092	.0765	.0537	.0135	.0036	.0010	.0002	.0000
39	.2166	.1491	.1031	.0715	.0497	.0120	.0031	.0008	.0002	.0000
40	.2083	.1420	.0972	.0668	.0460	.0107	.0026	.0007	.0001	.0000
41	.2003	.1353	.0917	.0624	.0426	.0096	.0023	.0006	.0001	.0000
42	.1926	.1288	.0865	.0583	.0395	.0086	.0020	.0005	.0001	.0000
43	.1852	.1227	.0816	.0545	.0365	.0076	.0017	.0004	.0001	.0000
44	.1780	.1169	.0770	.0509	.0338	.0068	.0015	.0003	.0001	.0000
45	.1712	.1113	.0727	.0476	.0313	.0061	.0013	.0003	.0000	.0000
46	.1646	.1060	.0685	.0445	.0290	.0054	.0011	.0002	.0000	.0000
47	.1583	.1009	.0647	.0416	.0269	.0049	.0009	.0002	.0000	.0000
48	.1522	.0961	.0610	.0389	.0249	.0043	.0008	.0002	.0000	.0000
49	.1463	.0916	.0575	.0363	.0230	.0039	.0007	.0001	.0000	.0000
50	.1407	.0872	.0543	.0339	.0213	.0035	.0006	.0001	.0000	.0000

4.10 Investment Analysis Criteria

Introduction

The preceding discussion has made it clear the net present value or worth, the net future value or worth, and/or the internal rate of return are the appropriate measures for evaluating streams of inputs and outputs. It is now necessary to discuss in detail how to calculate these criteria for farming investments.

The characteristics of primary industry investment projects

The methods must take into account that most farming and allied investments exhibit the following characteristics:

- A period over which developmental expenditure is occurring. These costs are usually not constant.
- A period over which gross returns are changing, possibly decreasing initially as, for example, stock numbers are built up, and then increasing until a stable level is reached. This level, on the average, remains at a constant level until further development is carried out. (In some projects, e.g. a hydro-electric dam, no further development will be possible. And of course, when, for example, governments change the rules, or world markets change, the 'stable' income will vary. But these perturbations can not, usually, be forecast).
- A period of time over which operating costs are increasing, but finally they will reach a stable level until further development occurs.

These statements are not strictly correct due to uncertainty and inflation, but where average, real monetary values are used, they represent the general situation.

Gross returns in some cases may be zero for the first few time periods. This would occur, for example, where forested land is being cleared for pasture development or in the case of the construction of an irrigation scheme. Further, gross returns often increase after capital expenditure has ceased as it takes a number of periods for new soil fertility levels to reach a relatively stable point.

In a farming investment situation it is doubtful whether development ever ceases, as most farmers are continually changing their farming systems. This means that gross returns are seldom stable for a period even given that uncertainty is assumed to be non-existent. This does not mean, however, that the effect of a particular development or investment process cannot be isolated for analytical purposes.

The basic implication of these characteristics is that period costs and returns will not be stable until development has ceased, so that individual period costs and returns must be treated separately rather than averaged out.

The criteria

Given these farming investment characteristics, the following sections show how the various criteria can be estimated.

The net present worth

This is the present value of the future stream of returns *less* the present value of the future stream of costs. Thus,

$$\text{NPW} = V - C$$

where

V = present value of future stream of returns;

C = present value of future stream of costs.

Note that in estimating $V-C$, capital and operating expenses are not treated separately. This is logical, as the farmer is concerned with the cash surplus and a cash outflow is a cost whether it provides an asset for a single, or several, time periods. This means that, *in general*, the analysis must consider a period which will encompass the life of the assets purchased. This kind of analysis is sometimes referred to as *cash flow analysis* as cash flows are considered rather than the *equivalent annual costs* of assets lasting several time periods.

As an example, consider a development project in which costs and returns fluctuate up to, and including, year 10, and thereafter are stable. *For this typical situation,* $V - C$ can be calculated in two parts:

1. The net present worth of the project up to and including year 10 (assume costs and returns accrue at the end of each period).

Let:

b_j = the gross return in year j

c_j = the cost in year j

Then:

$$V_o = \frac{b_1}{(1+i)^1} + \frac{b_2}{(1+i)^2} + \ldots + \frac{b_{10}}{(1+i)^{10}}$$

$$C_o = \frac{c_1}{(1+i)^1} + \frac{c_2}{(1+i)^2} + \ldots + \frac{c_{10}}{(1+i)^{10}}$$

where V_o and C_o represent year 0 present values.

Thus, NPW = $V_o - C_o$

2. The net present worth, at the beginning of year 11, of the project from year 11 until infinity (or disposal of the property).

$$V_{11} - C_{11} = \frac{b_{11} - c_{11}}{i} = \frac{b_{11}}{i} - \frac{c_{11}}{i}$$

Thus, the net present worth or value of the project as a whole is:

$$V - C = (V_o - C_o) + \frac{V_{11} - C_{11}}{(1+i)^{10}}$$

It will be noted that the *capitalization* formula has been used in order to simplify the calculations. Whether this simplification can be used depends on the number of future periods that must be included in the analysis.

In general terms, the net present value is given by:

$$V - C = \sum_{j=1}^{n} (b_j - c_j)\left(\frac{1}{(1+i)^j}\right)$$

Note that it is immaterial whether the period *net* returns are discounted or the costs and returns are discounted separately and finally summed and subtracted to give $V - C$.

The logical use of this criterion is in choosing between alternative farming systems. Given the funds at the farmer's disposal, the relevant costs and returns would be estimated for the feasible alternatives and a decision made on the basis of the greatest $V - C$. In many cases, however, the criterion is used somewhat differently in that it is used to determine whether a particular project is *worthwhile*.

A project is said to be worthwhile if the net present value is zero or greater. In this case the project will return at least the rate of interest used in estimating the discount factor. However, this use of the criterion is irrational in that the farmer's objective is to use the limited resources as efficiently as possible. All alternatives should be evaluated. The 'worthwhile procedure' may ignore alternatives that provide a greater net present value. In some cases, however, the problem may be one of deciding whether to develop a farm in the only feasible way available.

Net future worth

The calculation of *future net value or worth* follows the same procedures in general except that compounding rather than discounting is used.

The investment ratio (commonly known as the cost–benefit ratio)

The investment ratio is:

$$\text{Investment ratio} = \frac{V}{C}$$

where V and C are the present values of the return and cost streams as defined in 'Net present worth' above.

$$\frac{V}{C} = \frac{V_o + \left(\dfrac{b11}{i} \times \dfrac{1}{(1+i)^{10}}\right)}{C_o + \left(\dfrac{c11}{i} \times \dfrac{1}{(1+i)^{10}}\right)}$$

In some papers this ratio is defined as:

$$\frac{V}{C} = \frac{\Sigma(b_j - c_j)\left(\dfrac{1}{(1+i)^2}\right)}{C}$$

where C is the present worth of all capital costs and c_j includes only operating costs.

However, it is clear that:

$$\frac{V}{C} \neq \frac{V-C}{C}$$

The conclusion must be that capital costs should not be separated out, as to the farmer a cost is a cash outflow whether or not they are defined as capital or operating costs.

The criterion can be used to choose between projects by selecting the project with the greatest ratio. In terms of '*worthwhileness*', a project is said to be worthwhile, or profitable, if the ratio is greater or equal to unity, and vice versa. That is:

If $\frac{V}{C} \geq 1$, project worthwhile

If $\frac{V}{C} < 1$, project not worthwhile.

The internal rate of return

DEFINITION In the single input-single output case the IRR was defined as the rate of interest or return which equates the present worth of the return with the cost. In the case of multiple costs and returns, the internal rate or return is defined as *the rate of interest or return which makes the present worth of the cost stream equal to the present worth of the return stream*. That is, the rate of return giving:

$$V - C = 0$$
$$V = C$$

For the example presented above in which the project reaches stability at the end of year 10, the IRR is that rate of return (r) giving:

$$\frac{b_1}{(1+r)^1} + \frac{b_2}{(1+r)^2} + \ldots + \frac{b_{10}}{(1+r)^{10}} + \left(\frac{b_{11}}{r} \times \frac{1}{(1+r)^{10}}\right)$$

$$= \frac{c_1}{(1+r)^1} + \ldots\ldots\ldots\ldots + \frac{c_{10}}{(1+r)^{10}} + \left(\frac{c_{11}}{r} \times \frac{1}{(1+r)^{10}}\right)$$

Or, in general, the r giving:

$$\sum_{j=1}^{n} b_j \frac{1}{(1+r)^j} = \sum_{j=1}^{n} c_j \frac{1}{(1+r)^j}$$

CALCULATION OF THE INTERNAL RATE OF RETURN The rate of return, r, cannot be isolated from the equations (due to their complex nature) so a trial and error method must be used. However, there may be several positive values of r that will satisfy the equation $V = C$ so that a decision must be made as to which is the correct value. In most farming development projects, however, only a single r value eventuates. Multiple answers are theoretically possible due to the powered terms being greater than 2 in most cases (to the power of 'j').

Whether multiple solutions can occur depends on the nature of the equation equating the present value of returns to the present value of the costs. This can be demonstrated by graphing the net present value of possible projects against the interest rate, or as it is called, the discount rate (not to be confused with the discount factor). For example:

1. If the $V - C$ = NPV equation gives a graph of the following form, there is a unique r value as shown in Fig. 4.7.
2. If the $V - C$ = NPV equation gives a graph of the form shown in Fig. 4.8, there will be several positive r (i) values.

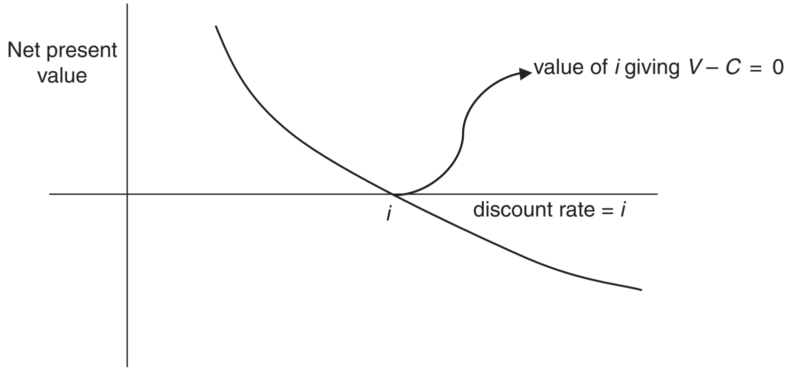


Fig. 4.7. Estimating the IRR graphically.

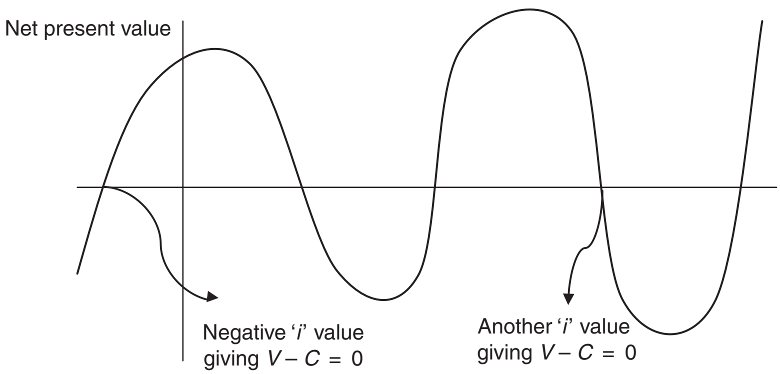


Fig. 4.8. Potential multiple values for the IRR (internal rate of return).

General observations on the number of solutions obtained provide the following rules of thumb:

1. If all the $(b_j - c_j)$ values are positive, there will be only one real solution and it will be positive.
2. If the first few $(b_j - c_j)$ values are negative and the remainder are positive, and
 (i) the Σ–ve $(b_j - c_j) > \Sigma$+ve $(b_j - c_j)$, then there will be one solution and it will be negative (the sign 'Σ' means 'the sum of');
 (ii) the Σ–ve $(b_j - c_j) \neq \Sigma$+ve $(b_j - c_j)$, then there will be a single positive solution.
3. If there are one or more changes in sign of the $(b_j - c_j)$ values through time, there can be more than one real answer and the number of solutions will depend on the number of sign changes and the absolute values of the negative and positive $(b_j - c_j)$ values. Fortunately, in most farming development projects, once $(b_j - c_j)$ becomes positive it seldom reverts to a negative value at a later time, though where development is carried out in a number of discrete stages this may occur.

Returning to the actual calculation of the IRR, the logical approach is to use an interpolation procedure. This involves calculating the net present value of the project using a high discount rate such that this net present value will be negative, and then repeating this procedure with a low discount rate to give a positive net present value. Using the degree of *positiveness* and *negativeness* of the net present values, the discount rate expected to give a zero net present value can be estimated. This trial and error process is continued until a rate is determined that gives a net present value equal to approximately zero.

This process is clearly time consuming. However, due to the logical nature of the calculations, the process is readily adaptable for solving on a computer and most computers have programs available to do this. Similarly, computer programs are available for calculating most of the other criteria.

As an example of calculating the IRR, consider the following investment cash flow:

Year	1	2	3	4
b_j (\$)	2500	2800	3000	3000
c_j (\$)	3300	2700	2400	2400

(a) at 15%:

$$\Sigma b_j \frac{1}{(1+i)^j} = 7978$$

$$\Sigma c_j \frac{1}{(1+i)^j} = 8035$$

thus:

$$V - C = -57$$

(b) at 10%:

$$\Sigma b_j \frac{1}{(1+i)^j} = 8890$$

$$\Sigma c_j \frac{1}{(1+i)^j} = 8674$$

thus:

$$V - C = 216$$

(c) Interpolating, r = 14% (approximately)

As the graph of the net present worth is curvilinear, where the negative and positive NPW estimates are well apart the interpolation will be very approximate. Thus, selecting interest rates which give nearly zero values will improve accuracy.

The annuity equivalent of the net present value

The net present value criterion requires choosing between alternatives on the basis of comparing lump monetary sums. While representing a particular farming system as a lump sum may seem reasonable, it is sometimes difficult for a farmer to comprehend that this monetary sum represents a course of action which provides a continuing stream of net returns. For this reason it may be useful to convert the net present value into an equivalent annuity. That is, find the equal sum received each period which has a present value equal to the net present value of the project. This annuity is given by the amortization factor.

$$\text{Annuity equivalent} = \text{NPV}\left(\frac{i(1+i)^n}{(1+i)^n - 1}\right)$$

4.11 Choice of Criterion

Introduction

Three basic criteria have been introduced:

$$\text{NPV}, \frac{V}{C} \text{ and the IRR.}$$

Each takes into account the effect of input and output timing on profitability. The question that remains is deciding which should be used in investment analysis. This question arises as it is possible for the criteria to rank alternative projects into different orders so that they cannot all be the correct measure to use. For example, consider two projects giving the following V and C values and therefore the criteria values shown:

	$V - C$	V/C
Alternative 1		
V = 10,000; C = 8,000	2,000	1.25
Alternative 2		
V = 19,000; C = 16,000	3,000	1.19

Selecting on the basis of the NPV, alternative 2 is preferred, but on the basis of $\frac{V}{C}$ alternative 1 is preferred. As will be shown later, using the IRR may give similar ranking inconsistencies.

The simple example above probably suggests which criteria should be used.

The requirements of a good criterion

In order to judge between the criteria a set of requirements is necessary. These requirements must distinguish the criteria on the basis of what a farmer is trying to achieve in a development programme. Leaving aside the implications of uncertainty, and objectives other than monetary ones, a farmer requires a criterion which, firstly, distinguishes alternative systems on the absolute level of profit surplus made. A farming system that provides, e.g. \$20,000 cash surplus is clearly preferred to a system returning \$15,000. Second, a criterion that distinguishes on the basis of the return in relation to the level of the investment is important. Even though a particular development programme provides a greater profit surplus than another alternative, this will not necessarily be preferred if this programme requires a greater level of capital investment. Summarizing for the general case, a useful criterion must indicate:

- the absolute value or worth of an investment; and
- the rate of return in relation to the size of the investment involved.

A comparison of the criteria

Net present value

The net present value measures the absolute worth of a particular development plan or system and, therefore, provides an indication of the profit surplus earned. It does not, however, directly indicate the rate of return achieved by the particular project (except in one special case). For example, given two alternative ways of developing a property they might produce a net present value of, e.g. \$35,480 and \$85,680. These figures do not indicate which alternative gives the greatest return per \$ invested unless a special condition holds. This is where both alternatives involve the *same* total investment. Another special case where the NPV will differentiate between alternatives successfully is where there are two or more alternatives with $V-C = 0$. In this case each alternative will have the same rate of return so that a farmer will be indifferent between them. A farmer, however, will not necessarily be indifferent between two alternative systems giving the same positive NPV.

The investment or cost–benefit ratio

The value of the ratio by itself indicates the rate of return achieved in that the value of the returns is expressed in relation to the value of costs involved in the development plan. The ratio by itself does not, however, indicate the absolute value of the profit surplus earned. Used by itself, this criterion is also unsatisfactory.

The internal rate of return

The IRR is clearly an indication of the rate of return achieved by the project but it does not, when considered by itself, indicate the absolute level of the profit surplus made. (The IRR also has other drawbacks as will be shown below.) Thus, this criterion taken by itself must also be classed as unacceptable.

4.12 Conclusion on Criteria

None of the criteria satisfy both requirements except in special circumstances. However, when the criteria are being used solely to decide whether a project is worthwhile or not, all of the criteria will achieve this requirement. This occurs as

$$V - C \text{ and } \frac{V}{C}$$

will indicate whether the project returns at least the rate of interest used as the discount rate in estimating

$$V - C \text{ and } \frac{V}{C},$$

and the IRR indicates the rate of return achieved, thus enabling a comparison with the opportunity cost of capital.

In other words, if

$$V - C \geq 0 \text{ and } \frac{V}{C} \geq 1,$$

the rate of return achieved will be greater than the discount rate used. Some qualifications, however, must be made for the IRR. These are presented in the next section.

In the general case, when alternative projects are being compared, two criteria must be used in conjunction to give the rate of return and the level of the absolute profit surplus. Thus

$$V - C \text{ inconjunction with } \frac{V}{C}, \text{or the IRR,}$$

will enable a general comparison. However, as will be noted below, in certain cases the IRR cannot be regarded as an acceptable measure, so that in general

$$V - C \text{and } \frac{V}{C} \text{ are preferred.}$$

Given, that

$$V - C \text{ or } \frac{V}{C} \text{ has been calculated,}$$

it is obvious what the level of the other criterion will be as the values of V and C are known.

The conclusion only holds, however, for the general case where the choice is between a list of alternatives which does not include *all* possibilities. Given a particular farm, the rate of return achieved is of no immediate interest. A farmer has a given level of funds at his disposal. His problem is to invest these in development programmes on the farm, or in off-farm investments, to *maximize* his return. Given that a farmer examines *all* alternative uses of his investment capital, the alternative with the greatest $V - C$ will clearly be preferred. The rate of return is not important as these alternatives all involve *the same input of funds*, although they may, of course, involve different levels of borrowed funds. In order to use all his funds, the farmer may need to invest in a combination of what can be classed as different investment projects. In effect, such a combination becomes another alternative course of action. For example, a farmer may have \$50,000 to invest and four different ways of using these funds:

1. Develop farm using system 1 – own capital requirement \$50,000;
2. Develop farm using system 2 – own capital requirement \$40,000;
3. Invest off-farm – capital requirement at any level desired;
4. Consume the funds.

Thus, the alternative uses of the \$50,000 are:

1. Develop farm using system 1;
2. Develop farm using system 2 *and* invest \$10,000 off-farm;
3. Invest \$50,000 off-farm;
4. Consume the funds.

The preferred alternative will give the greatest net present value. In reality, there would be many more possible alternatives and degrees of how much development is undertaken using a particular developmental system.

In many cases, however, a farmer does not have a fixed sum of capital available for investment as has been suggested above. However, there will always exist a number of alternative courses of action based on surplus income and borrowed funds. The preferred alternative should still give the greatest net present value, assuming the aim is to maximize profit surplus.

In the selection process the rate of return per dollar on expenditure can be confusing. An alternative with the greatest investment ratio may provide a lower profit surplus as indicated by the net present value. The example given in the introduction demonstrates this conclusion. It must be recalled, however, that where the expenditure involved in all alternatives is equal, the investment ratio will provide the same conclusion as the net present value.

Where the best alternative does not have a net present value greater than or equal to zero, the farmer should not invest in the project as the return is not great enough to compensate for his rate of time preference.

4.13 The Incorrectness of the Internal Rate of Return

In general the IRR cannot be accepted as a correct measure of profitability, though in some cases it will give the same conclusion as the other criteria. Use of a criterion, however, should be on the grounds that it is logically acceptable rather than it does sometimes give the same answer as other criteria.

The reason for its unacceptability has partially been covered above. That is, the farmer's primary aim is to maximize the profit surplus from farming and allied operations rather than maximize the rate of return. Secondly, even when a rate of return figure is required, $\frac{V}{C}$ should be used rather than the IRR as in some cases the IRR ranking order of projects will differ from the $\frac{V}{C}$ ranking order.

This ranking difference may occur because when calculating the IRR the rate of time preference assumed is represented by the actual IRR determined for the project; whereas $\frac{V}{C}$ assumes a rate of time preference represented by the discount rate used.

In other words, to determine the rate of discount necessary to give $V - C = 0$, it is assumed that the farmer's rate of time preference is represented by the

IRR. The only case where this will occur is where the IRR turns out to be equal to (or nearly equal to) the rate of discount representing the farmer's rate of time preference. Thus, for example, if 6% is the correct discount rate to use, and the IRR for alternative projects all turn out to be approximately equal to 6%, then the conclusion given by the IRR can be accepted as being approximately correct.

An example will help clarify the points made. Consider the following two alternative projects and the resultant criteria values:

		Cash Flows				
Project		Start of Year 1	End of Year 1	End of Year 2	End of Year 3	End of Year 4
A	Costs ($)	1000	0	4000	0	0
	Returns ($)	0	4000	0	0	3000
B	Costs ($)	4000	0	1000	0	0
	Returns ($)	0	7000	0	0	0

Both projects have no scrap value at the end of the period. Note that both projects have the same total costs and returns if discounting is ignored. While these projects have an unusual form, they were chosen in order to accentuate the conclusions that will be made.

	Criteria Value		
Project	$V - C$ at 6%	$\frac{V}{C}$ at 6%	IRR
A	$1,590	$1.35	168%
B	$1,714	$1.35	59%

All three measures provide a different conclusion. The net present value and the IRR rank the projects differently, whereas $\frac{V}{C}$ indicates indifference between the projects.

Firstly consider $V - C$ and $\frac{V}{C}$. Taken together these criteria indicate both projects are comparable in terms of the return or satisfaction produced compared with the level of investment involved, but that investment B involves a greater total investment.

Secondly, consider the IRR and $\frac{V}{C}$. In general it would be expected that these two measures should provide the same conclusion. The reason for the different conclusions is that in the IRR, costs and returns are discounted at the IRR and, at high discount rates, costs and returns occurring at the later stages of a project are discounted *proportionately* more than costs and returns occurring at earlier stages of a project relative to where a lower discount rate is used.

In this example $\frac{V}{C}$ is calculated using a 6% discount rate whereas to give $V - C = 0$, the discount rate necessary is very much greater than 6% for both projects.

To show the effect of low and high discount rates, consider the discount factors

$$\left(\frac{1}{(1+i)^n}\right) \text{ for the following cases:}$$

Discount Factors	5% Discount rate	25% Discount rate
Year 3	0.8638	0.5120
Year 10	0.6319	0.1074
Difference between years 3 and 10	0.2319	0.4046
Year 20	0.3769	0.0115
Difference between years 3 and 20	0.4869	0.5005

Thus, it is clear that at high discount rates, returns occurring at later periods are discounted heavily relative to low discount rates. The profit impact of this will also depend on the nature of the cost and return streams. In the example presented above, the discount rate necessary to give $V - C = 0$ (i.e. the IRR) was sufficiently high to give this distorting effect, so that $\frac{V}{C}$ and the IRR provided different conclusions. However, if the cost and return streams had followed a somewhat different pattern this may not have occurred.

4.14 The Value of Time and the Opportunity Cost

One of the reasons given for using discounting (compounding) was the opportunity cost argument: funds could be invested in a bank and return interest as an alternative to the farm investment. In some cases this is an invalid argument. This is where a farmer considers *all* possible alternatives when making a decision. In this case there are no other opportunities against which the alternatives being considered can be matched, so the reason for discounting is solely to take into account time preference.

Where the NPV is estimated for a single development programme and it is zero, this simply indicates that the investment is equivalent to a simple investment returning the rate of discount. (Note that this discussion assumes that interest charges on any borrowed funds are included as a cash cost as indeed they are. This point will be covered in more detail later.)

4.15 Characteristics of the Criteria

Introduction

To make the correct decision, *all* alternative systems should be evaluated. This is clearly a very time consuming operation. In most situations, general experience and a knowledge of maximization theory (production economics and other theories) enables an analyst to intuitively eliminate alternatives that are unlikely to be optimal. For example, a system using levels of fertilizer application that go beyond the point of marginal return equalling marginal cost should not considered. Clearly, production economics and cost–benefit theory indicate the general structure an optimal system should take and should be used accordingly. To assist in this process, this section will consider the characteristics of the investment criteria so it is clear how the criteria values will change as plans are changed.

Criteria formulae re-arrangement

To assist an analysis it is necessary to re-arrange the criteria formulae in order to make certain relationships clear.

Net present worth

The net present worth can be determined using:

$$V - C = (b_1 - c_1)\, a + (b_2 - c_2)\, a^2 + \ldots + (b_n - c_n)\, a^n$$

where

$$a = \frac{1}{(1+i)}$$

in general

$$a^j = \frac{1}{(1+i)^j} = \left(\frac{1}{(1+i)}\right)^j$$

Thus

$$V - C = \sum_{j=1}^{n} (b_j - c_j)\, a^j$$

Investment ratio

Following the above definitions, the investment ratio is given by:

$$\mathrm{V}/\mathrm{C} = \frac{b_1 a + b_2 a^2 + \ldots b_n a^n}{c_1 a + c_2 a^2 + \ldots c_n a^n} = \frac{\sum_{j=1}^{n} b_j a^j}{\sum_{j=1}^{n} c_j a^j}$$

Internal rate of return

Similarly, the internal rate of return is given by the r that gives:

$$\sum_{j=1}^{n} b_j a^j = \sum_{j=1}^{n} c_j a^j$$

where

$$a = \frac{1}{(1+r)}$$

The scale of investment

Many farm development programmes increase the number of stock units carried, or perhaps the number of hectares developed in some way. Such systems can be seen to consist of a basic unit, such as one cattle beast, used in multiples of itself (or units of irrigation, units of improved pasture, etc.). When development occurs for extra stock units each unit involves a range of activities such as pasture development, fence and water supply development and so on. A cash profile associated with each unit of development occurs. For example, the following might be such a profile:

Year 1	Year 2	Year 3	Year 4	Year 5	etc.
– $100	$20	$50	$90	$90	$90

The idea of the scale of investment can now be introduced. A programme involving, for example, 200 units and thus '200 cash profiles' is said to be of greater scale than a programme involving e.g. 100 units. Note, however, the idea that a cash profile for e.g. a single beast remains constant, for any number of units added, is not always correct. The lumpiness of inputs may mean the cash profile is different to what can be called an average cash profile. For example, a water tank may need to be purchased which is adequate for many beasts.

Using this definition of the scale of investment, the effect on the criteria values can be shown:

1. The net present value is directly proportional to the scale of investment.

PROOF

$$\text{NPV of 1 unit of investment} = \sum_{j=1}^{n} (b_j - c_j)a^j$$

$$\text{NPV of } x \text{ units of investment} = \sum_{j=1}^{n} x\,(b_j - c_j)\,a^j$$

$$= x\sum_{j=1}^{n} (b_j - c_j)\,a^j$$

Thus, the greater the value of x, the greater will be the NPV. If the NPV of 1 unit is negative, the reverse holds.

2. The investment ratio is independent of the scale of investment.

PROOF

$$V/C \text{ of 1 unit} = \frac{\sum_{j=1}^{n} b_j a^j}{\sum_{j=1}^{n} c_j a^j}$$

$$V/C \text{ of } x \text{ units} = \frac{x\Sigma b_j a^j}{x\Sigma c_j a^j} = \frac{\Sigma b_j a^j}{\Sigma c_j a^j}$$

3. The IRR is independent of the scale of investment.

PROOF The IRR of 1 unit is that value of r giving

$$\sum b_j \frac{1}{(1+r)^j} = \sum c_j \frac{1}{(1+r)^j}$$

The IRR of x units is that r giving

$$\sum xb_j \frac{1}{(1+r)^j} = \sum xc_j \frac{1}{(1+r)^j}$$

Dividing by x, the IRR is that r giving

$$\sum b_j a^j = \sum c_j a^j$$

Thus, the IRR is independent of x.

The commencement date of an investment

This refers to the time at which the particular project is started. The effect of varying timing on the criteria is:

1. The net present value of a project is reduced by the factor a for each time period the investment commencement date is delayed, assuming the per unit cash profile does not change when delayed.

PROOF Let W_t = NPV at the current time of an investment commenced in period t. Thus:

$$W_1 = (b_1 - c_1)a + (b_2 - c_2)a^2 + \ldots\ldots + (b_n - c_n)a^n$$
$$= \sum (b_j - c_j)a^j$$

$$W_2 = (b_1 - c_1)a^2 + \ldots + (b_n + c_n)a^{n+1}$$
$$= \sum (b_j - c_j)a^{j+1}$$
$$= a\sum (b_j - c_j)a^j$$

(i.e. the NPV of the project at year 2 discounted back for a future year)

$$\therefore W_1 - W_2 = \sum (b_j - c_j)a^j - a\sum (b_j - c_j)a^j$$

$$\therefore W_2 = a\ W_1$$

In general:

$$W_t = a^k\ W_t - k$$

where k = the number of time periods between the current time and t.

2. The investment ratio (V/C) does not alter as the commencement date of any investment is delayed.

PROOF Where the investment is delayed one period:

$V_2 = aV_1$; and $C_2 = aC_1$

Thus

$$\frac{V_2}{C_2} = \frac{aV_1}{aC_2} = \frac{V_1}{C_1} = \text{K, a constant}$$

In general

$$\frac{V_t}{C_t} = \text{K}$$

3. Similarly, the IRR does not vary as the commencement date of an investment is delayed.

PROOF

Let r at which $V_2 - C_2 = 0$ be r_1
Let r at which $V_1 - C_1 = 0$ be r_2
As $V_2 = aV_1$ where a is constructed using r_1
and $C_2 = aC_1$ where a is constructed using r_1

$\therefore aV_1 - aC_1 = 0$ at discount rate r_1

but

$V_1 - C_1 = 0$ at discount rate r_2
$\therefore r_1 = r_2$

as $aV_1 - aC_1 = V_1 - C_1$

The rate of investment

The rate is defined as the number of investment units commenced over a number of time periods. This rate can be constant or variable. Where a farmer increases, for example, the stock carrying capacity by x stock units per annum this would represent a constant rate of development. A fast rate of development would be a greater number of investment units per period relative to another investment programme.

The effect of the rate of investment on the criteria values can be simply determined. The rate is a combination of the scale and the timing of investment. A fast rate involves commencing investment in as many units as possible as soon as possible. Thus:

- The net present value will vary directly with the rate of investment. The nature of the variation will depend on the relative influence of the scale and timing.

- The investment ratio and the IRR are both independent of the rate of investment.

The effect of the discount rate used on alternative project ranking using *V* – *C* and $^V/_C$

One of the more critical problems in investment analysis is the decision on what rate of time preference to use. This question is made particularly important by the fact that the ranking of alternative projects may change if a different discount rate is used. In general terms, any problem in which a decision is particularly sensitive to the value of a parameter requires careful attention to the selection of the parameter value.

To demonstrate ranking variations, consider the following alternatives and the resultant criteria values for a range of discount rates:

Project		Year	1	2	3	4	5	6
A	b_j ($)		2	5	4	6	6	0
	c_j ($)		8	4	1	1	1	0
B	b_j ($)		2	8	4	4	0	0
	c_j ($)		8	1	1	1	0	0
C	b_j ($)		34	26	26	26	26	26
	c_j ($)		50	20	20	20	20	20

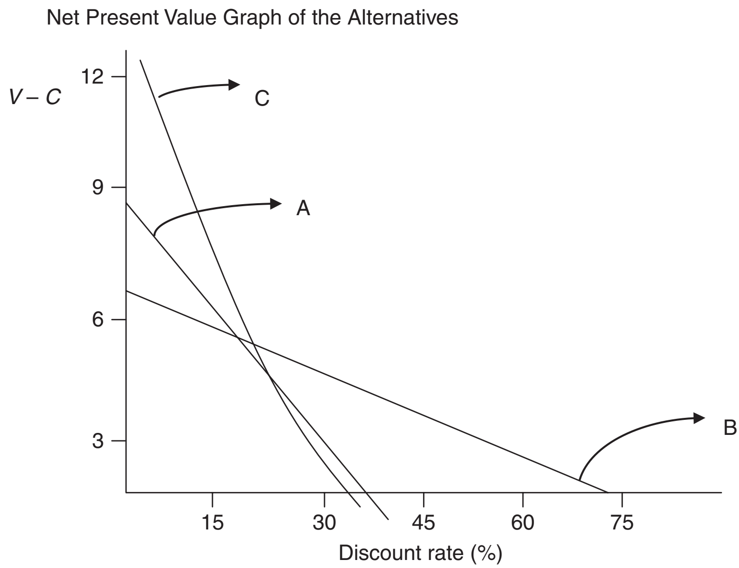
The graphs in Fig. 4.9 show that rankings can change quite markedly depending on the discount rate. These examples and discount rates at which the rankings change were chosen to accentuate the results. In practical situations, if the rankings change given relatively small changes in the discount rate it is probable that the alternatives will be approximately equally preferred. However, the output level of other commodities besides money may separate the alternatives more distinctly.

The reason for the ranking variations is the same reason as that given for the ranking inconsistencies that occur with the IRR. That is, as the discount rate varies the discount factor for different times varies other than proportionately when high and low rates are compared.

Conclusions

The basic conclusions that follow from the characteristics are:

- When analysing a number of alternative development programmes, the criteria should be evaluated using a range of discount rates. This will indicate the sensitivity of the results to the discount rate and thus the importance of



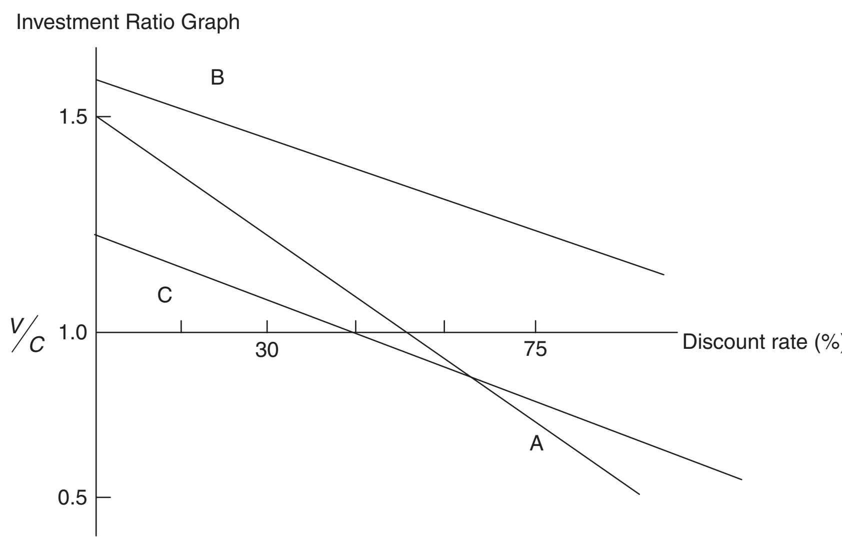


Fig. 4.9. Effect of discount rate used on project ranking.

the discount rate used to the conclusion or decision made. This approach is particularly important where there is uncertainty about what the correct rate of discount is.

- If a development programme can be seen to consist of a number of investment units of a similar type, then the IRR and V/C can in general be calculated from the cash profile of one of these units. Thus, to provide an indication of the farming system that is likely to justify a detailed analysis, calculate the criteria for unit cash flows of the alternatives. An idea of the likely NPV can also be obtained considering the effect of scale and timing.

This approach must be used with caution as the lumpiness of certain inputs may mean the cash profile per unit will vary according to the scale of investment. Further, the cash profile per unit for one particular property may be different from the profile for another property.

- The faster the rate of development the greater will be the NPV, provided $V - C > 0$. If $V - C = 0$, rate will have no effect and if $V - C < 0$, a faster rate will clearly increase the 'loss'. This means that in order to maximize the NPV, as many investment units as finance will permit should be commenced as soon as possible.

This statement requires some qualifications. The basic assumption is that the per unit cash profile will not change as the rate changes. In practice this may not be the case because:

- The cash profile may change according to the scale of investment due to the effect of lumpy inputs (e.g. labour) and possible size economies; and
- A fast rate of development may give management inefficiency so that the NPV per unit may decline.

Thus, in some cases too fast a rate may not be as profitable as a slower rate and vice versa, due to size effects.

4.16 Farming Investment Analysis

Introduction

The discussion so far has paid little attention to the practical details of estimating the relevant cost and return streams. In general there is little difficulty in deciding which returns should be included in the analysis but often considerable confusion over the costs. The answers depend on the approach taken in the analysis. This section outlines both approaches that may be used and the costs that should be included for the less obvious categories. Other practical problems such as the choice of discount rate are also discussed.

Types of analytical approaches

An ex-post study

This involves taking a farm, or a number of farms, *that have been* developed using a defined system and using the historical information to evaluate whether the development *has been* profitable. The results can be used to indicate to farmers and their consultants whether particular systems have been 'profitable'. Such results have limited use as they hold for the historic conditions which may not occur in the future. However, they have the advantage of being completely realistic as they reflect the profitability of actual systems and do not rely on non-certain forecasts.

A major difficulty can be in obtaining the relevant information, which is only possible where adequate records have been kept. The financial data must be deflated, or inflated, to a particular time to put them on to a *real* cost and return basis. Alternatively, if adequate physical information is available this can be used together with current prices and costs to estimate the b_j and c_j. This removes the need to inflate or deflate.

An ex-ante study

This involves evaluating *proposed* development programmes to determine an optimal system (more practically, this will probably be an *improved* system rather than the optimal system. It is doubtful whether the techniques are available to determine the optimal system without considerable cost). Plans and the resultant budgets for the proposed systems are calculated for each period until a stable position is expected to be reached. The major problem is obviously the prediction of technical input–output information and of expected prices and costs. Unless there has been good evidence to suggest otherwise, most analysts have used current costs and prices, though there is a good argument to increase costs every year by the inflation rate.

Ex-ante studies will fall into one of two categories: (i) where the farm is completely undeveloped and/or where a farmer is radically changing his total farming system. In this case a whole farm approach will have to be taken; and (ii) where the farm is partially developed. In this case only the *additional* costs and returns of the proposed systems need to be considered. The assumption in this case is that the costs and returns associated with the rest of the farm will continue at current levels.

General and specific studies

Method

In carrying out either an ex-post or ex-ante study, this could be for general use, or for a specific farm situation.

In a *general study* the method of funding the programme is ignored so that a defined physical development programme for a typical farm (and farmer) is analysed without the need to consider where finance is to be obtained and the relevant finance costs. In this case the relevant costs to include are *all* cash developmental and operating costs. Costs associated with borrowing funds are ignored. It is simply assumed that the farmer has available all the funds. The relevant returns are all the cash returns.

In a *specific farm study*, the analyst is dealing with a particular farmer and farm and thus a situation involving a specific level of available finance, and level of potentially borrowable finance, is assessed. *Only the farmer's cash costs and returns* should be included in the analysis. Thus, assets purchased using borrowed finance are not a cash cost to the farmer. The cash costs are the interest and principal repayments. There are two alternative ways of handling this problem, each of which will give the same answer:

- All costs *except* those paid for from borrowed funds are included. The costs must contain the principal and interest charges on the borrowed funds.
- All costs including those paid for from borrowed funds are included. But the funds obtained from borrowing must be included as a cash inflow, as indeed it is. As before, the interest and principal charges must also be included.

Interpreting the results

In a general study, the calculated NPV assumes that if funds are borrowed to carry out the programme, they are borrowed at the rate of discount used in calculating the NPV. In many cases this will not occur, so the NPV may give an incorrect ranking. And for a specific study, the results can only be directly applied to the particular property as financing arrangements (mainly level of borrowed funds) will need to vary from property to property. However, if the discount rate used is equal to the borrowing rate these qualifications will no longer hold.

The incorporation of depreciation in the cash profiles

Where all cash costs are included in the budgets depreciation must be ignored. The use of depreciation charges in a budget provides an ‘on average’ figure through apportioning capital costs to a number of years. A farmer, however, is not interested in theoretical accounting procedures, but in the actual cash returns (as well as other utility producing outputs). Thus, the cash costs must be included in the analysis when they occur, and this includes charges for new tractors, fences, land clearing and so on.

When the development programme reaches a *stable position* a different approach may be justifiable and is usually used. Thus, depreciation is then included as a cash cost. The alternative would be to continue the budgeting process for the life of the farm as, in reality, a stable cash return is never reached. A constant yearly capital asset replacement policy never (seldom) occurs. The charging of depreciation in the stable budget is, thus, an approximation.

Incorporating income tax into the cash profiles

Income tax, and other taxes, if any, are a cash cost to the farmer so must be included; however, in studies involving only part of a farm, only *additional* taxation charges need to be included. Note that in many cases total taxation may initially decline as many developmental costs are tax-deductible and thus reduce tax.

The need to include taxation charges means that two sets of budgets for each period must be calculated. One to indicate the cash situation, and the other to estimate the taxable income to enable the tax estimation. Thus, while it is not included in the determination of the NPV, depreciation charges must be estimated, adjustments due to stock and produce ‘on hand’ value increases, or decreases, must be estimated, the farmer’s personal tax allowances must be calculated, the developmental costs that are tax deductible must be determined and so on. Note that, however, the principal repayment charges must not be included in the tax budget, although they are, of course, included in the cash budget for a ‘specific farm’ problem.

The choice of a discount rate

The specific case with financing costs included

In this situation use:

- the opportunity costs of capital where the alternatives considered in the analysis do not include all the possibilities in which the farmer would be prepared to invest (in effect, the alternatives are ranked against this alternative opportunity);
- the farmer’s rate of time preference if all the possible alternatives are being included in the analysis (‘all’ in this context is taken to mean the list of projects that is most likely to include the ‘better’ possibilities). The problem here is determining the farmer’s rate of time preference. This must rely on questioning and observation. If there is no evidence to suggest otherwise, the rate of return on Government stock can be used (as a safe investment).

The general case (financing costs ignored)

In this situation use:

- the opportunity cost of capital if it is expected that the project will *mainly* be funded from the farmer’s funds and all alternative systems are not being analysed;
- the *typical* rate of time preference (reflected by Government stock rate) if it is expected that the project will be funded from farmer’s funds and all alternatives are being analysed;
- the borrowing rate if it is expected that the project will *mainly* be funded using borrowed finance.

The selection of a correct discount rate is clearly important. However, where the criteria values are estimated for a range of discount rate values, the problem of deciding which rate to use is not so critical in the general case. This approach puts the responsibility of making the correct decision on to the farmer and consultants, not on the analyst carrying out the study unless this person is the farmer’s adviser.

4.17 The Handling of Uncertainty

The techniques discussed in Chapter 3 on non-certainty can be used to estimate the cost and return distributions for each year, thus giving a yearly profit distribution. The farmer's utility function could then be used to give the expected utility for each year and the *net present utility* could then be obtained by discounting these yearly utility values. An alternative approach might be to determine the net present value distribution and to use this to determine the expected net present utility. However, there is considerable doubt whether utility values can be discounted when they cannot be 'comparably' measured. Certainly this approach is only possible for studies oriented to a specific farmer.

Given current knowledge, the use of utility functions cannot be recommended for general studies. This means that a passive approach must be used. This requires determining the NPV (or other criteria) distribution. This is a complex operation, as even where it is assumed the random variable can take on only a very small number of possible values, the possible number of variable value permutations that can occur over the entire length of a development programme is extremely large. In general, a more efficient technique for determining the NPV distribution is the use of Monte Carlo-type simulation, a technique which will be discussed in a later chapter (Chapter 14, Systems Simulation).

Whether an attempt should be made to estimate the NPV distribution also depends on the farmer's attitude to risk. In many cases *expected* NPV figures may be a sufficient basis for decision making. Further, alternative systems may not provide quite different NPV distributions, so risk may not be a deciding criterion.

As an alternative, analysts can use a number of simple *non-certainty hedging* techniques when estimating criteria values. These *do not* allow for non-certainty in a correct fashion, but do reduce the chance of over-estimation errors in estimating the criteria values. Three possible hedging techniques are:

- using conservative product price estimates with the degree of this conservatism increased for periods well into the future;
- using a higher discount rate than the correct one. This tends to reduce the effect of costs and returns occurring well into the future, it being assumed that there is considerable non-certainty attached to these costs and returns;
- evaluating the criteria for a range of prices and costs and determining whether the project ranking is sensitive to such changes. If not, greater confidence can be placed in the conclusions.

This latter approach does not correctly take into account uncertainty as it only considers an extremely small number of the possible price and cost permutations. A range of possible input–output coefficients might also be used: for example the NPV might be determined for a range of stocking rate assumptions, crop yields etc.

4.18 Technological Change

Technological change will undoubtedly increase the profitability of many development programmes compared with the estimates made using current technology. Conceptually, estimates of average technological change can be included in the budgets, but in practice such changes are very difficult to estimate. This aspect is, of course, part of the non-certainty problem.

Most studies ignore this problem. The justification given is that technological change will affect all the alternatives in the same way, so that the project ranking will not change. For most farm studies this is probably a reasonable assumption. A similar approach sometimes used is to ignore the presence of inflation and of technological change and assume one will counteract the effect of the other. The success of this assumption is clearly going to depend on the level of inflation.

4.19 The Planning Horizon

In a general planning sense, the maximum time to consider covers as many periods that are necessary to give a set of 'current time decisions' (those implemented *now*) that will not alter if a longer period were taken into account. However, in practice it is very difficult to determine this minimum horizon, so in most practical situations the planning horizon should at least encompass the period defined by the following:

1. In situations where it is probable that the farmer has no immediate intention of selling the property, then use a planning horizon of infinity. This means that the stable budget should be capitalized as previously suggested. In practice, many farmers approach development programmes under the assumption that the farm will probably remain in the family after the farmer's retirement. In many programmes, taking a period of, say, 50 years instead of infinity seldom alters project rankings.
2. In situations where the farmer intends to sell the property within the near future this sale time should be used as the horizon. In this case the expected sale price must constitute part of the programme cash returns. (People investing in properties for tax purposes may fall into this category.)

In many cases, however, the results obtained under categories (**1**) and (**2**) will be nearly identical. When a stable budget is capitalized, this approach, in effect, puts a value on the resources held at this time equivalent to what their sale value could be. Whether the results will be identical depends on whether the estimated market value is similar to the productive valuation.

4.20 The Effects of Inflation Again

If it is assumed the estimates of yearly cash surpluses will represent the actual cash surpluses, these should be deflated by the inflation rate before an NPV is determined. The reason is the real value (or purchasing power) of these cash surpluses will be considerably less for periods well into the future than an equivalent cash surplus occurring at the current time. Many studies have ignored this aspect of the effect of inflation. The assumption has been that as current product prices are used in the study, future product price rises will counteract the declining purchasing power of the surpluses. In fact this seldom occurs. A counteracting effect, however, is the rising land values, though such value changes are of no benefit unless the property is sold or used as security for borrowing purposes.

4.21 Concluding Comment

In stable production systems there will be little use for investment analysis. In this changing world, however, very few optimal farming systems will be static. Price changes, government policy changes and changing weather patterns, all mean optimal farming systems should constantly be reviewed and improved. As the years pass, farmer's objectives are also under review, and more recently there has been an interest in sustainable systems, which is again a harbinger of change. Agriculture involves biology and, when perturbed, biological systems take many years to reach a new equilibrium. For all these reasons, it is very important to allow for the time value of money, and liquidity preference, in analysing proposed changes. Valuable improvements take time.

Further Reading

Hanley, N. and Spash, C.L. (1993) *Cost-benefit Analysis and the Environment*. Edward Elgar, Northampton, Massachusetts.

Mishan, E.J. and Quah, E. (2007) *Cost Benefit Analysis*, 5th edn. Routledge, Abingdon, UK.

Sinden, J.A. (1994) A review of environmental valuation in Australia. *Review of Marketing and Agricultural Economics* 62(3), 337–367.

5 More on Decision Making and Utility (Objectives)

5.1 Introduction

Only passing reference has been made to incorporating other than cash and profit as outcomes in decision making. The knowledge on quantitatively measuring other objectives is probably too inadequate to rigorously incorporate them in a formal manner. Leisure time is one exception. Overall, this means subjective approaches must continue. However, to round off the discussion this brief chapter contains comments on incorporating other objective forms.

5.2 Active Approach

If 'all factors' utility functions could be estimated, decisions could be made on the basis of maximizing expected utility as demonstrated for monetary utility (Chapter 3). This would not only account for factors such as the leisure hours supplied by a particular farming system, but would also allow for farmers' attitudes to risky situations involving these factors, and all other components of objective functions. Any component, including leisure hours, is just as much a random variable as monetary outcomes and needs to be similarly incorporated. Risk attitudes will impact on leisure as it does for monetary incomes from alternative systems.

Assuming it could be measured, an 'all factors' utility function would have the general form:

$$U(C) = f(U_1\ (C_1)\ldots..\ U_n\ (C_n))$$

where $U_1(C_1)\ \ldots\ U_n(C_n)$ represent the utility functions for individual satisfaction-producing commodities, and $f\,(\ldots)$ represents the relationship bringing together the individual functions.

The important imponderable is the form of the relationship between satisfaction-producing commodities. What are the rates of substitution between the commodities and how do they change? For example, is 1 h of leisure equivalent to \$40? Perhaps this might be the case when the farmer has little leisure, whereas if his current leisure level is high, the relationship might be 1 h to \$22. No answers or recipes can be provided.

Considerable research has been performed by farm management workers through to psychologists in attempting to estimate the functions. Methods used range from questioning procedures, such as that used in the monetary utility function system, to observing how individuals make decisions and then using the results to fit a function in the same way as a production function would be estimated from plot trial data. This work will undoubtedly continue.

5.3 Passive Approach

This approach is more practicable as no attempt is made to measure the utility functions. Efficient frontiers are developed for cash and leisure, and any other objective that is measurable, and the farmer is left to choose between the possibilities. This kind of approach is partially used by many consultants and advisers. When a particular system is suggested, the farmer may indicate the system is not acceptable for some reason or other. The consultant then makes adjustments to comply with the requirements. Effectively the farmer is forcing the consultant to come up with another more acceptable plan that represents a point on the efficient frontier.

The efficient frontier will be multidimensional in the ‘all factors’ case and, where outcome variability is an important part of the farmer’s objective, one of the axes should measure variance (assuming only the expected and variance values are the relevant distribution parameters). Again, the difficulty is in measuring the non-monetary outcomes, but at least items like the type and numbers of stock, crops and other activities can be measured under the assumption they confer other attributes such as enjoyment in having, e.g. registered stud stock, or perhaps a field of peony roses. The level of debt is also likely to be important.

It is impractical to estimate graphs for each individual farmer due to the work relative to the potential benefits. A more practical approach is to obtain some idea of what the farmer’s objectives are through questioning and then produce a number of systems with the important satisfaction-producing outputs quantified. The farmer can then choose. An experienced consultant can often produce a good farming system from the outset. He can judge the objectives and knows the alternatives. In most cases the consultant will use the current farming system as a starting basis, for it has been acceptable in the past.

Finally, it must be noted that where an *expected profit/variance* frontier is estimated, the *farmer’s probabilities* should be used in the analysis. This is in contrast to someone else’s or objective probabilities (though these may be the farmer’s subjective probabilities). This does not mean a consultant should not debate a farmer’s estimates through discussions. Every farmer should seek, and listen to, all ideas and information that is available.

5.4 Lexicographic Utilities

It has been assumed increasing quantities of an output produce greater satisfaction. For some products this assumption should be questioned. Researchers have suggested that in some cases once a critical level of a product is produced, greater production will not provide greater satisfaction. This could well be the case for a commodity such as a farm dam for the swimming, fishing and fire precaution services it provides. One dam will provide all these requirements. More are not necessary, except for other reasons.

This idea is known as a *lexicographic utility* system. An individual ranks possible satisfaction-producing goods, or factors, in order of priority, and for each has a critical satisfaction level. The individual makes decisions so that the first priority's critical level is attained. This factor is then removed from primary focus other than in ensuring at least this critical level is always attained. Attention is then focused on the second priority and a farming system developed which will satisfy this requirement at the same time as satisfying the first requirement. In developing a whole farm system attention is paid to lower and lower priority factors until all resources are utilized. Clearly, monetary gains is likely to be the first factor considered and once a plan has been developed to ensure a minimum level of cash return, the farming system is adjusted to see if a critical level of, e.g. leisure hours can be attained at the same time as ensuring the required cash income is forthcoming.

There is undoubtedly an element of this process in the way most people make decisions, and some farmers largely operate in this way. However, the majority probably operate according to the maximum expected utility theory, but with some minimum requirements at fairly low levels for other satisfaction-producing goods, particularly cash and leisure.

5.5 Objectives and Decision Making – an Overall Conclusion

What is your conclusion? As an analyst or consultant, which system do you believe should be used in developing efficient farming systems? The answer must depend on the particular farm and farmer/household complex. As a researcher the same applies, but with respect to the particular problem that is being solved.

Your conclusion over using risk and uncertainty in an analysis may well be as good as many other people's conclusions. However, it is fair to note few farmers actually achieve maximum profits for their particular resource endowment. Thus factors such as the shape of the cash income distribution and leisure hours must be important, as it is unlikely all farmers are irrational in their decisions. Some researchers even go as far as suggesting all farmers are rational, and only appear to be irrational when judged according to the wrong objectives. Even if it is decided to analyse systems using, e.g. single valued expectations to maximize simple profit, an understanding of the kinds of errors that will be made in taking this approach should be evident.

While a consultant might base most of his work on single valued expectations, on a few typical farms analyses using more sophisticated risk analysis techniques should be carried out to appraise the degree of error in using the simple techniques. Conclusions can then be used to create farming systems, suitably modified, for other cases.

A conclusion on incorporating time into investment analyses (discounting, and similar procedures) is easier to make. The additional work involved in using investment analysis methods in appraisals is relatively small and so should be carried out for all major investments. And when it comes to all the other satisfaction-producing outputs, due to measurement difficulties, subjective methods must be used in general. The useful approach is to create a wide range of alternative systems and let the farmer decide which is most appropriate (the passive approach). If none meet the requirements then more analysis is clearly necessary using the preferences the farmer displays to provide clues on the direction of this further analysis.

6 Farm Surveys – Uses, Procedures and Methods

The idea of a survey is simple. It involves exploring problems through observing, recording and analysing farm and related information. The problem might be finding an optimal whole farm system, right through to, e.g. comparing the profitability of different levels of irrigation. While the processes of data collection and analysis appear quite simple, surveying is in general an exacting and time-consuming job. The most difficult areas are sampling theory and its implementation, extracting the correct data, and its analysis and successful interpretation. In some cases this latter area may be quite simple as only a simple recording of observed facts may be necessary. Some surveys concentrate on collecting facts and figures (quantitative), others on opinions and ideas (qualitative) and many collect some of both. The discussion will concentrate on quantitative surveys, though a review of qualitative concepts and uses will also be provided.

More precisely, a survey can be defined as:

> The collection of facts, figures and opinions from a group of farmers and farms which make up a whole population, *or* a sample of farms from a defined population, for the purposes of inquiring into the numerous aspects of a situation as it exists in a given population of farms over a defined period of time; *and* the analysis and interpretation of the information obtained.

This definition makes it clear the steps involved in carrying out a survey include:

1. Defining the problem to be solved (this may be quite general in some cases);
2. Defining the population of farms for which this problem is to be solved or answered;
3. Selecting the farms from which the information is to be collected;
4. Collecting the information;
5. Analysing and interpreting the information collected.

The sections in this chapter cover each of these areas. Their aim is to introduce survey method rather than to provide a complete theoretical and practical knowledge of surveys. Researchers involved in rigorous survey work would require more detail in some sampling and analytical methods. Specifically, the sections will cover:

1. Possible uses of farm surveys;
2. Methods of obtaining information;
3. Sampling;
4. Questionnaire design and response errors;
5. Organization of surveys and the coding and analysis of survey data;
6. Limitations and problems of farm surveys;
7. Qualitative surveys;
8. Conclusions.

6.1 Farm Management Uses

While a farm survey can be used to explore almost anything to do with farms and farmers, in exploring and improving farming systems there are a number of more common uses as listed below.

Determining the profitability of alternative farming systems

This is probably one of the most common uses of farm surveys. Some examples are:

- irrigation farming compared with dryland farming;
- lamb production compared with cattle farming and any other combination of stock;
- comparison of various crop combinations;
- use of computer software options in efficient decision making.

These four examples are general comparative studies. At the other extreme a survey might be used to compare the comparative profitability of small variations in farming systems. For example:

- alternative lamb weaning ages;
- alternative winter feeding systems;
- factors influencing farm computer use levels.

Further, rather than specifically comparing defined farming systems (a predetermined hypothesis), the survey may simply be to collect information on the farming systems used by a group of farmers to determine which system has high profitability. For example, the information may show farms with at least *x*% of their area in certain cash crops are the most profitable, or as another example, those that breed their own stock replacements are the most profitable. Clearly the list is endless, with any difference between the farms being investigable.

In all cases, the general procedure is as follows.

1. The selection of one, or more, homogeneous area/s containing a 'reasonable' number of farms with the systems to be compared;
2. The collection of physical and financial information so the farming systems can be related to financial results (or whatever the objective is, e.g. physical yield relative to irrigation level);
3. Comparing and determining the most profitable systems, or most efficient, depending on the objective.

In most cases in the developed world, farm accounts will provide the financial information while some kind of questioning procedure will be necessary to obtain the physical information required to group farms according to farming system. Where financial records are not available, questioning will be necessary or the use of source documents (e.g. invoices).

The analysis requires grouping the farms on their farming systems and the estimation of the profitability (or stated outcome) of each group. In some cases the procedure will operate in the opposite direction. Farms will be grouped according to profitability and then analysed to see if the more highly profitable farms are using a particular farming system compared with the less profitable groups. Logically, the first procedure should be used as the grouping should be on the independent variables (farming systems used), not the dependent variable profit, as the farmer's choice lies with the independent variables, not the outcome.

Determining farm standards (benchmarks)

Rather than simply comparing farming systems, comparative information can be obtained from the most profitable farms. These farms provide *farm standards* or *benchmarks*. These can be a wide range of statistics such as 'labour use per stock equivalent', 'fertilizer expenditure per hectare', 'stock equivalents carried per hectare', 'ratio of crops produced' etc. The idea is that the managers of the less profitable farms adjust their systems so that they use and produce the same input and output ratios (benchmarks/farm standards) as the 'successful' group. This 'farm standards' system of determining improved systems, and of providing advice, will be further discussed in the subsequent chapter, which contains comments on various other uses of survey information.

The collection of input–output information for use in planning

Planning needs a large body of technical knowledge to provide the production functions of alternative products, including intermediate products such as pastures, turnips, lucerne etc. (these are called 'intermediate' as they provide input to a saleable product). These are required for each set of soil, climatic and topographical conditions. One source is small plot trials. However, such information is usually limited and does not cover items such as the labour requirements per 1000 ewes, machinery operation rates and a host of other production statistics at a practical farm level. As most populations of farms (i.e. a group with defined characteristics) will have a wide range of products and farming systems, they can provide a large amount of input–output information for planning purposes.

Success in collecting what is, mainly, physical information will depend on the farmer's records. In developed countries, as most farmers have accounts prepared by accountants, financial information is usually not difficult to obtain. Particular attention must be paid to ensuring that the physical conditions to which the input–output information applies are clearly tabulated and that different environmental areas are kept separate. Sometimes it will be necessary to set up special recording systems, perhaps with outside help.

Collection of information to familiarize professionals with an area

There is a continuing need for advisers and researchers to have background information on the farms and farming systems in their particular areas. In many cases information is obtained from normal contact with farmers, but where this contact is small there may be a need for a survey to obtain this information. The data collected will depend on the situation but should encompass all the information necessary for a consultant or analyst to operate. Particular attention should be placed on obtaining information on what the farmers *think* are their problems, and on such items as the level of indebtedness, level of profitability and degree of development. This information indicates the scope for change.

Further, where a consultant conducts the survey himself it is an opportunity for meeting the farmers.

A 'familiarization' survey should also have a positive aspect that can provide useful feedback to farmers to engender greater cooperation.

Information for selecting representative or typical farms

To carry out optimal farm system studies it is important to have a case farm that is typical of as many farms as possible. To determine the characteristics of a typical farm a survey may be necessary. Both financial and physical information should be collected, as they determine the opportunities, or alternative farming systems, that are feasible.

The use and selection of representative farms will be discussed in more detail in a later section.

Other farm uses

While categories described above cover the main uses of surveys in farm management, many other specific problems may be answered through surveys. Any problem that can be answered using information available from a population of farms is solvable with a survey. Examples include determining the following.

1. Minimum farm size for economic viability. The procedure would be to: (i) obtain financial information from a wide range of farm sizes for the particular environment under study; and (ii) compare profitability with size.

2. Cost of running various sized headers (combine harvesters) per hectare harvested. The procedure would be to: (i) isolate a number of farms using a range

of header sizes; (ii) collect the cost information (a recording scheme may need to be instituted); and (iii) calculate costs/hectare.
3. Working or operating capital requirements for different farming systems (this information might be necessary for budgeting purposes).

Macro uses

While not directly related to finding improved farming systems, it is useful to note there is a wide range of topics and problems for which a government, and other planning organizations, require up-to-date information for making policy decisions. Farm surveys can be used for many of these. The following examples indicate the more important.

Cost of production estimates

Where a government wishes to ensure that a product is produced at some minimum level there may be a need to fix the minimum price. One approach is to estimate the cost of production and set the product price accordingly. Another may simply be to set a product price so that farmers' incomes will be at least a certain level. The procedure in these cases should be:

1. Select farms in the various physical environments producing the product;
2. Collect information giving costs and the production level of the product (information may need to be collected for other products where the farms are multi-enterprise units);
3. Allocate costs between products and thus determine the cost of production for the product of interest for the various environmental groups.

Note, however, such estimates often average the cost of production figures whereas marginal estimates will be of more use. And with multi-product farms, allocating costs can be logically difficult. This problem is discussed further in another section.

Policy measures to achieve a stated objective

A survey can determine, potentially, how successful particular measures might be. For example, many small sheep farms might be becoming sub-economic units. The government may have decided these farmers should be encouraged to move out of farming so the land can be used by larger units. However, a survey may turn up that an injection of development capital would correct their current economic problem. The problem might be to determine how to move into another industry. The procedures might be:

1. Survey a range of small farms and ask questions to estimate the number of farmers prepared to leave their farms if, for example: (i) a government job was provided for the farmer at a defined income level; and (ii) at least a defined minimum sale price on their land was guaranteed.
2. Survey larger farms in order to estimate the likely demand for land given up by the small farmers. Such a demand will depend on, amongst other factors:

(i) the farmers' objectives; (ii) their current profitability level; (iii) current cash assets and access to borrowed funds; and (iv) the price set for the land.

Many other questions should also be considered. For example, the effect of farm amalgamation on the country's sheep product export earnings. This would depend on the current production of the small farms in relation to potential production of the smaller number of larger farms.

Ongoing effectiveness of new policy measures

The answers obtained from surveys in the previous category may depend on opinions and estimates provided by farmers. For this reason it is important a government has available up-to-date information on the effectiveness of policies. For example, if a farm amalgamation scheme is introduced, the government should know the effect and, thus, whether other policy measures are required. In such a survey the general procedure would be:

1. Select a sample of farms prior to the introduction of the policy measures and collect information from these farms;
2. At successive points in time re-survey the farms.

In some cases, however, information normally collected by the government and local body statisticians may provide the relevant information.

Assessment of the economic viability of farming sectors

To determine when policy measures may need to be introduced a government must have available continuous information on the financial situation of each sector. The procedure in these cases will be:

1. Select a sample of farms from each general farming and environmental category.
2. Obtain as much physical and financial information as possible from these farms at successive intervals.

Such general surveys have extensive use throughout the community. For example, a new adviser can use the results to familiarize himself with the situation in an area, or a researcher might select an area for a study on, e.g. indebtedness levels.

Marketing strategies

Marketing groups and selling organizations require continuous information on product quantities available to be sold and/or stored. Such information will indicate, for example, what shipping arrangements are necessary. The general procedure in this case would be to:

1. Carry out a number of farm surveys throughout a particular season.
2. Estimate product areas (crops) or numbers (stock) on the sampled farms.
3. Estimate expected yields as the season progresses.

The information on yields and areas enables total production estimates. Similar demand information will also be required, perhaps from consumer surveys.

Improved extension techniques

To develop successful extension programmes, information on where farmers obtain their information (e.g. farming papers, radio broadcasts, talking to 'leading' farmers etc.) and on the effectiveness of alternative techniques is required. Surveys are the logical means of determining the information. For the comparisons the procedure could be:

1. Select a sample of similar farmers and farms and collect farm system and financial data.
2. Expose sub-samples to a range of techniques (e.g. use of printed material, use of discussion groups, use of field days etc.)
3. Re-survey the farms after various time periods to determine changes in farming systems and profitability.

A major problem will be deciding whether any changes are due to the extension techniques or to other contacts the farmers have had. Isolating comparable sub-samples (particularly with respect to the farmers themselves) may be difficult.

Further comments

The list of survey uses could be enlarged further, but imagination on possibilities will suffice. However, whether the survey technique is the best system for solving the various problems has not been covered. Some of the answers will emerge in later sections, but a complete appraisal is not possible until the very end of the book. The problem, as always, is to decide which of the techniques is the most appropriate to determine improved farming systems at least cost. For some problems few alternatives to a survey exist. For example, if information on current production levels is required, a survey is likely to be the only choice.

A search of the literature will provide many examples of surveys. For example, Anon (2010a, b) provides examples of regular yearly surveys for keeping professionals up to date on what is happening in two industries, Nuthall (2002) provides an example of a survey to determine farmers' views of skills required for success, Nuthall (2006a, b) covers the use of surveys in creating tests to assess managers' characteristics and Nuthall (2009a) shows how to use a survey to obtain information on factors that influence a farmer's managerial skill.

6.2 Methods of Collecting Information

Introduction

While potentially there are a number of different methods, for a particular survey the range of alternatives may be restricted. This will depend on the type of information required and the records available from farmers and other sources.

A few problems, though more generally only part of a problem, can be answered through the use of records and documents that are normally collected and held by various government and local-body departments. Producer boards, farmer and allied organizations may also hold helpful records. For example, the areas of farms in a defined locality may be obtained through searching land registry titles, or perhaps through valuation records in countries where valuations are carried out regularly for tax purposes. Another common example is the use of farm accounts to provide information on costs, returns and the taxable profit.

In order to reduce the work involved in a survey, and to increase its accuracy, as much use as possible must be made of such records, though in a few cases record searching may involve additional work compared with directly questioning the farmer concerned. In general, however, the main methods of obtaining information include direct observation, mail surveys, telephone surveys, personal interview surveys and specialist recording schemes.

Direct observation

Some information can be obtained through direct observation. For example, a list of machinery and buildings can be made through observation (similarly, their values), the 'quality' of pastures can be assessed and so on. Many pieces of information, of course, cannot be collected in this way. For example, it is difficult to accurately observe fertilizer levels. As a general rule, where possible obtain information through observation provided the information collector is trained to make such observations. If not it may be better to let the farmer provide the information and rely on his judgement and memory. This depends, in part, on the type of information being collected and its importance.

Mail surveys

A mail survey involves providing a questionnaire, usually through the mail, to be filled in by the farmer in his own time. Alternatively the questionnaire could be delivered by the internet, or perhaps even by hand. The most common systems use mail for both delivery and return, though sometimes either delivery or collection is by hand, especially where a verbal explanation may help, or 'on the spot' checking is valuable before collection. Of course, this approach is only possible where a good mail service/internet exists.

The advantages of mail surveys are:

- they are usually cheaper than other methods and sometimes quicker;
- the respondent (the farmer) can spend time considering each question and looking up information. Some farmers, however, may spend even less time thinking about a question compared with the case where a surveyor asks the question;
- for personal questions a more accurate reply is likely compared with the case where a surveyor (enumerator) is present;

- the farmer is easily contacted. In an interview process it may be difficult to arrange a suitable time.

The disadvantages of mail surveys are:

- they can only be used where the questions are sufficiently simple to enable a full understanding without personal help;
- there is no chance of a discussion to ensure that the ‘correct’ answer is provided. Thus, answers provided must be accepted and in general there is no opportunity for anomalies to be investigated;
- the number of questions asked must be small, as otherwise the respondents will tend to discard the survey;
- similarly, the amount of work involved (calculations, looking up information) must be kept at a low level.

In general, the response rate to mail surveys is low. Usually a rate of 30–50% is all that can be expected but this will depend on the topic being investigated. Where there is a high level of interest in the particular topic and the farmers involved are well ‘educated’, the response rate will be much higher and therefore a mail survey may be a satisfactory method of obtaining the information. An example would be where some kind of subsidy or government handout is as a result of the survey (provided the farmers believed they have a good case).

As in all surveys, bias can occur in mail surveys. As the farmer can more easily refuse to cooperate in a mail survey, it is usual to find considerable bias in some kinds of surveys. This will depend on the topic under study. For example, in a survey designed to determine the profitability of alternative systems, farmers who *think* they are poor farmers are less likely to respond.

To maximize the response rate, experience has shown that the following procedures tend to help:

- enclosing a stamped, addressed return envelope;
- enclosing a good covering letter from a respected person explaining the objects of the survey, details of the agency carrying out the survey and other background information;
- the use of reminder letters. Later ones should include another questionnaire.

Dillon and Jarrett (1964) and Freebairn (1967) discuss response rates in mail surveys and the impact of various factors.

Telephone surveys

Many of the comments made for mail surveys apply to telephone surveys. The major advantage of a telephone survey is that it is quick and inexpensive.

The disadvantages are:

- the information sought must be at the farmer’s fingertips;
- only a very small number of questions can be asked as farmers, in general, will not want to spend a long period of time on a telephone (however, several calls may be made);

- it can be easy for a farmer to provide an unconsidered answer as he is not facing the enumerator in person, but equally the opposite may hold – it depends on the type of information;
- farmers tend to associate telephone surveys with commercial-type purposes. Thus, the farmer may not believe you, considering your story to be a way of making contact. This problem can be partially overcome by an introductory letter;
- where the telephone is a shared line, the farmer will be reluctant to divulge certain information.

Personal interview surveys

As the name implies, these surveys involve an interview with the farmer by an enumerator. Questions are asked and the answers recorded. Questions can take several general forms, details of which are discussed later under questionnaire design. This type of survey is the most common, though invariably some of the information is also collected using available records and observations. Where a considerable amount of information has to be collected there is usually no alternative to using a personal interview approach. In some cases as much as a complete day may be required, though probably not all at one time.

While personal interview surveys are the most expensive, they have few disadvantages. The biggest problem is that the enumerator is a key link. Given good training an enumerator can extract information efficiently and check answers with observations, and also collect base information through observation. Poorly trained enumerators and enumerators with poor personalities can introduce bias through unfavourable reactions. Further, they may not use sufficient effort to extract correct answers.

The response rate can be high, and usually higher than other systems, but it does depend on the nature of the information collected. If the farmers have much to benefit from the survey outcome (e.g. a drop in water charges might be possible) they will probably cooperate. Response rates of 80% are common but 95% is not unusual given good preparation and enumerators. Clearly, the skill of an enumerator in convincing farmers of the value of cooperating is important.

Recording schemes

Where continuous and detailed information that is usually not recorded, nor can be remembered, is required there may be a place for providing farmers with recording books. For example, if information on the labour requirements of various activities is required, probably the only way is to provide recording books which are filled in by the farmer once per week, or even daily. These books would be periodically collected and the information extracted.

Alternatively, an enumerator could visit the farmer at frequent intervals and, as a result of questioning the farmer, record the information. Telephones, or the internet, might also be used.

A difficulty is finding sufficient farmers prepared to spend the time to fill in the books. In general, continuous recording schemes can only be used where a totally random survey is not required. Cooperators are likely to be the better farmers, though offering incentives may encourage others. Further, as a result of recording and the continuous contact with enumerators, cooperating farmers tend to become more efficient. This can be overcome in part by constantly changing the sample.

6.3 Sampling Background

Introduction

It is usual to obtain information from only a limited number of farmers and farms from within a population. (A *population* is the total number of farms (units) that satisfy some defined category. For example, all the farms on a particular soil type form a population.) This section contains a discussion on the alternative ways of selecting a sample. There is a large number of sampling methods, many of which are not suitable for farm surveys. Only the more useful are covered. Further, the attributes of a good sample, the sample size necessary to give a defined level of accuracy and the mathematical determination of an optimal sampling technique are all discussed.

The reason for using a sample

A sample is used because the *cost will be less* compared to collecting information from the whole population. However, the situation is not this simple. The money allocated to a survey should be related to the expected gains resulting from the conclusions; thus in some cases it may pay to survey the whole population. Further, the expected *variation between farms* also affects the decision. If little variation exists, a small sample may provide as good an estimate as sampling nearly the whole population. The reason, of course, is with each extra farm added the information obtained will probably change only marginally, if at all. Where extreme variation exists the opposite applies.

Another factor in favour of using a sample is that with a small sample it may be possible *to obtain more accurate information*. More time can be spent with each farmer and, if necessary, they can be revisited to clear up anomalies. In many cases attention to accuracy rather than attempting to cover a large proportion of the population provides better results. Expected sampling errors can usually be statistically assessed, but mistakes in recording are totally unknown. The conclusion must, however, depend on each particular survey and the nature of the population.

Only where there is extreme variation in the information to be collected from the farms and where the results of the survey are expected to have considerable benefit would the whole population be surveyed.

A good sample

A good sample is a group of farms that accurately reflect the characteristics of the whole population. In selecting a sample a method must be used that gives a high probability of a good sample being selected.

When a sample is used, two types of *sampling errors* may be introduced:

- *Random sampling error*. This is the error introduced where a non-representative sample is selected solely due to *chance effects*, e.g. when selecting a sample most of the profitable farms might miss out simply due to chance.
- *Sampling bias*. Where a *sampling technique* is used that gives a non-representative sample, conclusions drawn from the sampled farms will provide an inaccurate estimation of the population characteristics.

Sampling bias is a *constant component of error* in that it will not decrease as the size of the sample is increased. The random sampling error, however, *will decrease as the size of the sample is increased*, as there will be less chance of obtaining a non-representative sample. Further, the random sampling error can also be reduced by selecting *larger proportions of the sample* from sections of the population in which *the variability of the information to be collected is high*. These techniques provide a greater chance of all farming systems being included in the sample.

Some of the methods that give rise to bias are listed below:

- The deliberate selection of a sample using a non-representative factor, e.g. farmers answering an advertisement in a farming paper (this may be unconscious);
- Using a selection procedure that depends on some characteristic that is correlated with particular values of the information to be collected. For example, if members of a farming organization are selected, they are likely to be the more progressive farmers and will give an unrepresentative estimate of population profitability;
- Substituting a member of the chosen sample with another farmer if the chosen farmer is not readily available. The farmer 'not readily available' may be providing excuses as he does not want his poor farm inspected. Thus, every effort must be made to interview the whole chosen sample;
- Failure to cover the whole of the chosen sample. This may lead to bias for the reasons given above.

To avoid sampling bias it is clear that:

- The sample must be selected entirely at random; or
- Selected at random subject to restrictions that ensure bias will not be introduced. Such restrictions are designed to reduce the random sampling error. Examples will appear under the sampling methods discussion.

Where bias is required

PURPOSE BIAS In some surveys there may be a need to introduce bias. For example, the object of the survey may be to determine the farming systems being used by the most profitable farmers. In this case attempts would be made to include only the more highly profitable firms in the survey. However, attempts must still be made to obtain a good sample from within the population of profitable farms.

ACCEPTABLE BIAS Where the object of a survey is to measure the relative differences between groups, bias may be acceptable provided it gives constant error. For example, where the survey requires farms to be ranked in order of profitability a constant bias will not affect the ranking. However, it is extremely difficult to decide whether possible bias affects all groups in the same way. Thus, it is usual to attempt to prevent bias even where a constant bias is acceptable.

Sampling methods

The more common alternative sampling methods are listed below.

Simple random sampling

The required number of farms (units) are selected at random from a list of all the farms in the defined population. Each farm has an equal chance of selection. By the 'defined population' is meant all farms within a particular grouping. For example, the object of a survey may be to estimate the average level of indebtedness per stock unit on fat lamb farms on Lismore soils in the 62–70 cm rainfall belt. Thus, the defined population is all farms satisfying these conditions. Computer programs are readily available for random selection; each farm is given a number which is assigned to the random number generation system and a selection is then made.

Stratification with a uniform sampling fraction

The population is divided into groups, or strata, according to factor/s such as farm area and soil type, and then an *equal* number of farms is randomly selected from each strata. The total number of population units in each strata may not be the same.

Stratification is a technique to reduce the random sampling error. There is a greater chance of the sample being representative as the strata ensure there will be at least a certain number of farms from each group. In simple random sampling there is no guarantee this will occur. As shown below, there are several alternative methods of creating strata, each of which will be appropriate for certain requirements. In general, the principle is to have the members of the strata as homogeneous as possible. Thus, given sufficient strata, the sample selected must be representative.

Multiple stratification with a uniform sampling fraction

This is the same as the previous method except the population is divided into strata according to *several* factors before a sample is selected. For example, groups of farms with a similar area *and* stock system might constitute the strata. Thus, if six area and four stock system classes were constructed, there would be 24 strata. Whether multiple stratification should be used depends on the number of factors correlated with the information being collected. However, considerable prior knowledge is required before extensive multiple stratification can be used.

Stratification with a variable sampling fraction

If the information collected exhibits a different level of variability between strata the random sampling error will be reduced if more farms (or sampling units) are selected from the strata exhibiting the greatest variability. To give the greatest accuracy, the number of units selected from each stratum should be proportional to the variability. That is, where $f_1, f_2, \dots f_n$ represents the proportion of the total sample taken from the 1st, 2nd, … nth stratum, and the standard deviation of the factor being determined from the survey in each stratum is $\sigma_1, \sigma_2, \dots \sigma_n$, then the number of units selected from each stratum should be such that:

$$\frac{f_1}{\sigma_1} = \frac{f_2}{\sigma_2} = \dots = \frac{f_n}{\sigma_n}$$

Such a sampling method requires prior knowledge of σ, the standard deviation. This may come from previous surveys, or a small pilot survey. Where estimates are not available, the variability of the factor used to create the strata can be used (e.g. farm area) under the assumption that this factor will be correlated with the variable being estimated from the survey (e.g. profitability).

In many farm surveys, however, several variables are being estimated. Thus, a decision must be made on which to base the selection. Alternatively, a weighted combination of the standard deviations of each variable could be used, with the weights depending on the importance of each. Given a strong correlation between two variables, only one need be used in estimating the proportions.

Sampling from within strata with the chance of selection proportional to the size of unit (or some other variable)

This is a special form of variable proportion sampling. Where the factor being estimated is positively correlated with, for example, farm size, the accuracy of the estimate will be improved if a large proportion of the sample is selected from large farms. If a survey, for example, is for estimating the total assets of farms this method will give greater accuracy as the larger farms contribute proportionately more of the total assets. One method of obtaining a sample is assigning random numbers in proportion to size so larger farms have a greater chance of selection.

Sampling from within strata with the chance of selection proportional to the size of unit

This is the same system as above, except the proportional sampling occurs from within strata. This may be of use where several factors are important to the result. For example, where total farm assets are being estimated, the population could be divided into farm type strata before the selection based on farm size.

Multi-stage sampling

This involves defining a number of groups of farms on the basis of a factor and randomly selecting a number of these *groups*. Using the farms in the selected groups, another set of sub-groups would be constructed within each selected group on the basis of a factor and further random sampling carried out. The process is continued for as many *stages* as required.

This method has little use, and may in fact introduce bias. This will occur where there is a correlation between the variable/s being estimated and the factor used to provide the, mainly, first group. By chance, the more important groups may not be included in the sample.

The only argument for this method is where the first-stage groups are based on locality. This permits the concentration of field survey work to a smaller area, thus reducing costs.

Multi-phase sampling

In some cases the variability of the information being collected varies. For example, if information on stocking rate and breed of animal is being collected it is probable that stocking rate will exhibit considerable variability compared with breed of stock information. Thus, in order to reduce costs, multi-phase sampling can be used. This involves collecting the extremely variable information from the whole sample and other, less variable information, from part of the sample and so on. In the example given, to get an accurate estimate of stocking rate the information would be collected from all farms whereas it would only be necessary to collect breed information from a small sub-sample to give an accurate estimate of the most important breed. To obtain the sub-sample the required number of units would be randomly selected from the major sample. This method can be combined with stratification techniques.

Procedures for surveys on successive occasions

Where a survey is repeated at regular intervals, there are a number of alternative systems that can be used on each occasion. These include:

- use the same units on each occasion;
- select a completely new sample on each occasion;
- replace part of the sample on each occasion.

The best system depends on whether the surveyed farms change their methods and techniques as a result of being surveyed. Being surveyed often provides stimulation, giving rise to change.

Further, rather than carry out an extensive survey on each occasion, small samples might be used in most years and a major survey carried out, say, every 5 years. The small samples will indicate changing situations and, if the change appears to be large, a major survey can be conducted. With the continuous information that this system provides, it should be possible to estimate the sample size necessary to give a certain degree of accuracy as prior knowledge is available.

Sampling method to use

The discussion indicates this must depend on:

- the accuracy required;
- the degree of variability existing in the population;
- the extent of the information currently available on which to base the selection of the sample;
- the funds available for the survey.

For further information on sampling, refer to Yates (1981).

Size of sample

General comments have been made that the size of sample depends on the level of accuracy required (to obtain perfect accuracy, of course, the whole population must be sampled). This section will quantify these comments.

The *standard error* of the variable/s being estimated indicates the extent of likely random sampling errors. The standard error (the name given to the *estimate* of the true population standard deviation of a sample statistic) indicates the range over which the true population value is likely to lie. For example, given a normal distribution there is a 95% chance that the true population value will lie within $X \pm 1.96\sigma_X$ where X is the sample estimate. Given a small standard error, greater confidence can be placed on the sample estimate as the range over which the true value could lie is much smaller.

Where the size of the population is infinite, the standard error of the sample statistics (e.g. mean debt/stock unit) is given by:

$$\text{Standard error } = \sigma_{\bar{x}} = \frac{\sigma}{\sqrt{N}}$$

where

σ = true population standard deviation of the statistic;

N = sample size;

$\sigma_{\bar{x}}$ = the standard deviation of the mean of the sample statistic estimated from the sample (a standard deviation for a mean exists, as many estimates can be made from repeated samples).

Clearly, the larger the sample used, the smaller will be the standard error. This formula can not be used, however as σ is not known.

Where the population being sampled from is finite (the usual case in farm surveys), the standard error must be proportional to the percentage of the population included in the sample. Thus:

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{N}}\sqrt{\frac{P-N}{P-1}}$$

where P = the number of units in the population.

For very small populations this reduces to:

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{N}}\sqrt{1-F}$$

where F = sampling fraction = N/P

Thus, the 95% confidence limits for the sample mean of a population statistic will be:

$$\overline{X} \pm 1.96 \frac{S_x}{\sqrt{N}}\sqrt{1-F}$$

where S_x = standard deviation of the statistic (x) estimated from the sample information, and the *best* estimate of S_x is given by:

$$S_x = \sqrt{\frac{\sum (X-\bar{X})^2}{N-1}}$$

where

X = possible values of the sample statistic;

$\bar{X}$ = mean of the statistic estimated from the sample.

Note that the normal distribution can be used to give the confidence limits as, by the central limit theorem, the distribution of the *mean* of a sample statistic approaches normality given many estimates of the mean from repeated samples.

Thus, it can be seen the larger value of F, the smaller will be the sampling error. That is, a large sample gives a large F which gives a low value of $\sqrt{1-F}$. Where $P = N$ then $F = 1$ and the sample (whole population in this case) mean of a statistic is the true population mean, i.e. the standard error will be 0.

The relationships show how the sample size can be estimated to give a defined level of accuracy. A decision is made regarding the maximum acceptable standard error and then solve for N in the standard error equation. This procedure, however, requires prior knowledge of S_x. A pilot survey may be necessary to obtain this if no indications are available from previous surveys.

As an example, consider estimating the mean profit of a population of farms with a tolerance of ± \$1000 with 95% confidence. The sample size necessary will be that N giving:

$$1000 = 1.96 \frac{S_x}{\sqrt{N}}\sqrt{1-F}$$

In many farm surveys, an estimate of more than one population statistic will be required. In this case the sample size can be determined to give the required accuracy for the most variable factor or variable and:

- Information for the less variable factors is collected from sub-samples, the number of units depending on the variability; or
- Information is collected from all units and a greater accuracy than is required is accepted for the factors exhibiting a lower variability.

Effect of the sampling method used on the standard error

Given the value of a population statistic is to be estimated (e.g. mean profit) with a given degree of accuracy, the least cost sampling method providing this accuracy should be selected, or, given certain funds, a sampling method should be selected that will give the lowest standard error within the budget. This section will show how to estimate the standard error for a stratified random sample compared with a simple random sample. This indicates *the general methods used in determining an optimal sampling method*.

Consider the estimate of a population statistic using a stratified random sample. The mean value of a *population* statistic is given by:

$$\mu = \frac{\sum P_s \mu_s}{P}$$

$$= \sum \alpha_s \mu_s$$

where $\alpha_s = \frac{P_s}{P}$

μ = population mean for the particular statistic;
μ_s = mean for the Sth stratum;
P = population size;
P_s = number of population units in the Sth stratum;
N_s = size of sample taken from the Sth stratum.

The estimate of the mean calculated from the sample will be given by:

$$\overline{X}^* = \alpha_1 \overline{X}_1 + \alpha_2 \overline{X}_2 + \ldots\ldots$$

$$= \sum \alpha_s \overline{X}_s$$

where $\overline{X}^*$ = estimate of the population mean;
$\overline{X}_s$ = mean obtained from the Sth stratum.

The standard error of $\overline{X}^*$ is given by:

$$\sigma^*_{\overline{X}} = \sqrt{\alpha_S^2 \, \sigma^2_{\overline{X}_S}}$$

where:

$$\sigma^2_{\overline{X}_S} = \frac{\alpha_S^2}{N_S}$$

and the best estimate of σ_S

$$= \sqrt{\frac{\sum (X_S - \bar{x}_S)^2}{N-1}}$$

The question is whether, for a given N, this standard error will be less than a simple random sample standard error. It is clear that the number of units sampled from each strata will affect the standard error of estimate. Before a comparison can be made, the number of units in each strata to give the minimum standard error must be estimated. It can be shown that the optimal sampling fraction is given by:

$$N_S = \frac{\alpha_S \sigma_S}{\sum \alpha_S \sigma_S} N$$

This formula indicates the important factors determining an optimal N are:

- the within stratum standard deviation (the greater the variability, the greater N_s should be);
- the importance of the stratum in relation to the whole population, i.e.

$$\alpha_S = \frac{P_S}{P}$$

In general, the following rather obvious relationship between $\alpha_{\overline{X}}$; (standard error for a simple random sample) and $\sigma^*_{\overline{X}}$ (standard error for a stratified random sample) can be stated:

$$\sigma^*_{\overline{x}} < \sigma_{\overline{X}}$$

if either:

- the population can be stratified into groups that have different means; and/or
- the population can be stratified so the within strata standard deviations have different values.

In most cases these conditions will hold so that stratification is a preferred sampling method. However, optimal stratification requires prior knowledge, which, in many surveys, is not available. However, stratification can also occur when the results are analysed to improve the clarity of result presentation.

Estimates of profitability, for example, might be presented for farms with a total investment greater than, and less than, a certain level, and so on. Similarly, standard error estimates can be made for each group so it is clear for which groups the estimates are more accurate.

6.4 Questionnaire Design and Response Errors

Introduction

No matter which method is used to collect the information, some kind of questionnaire will be necessary to list and record the information. The detail in the questionnaire should depend on the problem, the method used to collect the information and on the experience of the enumerators. Questionnaire design is an important aspect of survey work, as poorly constructed questionnaires will give rise to errors (bias). This section will contain a discussion of the principles to be followed and on response errors, or bias, as this can in part be associated with questionnaire design.

The alternative questioning procedures include the following.

Question form

There are two forms:

1. Mutiliple choice questions. All possible answers to a particular question are listed and the enumerator (or the farmer, depending on the information collection method) simply ticks the appropriate answer, e.g.

What is the breed of cow?

1. Jersey	x
2. Friesian (Holstein)	
3. Other	

2. Open-ended questions where possible answers are not provided. The answer is recorded, e.g.

1. What is the breed of cow?
Answer: 'Jersey'

For many questions, open-ended questions will be necessary as the possible answers are too numerous (e.g. what was the size of the ewe flock on 30 June?).

Question asking method

The two possibilities are:

- The question is asked precisely in the form it is written in the questionnaire. This is referred to as a *set form* question.
- The enumerator decides the exact wording as the interview proceeds. The questionnaire may contain a suggested question form.

The best method depends in part on the experience of the enumerators. Experienced enumerators will be able to obtain better answers through a discussion process whereas set form questions should be used if the enumerators are not professionals. Where *opinions* are being obtained, however, set form questions are preferred as variation in the way questions are asked may provide bias (assuming you know all the possible opinions that might be held).

Question wording and types of questions

No matter whether set-form or enumerator-worded questions are asked, the following principles should be adhered to:

- Familiar words and phrases must be used.
- Questions must be precise and unambiguous.
- Questions must be phrased in specific terms.

- Questions must be 'non-emotional'. For example, if possible avoid asking a farmer about drought effects if he is currently experiencing severe difficulties due to an extreme drought.
- Questions must not encourage defensive attitudes; e.g. Question: 'Research has shown that the use of X kg/ha of N is profitable. Why haven't you adopted this practice?'
- Leading questions must be avoided. An answer should never be suggested. To appear normal the farmer will tend to give the suggested answer. Unfortunately, an interviewer sometimes suggests answers simply to speed up the process. This also tends to happen when advisers inspect a property.
- Where possible, hypothetical questions should be avoided as the answers will be of limited use unless the farmer has actually had some experience of the topic; e.g. 'How would you develop your property given $*X*?' Having never had $*X* the farmer has probably never considered the possibility.
- Where possible, questions that are quite personal should be avoided as they may embarrass the farmer, or receive a censored answer; e.g. 'What is the amount of family money you have borrowed and at what cost?'
- Questions should obtain basic facts rather than summary information that relies on the farmer's own calculations; e.g. rather than ask the farmer for his average wheat yield, obtain the yield paddock (field) by paddock (better still, the total quantity harvested/paddock) over a number of years. Farmers tend to forget the poor years and poor paddocks in calculating their averages.
- Introduce check questions to verify the answers given; e.g. on a dairy farm ask: (i) How much hay do you feed per cow? Then later in the questionnaire ask: (ii) How much hay do you make and purchase? Knowing the cow numbers, the answers enable a consistency check. If the answers are not consistent the farmer can be re-questioned. If possible, such checks should be compared during the interview so that anomalies can be cleared up.
- Combine as many questions as possible into one question; e.g. instead of asking '(i) Is winter feed purchased?' '(ii) How much winter feed is purchased/cow?', ask only the second question as the answer will provide the answer to both questions.
- Before including a question, efforts should be made to determine whether the farmers are likely to have the required information. Farmers (and others) tend to make guesses rather than say they do not know.

Question ordering and questionnaire format

To minimize the time required to both fill in a questionnaire and the eventual analysis time, and to obtain correct answers, the following principles should be followed:

1. The first questions must obtain identification information, e.g. farmer's name, address, telephone number/email (for possible re-contact), the group of farms that his farm falls into if several groups are being surveyed, and other identification and background information.

2. The next set should be simple and of interest to the respondent. This creates confidence and interest.

3. Questions requiring the maximum cooperation must be placed next. They should not appear at the end of the questionnaire as by this stage the respondent may be bored and not prepared to look up information.

4. Questions must be arranged in logical order within each section, e.g. all stock/animal questions should be grouped together so that the farmer will be thinking about stock matters. Further, he can make use of any records produced at this stage without having to re-find them. Following the stock questions, feeding questions might be posed.

5. The questionnaire length should be kept to minimum. As a rule of thumb, a maximum time of 3–4 h should not be exceeded. If this is necessary, two sessions will usually provide greater accuracy.

6. In respondent-completed questionnaires (mail surveys), instructions should accompany each question rather than be on a separate sheet.

7. 'Personal' questions should be included at the end of the questionnaire, so that any reaction will not affect the answers to other questions.

8. The areas on the questionnaire provided for recording should be designed to allow easy extraction of the information, e.g. figures that need to be added up should be entered in columns.

9. The physical characteristics of the questionnaire must ensure ease of use, e.g. (i) it should fit easily onto a workboard; (ii) the paper must be easy to write on; (iii) it should be as compact as possible in order to minimize the amount of page turning when checking back.

Response errors or bias

Another source of error (besides sampling bias) is response bias. This comes from incorrect answers ending up in the questionnaire. The origin of this bias can be:

- Enumerator introduced;
- Questionnaire introduced. To reduce this bias to a minimum, the questionnaire must be constructed according to the principles outlined and carefully pre-tested;
- Respondent (farmer) introduced.

Enumerator-introduced bias

Some possibilities are:

- The enumerator mis-records an answer. In general, these errors tend to even out over the whole sample.
- The personality and opinions of the enumerator affects the answers given, e.g. the approach taken by the enumerator will affect how seriously the

farmer takes the survey. A flippant attitude, for example, will encourage the farmer to provide unconsidered answers.

To minimize this source of bias the enumerators must be adequately trained and, where possible, have a good personality and character.

Respondent-introduced bias

This will occur where:

- The farmer 'deliberately' gives incorrect information. This action may be subconscious. For example, farmers tend to remember their good lambing percentages etc. In some cases farmers may 'improve' their achievements to save face.
- The farmer does not have the knowledge or records necessary to give a correct answer, so a guess is provided.

This form of bias tends to introduce a *systematic error* as 'wrong answer' questions tend to be the same for all farmers, e.g. a group of pedigree breeders may indicate a lower level of purchased feed use than they in fact use to make the animals appear more efficient than in reality.

Little can be done to overcome this bias except by:

- Using good question design, and checking to see if the farmers have the information necessary;
- Using check questions;
- Checking general answers with previous surveys and advisers in the area.

6.5 The Organization of a Survey

The steps for carrying out a survey, in general order of occurrence, are as follows.

Defining the problem/s to be solved (formulation of the hypothesis)

In management work the problem is often quite general. For example, the problem may be to determine the most profitable farming system from those currently used, or perhaps a hypothesis that a particular farming system is the most profitable needs testing. In other cases the problem can be more open ended, e.g. determining the minimum farm size for economic viability.

Given a clear problem statement, this dictates how the survey should be carried out.

Define the study region or locality

This may have been defined above, as the hypothesis often relates to a particular area. The selection may be affected by the availability of information, particularly information required in selecting a sample. For example:

- soil survey information and soil maps;
- rainfall and other climatic information;
- topographical maps.

Define the type of farms to be included

A region will probably have a range of farm types, so those to be included must be defined. This may have been achieved when the problem was first defined. Some of the questions to be answered include:

- the farm sizes to be included;
- the product types to be included;
- whether part-time farms are to be excluded;
- whether absentee owner farms are to be excluded.

Define the information to be collected

The problem to be solved dictates the general nature of the information, but a decision must be made on:

- the specific items of information to be collected; and
- the years of information to be collected.

To determine the specific items it is useful to carry out a hypothetical calculation working backwards from the final figures that the survey must provide. This defines the information required.

Due to the chance effects of weather and other random variables, for many problems it is necessary to collect information for several years to provide an accurate average. A common number of years is 3. In selecting the specific years abnormal years should be excluded, though there is a limit to exclusions as farmers' records and memories may not allow going back far. Where the objective is to record what the current situation is, this procedure may not be necessary.

Define the method to be used in collecting the information

Given the information required, currently available statistics would be examined to determine the additional information to be collected from the farmers. The type and quantity of this remaining information will then dictate which method to use.

Prepare and test the questionnaire

The questionnaire should be pre-tested on a small sample of farmers before it is finally printed. This pre-testing enables:

- testing of alternative question forms, and checking farmers' understanding of the questions and ambiguities;
- determining whether the farmers have the required information;
- testing whether the enumerators are likely to have any problems with the questionnaire.

Determine the sampling method

This should occur earlier in some cases, for there is an interaction between question form, sample type and collection method. This may involve:

- using previous surveys to obtain information on which to make these decisions; or
- carrying out a small pilot survey to obtain this information (local advisers may also be questioned).

The analyst's experience of the area and farmers may also suggest answers.

Obtain the information for selecting the sample

This information includes:

- the names of farmers in the locality;
- the location (address) of these farms and possibly the farm area;
- possibly the type, or class, of each farm.

The first two items above are essential. Possible sources of this information are:

- organizations responsible for valuing properties for local tax purposes;
- local government lists of household and farm owners;
- cooperatives and other producer boards/committees;
- farmer organizations;
- government departments responsible for lists of land titles and legal descriptions.

Select the sample

Select and train the enumerators

Arrange and test the field organization

A pilot survey may be necessary, or at least the survey should start with a small number of farms so that changes can be made if necessary. Details such as the allocation of defined areas to each enumerator, the enumerator's accommodation and the availability of supervisors to solve any problems as they arise, must all be considered. For mail and telephone surveys some of these tasks will have a different form.

Advertising the survey in the relevant areas

This process must indicate:

- the purpose of the survey;
- what the farmers might gain from the results of the survey;
- the organization responsible for carrying out the survey;
- the nature of the cooperation necessary if they are selected;
- that *all* information collected will be treated as *confidential* and their names will not be directly associated with any of the results (unless there is a special need for identification, e.g. perhaps supporting grants are available for qualifying farmers. In any special case the farmer's permission should be a prerequisite for publishing identifiable information);

- the ethics committee checks that have been carried out (most research organizations, and countries, have social science ethics committees).

Possible advertising methods include:

- letters sent to all selected farmers giving detailed information;
- general information on the internet, radio, television and in the local press;
- talking at farmer organization meetings.

Conduct the interview

The principles to be followed include:

- pre-arrange farm visits;
- interviewing in the farmer's slack time of the year, and in the slack time of day, e.g. not at milking time;
- interview, if possible, when the information is fresh in the farmers' minds, e.g. just after harvesting for a survey to obtain crop input/output information;
- interview first the farmers considered to be the district leaders. The best enumerators should be used. These farmers will tend to influence others on whether to cooperate;
- the enumerator must initially attempt to create a good relationship. This may involve general farmer talk on current topics and a brief look around the farm to see aspects in which the farmer is particularly interested;
- where possible, arrange for a local adviser, or a leading farmer (popular one), to initially go around all the farms and introduce the enumerator. Farmers' meetings may also be used for this purpose.

Obtain advice

Advice should be sought for all phases from survey experts through to statisticians. When constructing the questionnaire and selecting the sample, advice on statistical methods should be sought to ensure a proper analysis is likely to produce statistically significant results. If a sophisticated statistical analysis is planned it is most important to test the procedure to ensure the data collected will allow significant results. Of course, the nature of the problem being solved will dictate to a certain extent the type of analysis that will be appropriate.

Data coding

Before the information can be extracted from the questionnaires and the calculations performed, the following procedures are necessary:

1. The questionnaires must be examined to check on their completeness and suitability.

(i) Questionnaires with insufficient information (though questions can sometimes be completed from other answers such as those given for check questions) must be discarded, or returned for completion (enumerators should check completion at the time if possible).

(ii) The farms that do not satisfy the initial requirements (e.g. part-time farms) must be discarded.

2. The accuracy of the answers must be checked using the check questions, and where required, farms re-visited (a phone call may solve the problem).

To simplify analysis some non-numerical items may need to be *coded* with a numeral. For example, where a question asks about the ewe flock replacements the answers might be coded as follows:

	CODE
Breed replacement stock	1
Buy ewe lambs	2
Buy two-tooths	3
50/50 Buy/rear combination	4

The major problem in coding is in ensuring sufficient categories are created and logically assigning codes.

In some cases numerical information may need to be divided into groups. In this case each group would be given a code, for example:

Farm area (ha)	CODE
0–100	1
101–200	2
201–300	3
...	

Note that, where possible, a coding system should be logical as in this example of farm areas, the bigger the area the larger the numerical code. In an opinion survey the same applies so, for example, if the farmers were asked to express their belief in the value of a newsletter, code 1 might be used for ‘no value’ through to, say, 5 for ‘highly valuable’. It would then be possible, for example, to develop an equation predicting value based on, e.g. the content types. A scale like this is known as a ‘Likert scale’.

Data analysis

Introduction

Little can be noted about this operation as the calculations required will depend on the particular survey. In most surveys a computer would be used to carry out the tedious sorting and calculations, thus the need to give numeric codes in general. Where a computer is used the information extracted from the questionnaires would be entered straight into a database or spreadsheet from which most analysis packages will accept data.

The most important step is to ensure that *adequate statistical techniques* are used in testing and describing the results so it is clear where significant differences occur for comparative work (for a text on the methods see, for example, Keller, 2008).

Surveys can generally be grouped into the following classes:

1. Surveys to obtain descriptive information of a population of farms (e.g. What is the average level of indebtedness? What is the expected yield of wheat if *X* kg superphosphate/ha is applied? What is the cost of production per litre of milk produced?);
2. Surveys to compare groups of farms (e.g. which of the following systems gives highest profitability: (i) Rearing stock replacements; (ii) Buying stock replacements; (iii) … etc. And which farm size group is most efficient?)

In this example the farms would be grouped according to their stock policy and then averages for each group of farms for each statistic calculated. The comparison will then indicate whether the different groups have different profits, or whatever criteria are being used. However, before a conclusion is possible, statistical tests are necessary to decide whether the differences are significant.

Also note that rather than comparing discrete groups, multiple regression techniques might be used on all the data to determine the relationship between, for example, profit and the various inputs. Thus, a whole farm production function is determined. This approach will be discussed further in the next chapter.

Analysing data in descriptive-type surveys

As a sample has been used in contrast to the whole population, average statistics calculated are only an *estimate* of the true population figure. To give some idea of the accuracy of the estimate the *standard error* must be calculated to allow the estimation of *confidence limits*. Further, it is useful for frequency distributions to be presented, indicating the likelihood of the estimate of the mean value. For example, where a survey has shown that the average indebtedness per stock equivalent is \$20, this information is misleading where the frequency distribution is skewed like that shown in Fig. 6.1.

Analysing the data in group comparison surveys

Drawing conclusions about group differences (e.g. different profitability) based on *average* statistics can give incorrect conclusions. Any difference that occurs may solely be due to chance, with the samples having been selected on a chance basis. Thus, group differences must be statistically tested *for significance* by the use of t, F and chi squared tests. As the random variable 'weather' affects results, tests must be carried out to ensure that group differences are due to farming system differences and not simply due to weather and other chance effects.

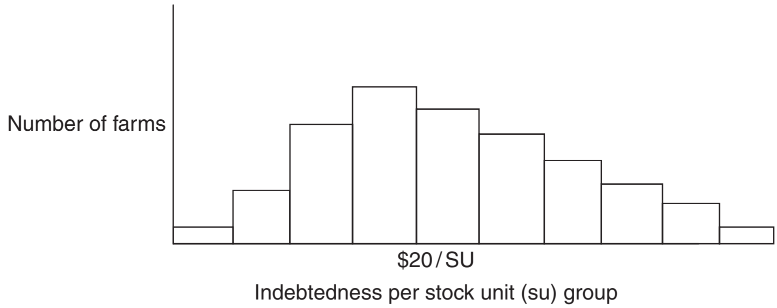


Fig. 6.1. Distribution of indebtedness levels per stock unit.

It must be heavily emphasized that when comparing farming systems only farms in *similar environments and on similar soils* should be compared, as otherwise profit difference will be due to more than the different management systems (a confounding factor here is managerial ability variations as it is difficult to measure). Similar comments apply when descriptive-type survey results are being presented.

In surveys using farm accounts it is usually necessary to adjust the data before use, as accounts give taxable income in contrast to the true profit. Thus, for example, the accounts may contain repairs and maintenance costs, which are really development expenditure (e.g. new fencing for a new irrigation system), which should be removed from the operating costs. Similarly, in some tax legislation assets are valued with the difference between the year starting and ending valuations being taxable. Often the valuations are not market based, so an adjustment is necessary to ensure the values are true market values. Another adjustment sometimes necessary is the depreciation charged where the rates allowed for income taxation are unrealistic. Effectively, taxation accounts must be adjusted so the profit shown is a true reflection of the economic surplus.

Calculation bias

A further form of bias can be introduced where systematic calculational errors occur. This might be due, for example, to an assistant consistently adding up the wrong columns when entering data, or, as another example, the computer program used might have an error.

This type of bias or error can be completely overcome by rigorous checking of methods, results and calculational systems.

Additional analysis methods

Modern statistical packages, such as the Statistical Package for the Social Sciences (SPSS), provide a means to check, sort, fill in missing information and analyse data in many ways. Mention of some of the techniques will be given in later chapters, but the details can be found in most modern econometric and statistical texts (e.g. Hair *et al*., 2005). Mention has already been made of regression analysis in which both linear and curvilinear equations explaining the relationship between a dependent variable (e.g. net profit) and independent (decision) variables, such as all the farm inputs, can be developed. The estimated equation can then be used to find optimal input levels and output mixes. The raw data would come from, of course, a survey of the farms. Such data can also be used to estimate what is known as the efficient frontier, which is then used to produce benchmark data for comparative purposes. The efficient frontier is a production possibility curve defined by the farms which produce the most output for given inputs, or use the least inputs for given output. Various statistical techniques allow this frontier to be estimated and to identify the farms which define the frontier. These farms provide the benchmark statistics for comparison with all other farms, with the differences being used to suggest ways of improving efficiency.

Special forms of regression analysis might be used in specific cases. One example is *logit analysis*, which is used when the dependent variable can take on one of two values, e.g. did a farmer use a particular technique, or not (yes/no). Indeed, any decision situation where the outcome is one of two possibilities requires logit analysis to ensure the most accurate and lowest variance equation parameters. Logit analysis is readily available in most computer packages, as is probit analysis, which is similar.

Another technique that might be useful is *factor analysis*, in which a large number of variables are analysed to find unobserved core variables (linear combinations of the observed source variables). This process reduces the number of variables that need to be introduced into, e.g. a regression analysis. For example, 20 variables on attitudes to extension systems might be collected from farmers, so a factor analysis might show that there are only, say, four core variables expressing the farmers' core values; these might be, for example, visual approaches, group meetings where discussion takes place, written material to take home and follow-up from experts.

Related to factor analysis is *cluster analysis*, which examines each farm's statistics to find groups that are similar for specified variables. The farms in each cluster might then be compared, for example, to explore different management approaches and the efficiencies they confer. And finally, but certainly not least, is *structural equation modelling*, which is used when it might be hypothesized a dependent variable of interest (e.g. net profit, or perhaps leisure hours) is generated from several base variables (e.g. managerial ability, farmer's experience, soil quality and net assets), which in turn are products of even more basic variables (e.g. farmer's education, whether he is raised in the country, soil depth, soil type, rainfall total and pattern, mortgage type and years on the farm etc.). Structural equation modelling (SEM) computer packages take the survey data and estimate the parameters of each of many equations connecting each set of variables. The results then allow working out the important variables that give rise to the dependent variable and, consequently, provide a conclusion on their desirable values (e.g. having high level education, years of experience etc.).

It was noted earlier that surveys can be used to answer a very wide range of problems, many of which are related to developing improved farming systems. Each use leads to a unique survey and method of analysis. The discussion has outlined both the general procedures that must be followed and introduced some of the more common methods of analysis. However, it is always valuable to study specific examples to further enhance farm survey understanding. The literature abounds with examples and should be searched for previous studies of a specific problem and hypothesis that an analyst might be concerned with. For an example of a simple analysis, consider Nuthall (2009a), for the use of regression and factor analysis Nuthall (2002) and for SEM Nuthall (2009b). However, a search of the World Wide Web will provide examples of very many other situations and examples of the questionnaires used. There are many templates available.

6.6 The Limitations and Problems of Farm Surveys

Introduction

Given a problem, the researcher/adviser must decide which of the available problem-solving techniques should be used. For some the survey method is a possibility. For example, the problem may be deciding whether irrigation is profitable and which irrigation technique is best. This problem might be solved using a survey *or*, for example, doing a series of budgets to compare the alternatives. However, to do a series of budgets the basic input–output information must be known, possibly necessitating the use of a survey. The problem-solving technique to use will, of course, depend on:

- the costs involved; and
- the expected improvement to farming systems in an area as a result of carrying out the study.

For some problems there are no alternatives to the survey technique. For example, determining the *existing state* of a population of farms, a descriptive-type survey, is unlikely to have an alternative. Similarly, there is no alternative for determining actual cost of production estimates.

In deciding whether to use the survey technique an understanding of the limitations of farm surveys is clearly necessary, as is a knowledge of the alternative methods. Survey limitations and problems are given below.

Limitations and problems

Survey information is historical

When survey conclusions are used to decide *future* courses of action it is largely assumed future conditions will be similar to the time of the survey (prices, costs, technology etc.). In many cases, however, the historically best farming system may need changing, though where farms mainly produce a single product the survey conclusions may apply as product price changes would have to be large to justify a change in products. Whatever the case, care must be exercised in deciding the validity of the information. For example, a survey right at the beginning of a wheat season to predict wheat production will very likely provide an inaccurate prediction. Constant updates are required, if affordable, or the survey should be conducted as late as possible to provide useful information. The important point in all surveying is recognizing the likely inaccuracies and adjusting accordingly.

Where historical information *is* the objective, clearly no problem exists. For example, a survey to estimate *current* debt levels. Similarly, if the objective is to obtain physical input–output information to be used with predicted prices and costs to arrive at contemporary conclusions, the historic nature of the data is not a problem as these physical parameters are unlikely to change to any great extent.

Profitability studies and alternative systems

When a survey is used to determine improved farming systems it is assumed there are an appreciable number of farms actually operating 'improved' farming systems. While there will always be a *range* of farming systems, the question is how good are the better systems relative to those lying on the possibility boundary. The best answer obtained is only as good as the best system found on the sample farms, so if the leading farmers are better decision makers than the consultants/researchers in a general sense, then this approach is acceptable. However, in many cases this assumption is unlikely and conclusions will not be so good as those available from more prescriptive research techniques. But where the local professionals *are not well trained in these techniques* there is no choice but to use a survey, though a series of simple budgets may be more useful than survey conclusions. However, note that a survey may provide suggestions that enable, through budgeting, an improved system to be developed.

Farm group homogeneity

A major problem is ensuring that homogeneous populations are defined and located, particularly in comparative studies. Where environmental homogeneity (e.g. soils, climate, topography) does not exist, variation between farming systems will be confounded by variations due to the environment. Prior to a survey there is, in many cases, insufficient information to ensure homogeneity, so grouping into similar categories must occur at the analysis stage. Alternatively, in developing equations to explain variations, variables reflecting environmental categories should be included (e.g. rainfall, soil quality).

Accuracy of the information available

Like most things in life, the results are only as good as the input. Accuracy will depend on:

- how well the farmers keep records, and the percentage of the population keeping consistent records; and
- the accuracy of farmers' memories.

In some cases the only reasonably correct source of information is the set of farm accounts. Clearly, many potentially useful surveys are limited in scope by the information available, thus, in some cases, the need for special recording schemes.

The influence of managerial ability and labour quality

Given two farms operating the same general farming system, their profitability may be different due to the effect of managerial ability and the quality of the labour. Thus variations in managerial and labour ability may confound the analysis. The way to overcome the problem is to measure ability and compare alternative systems for groups exhibiting similar levels, or having a variable for ability in any equation used. However, measuring ability is difficult, and even then there may be insufficient farmers in each similar ability group to give statistically acceptable comparisons. In most surveys managerial ability and labour quality is ignored, so small differences in profitability between farming systems must be considered in this light. This problem of accounting for managerial ability is not, of course, peculiar to survey work. Recent work on explaining the origins of ability may eventually lead to improved measurement (see Nuthall, 2009b).

The fallacy of averages

Results presented as simple averages are not particularly informative about the nature of the population. Depending on the problem, an analyst must consider the best method. Remember simple averages may suggest different farming systems have different profit levels – but the differences may simply be the result of chance. Erroneous conclusions will be prevented through using appropriate statistical techniques, making clear which of the differences are worth noting. Presenting the parameters of the variable distributions will also most likely be useful, i.e. the mean, standard deviation and third moment giving the skewness.

Data requirement

As noted, to overcome year to year chance effects (e.g. weather, the particular auction the products end up in and all the other random impacts) data for a number of years must be collected. This is often a problem due to the inadequacies of farmers' records, memories and locating copies of farm financial accounts. The farmer's accountant, however, will probably retain copies for several years.

Varying objectives

In a survey that groups farms according to a defined objective (e.g. profit), when recommendations are made it is assumed all farmers will have the same objective. This is clearly incorrect. However, to overcome this problem the farms could be grouped by a number of objectives, so the conclusion might be that farming system B is preferred for a particular objective while system E is preferred for another objective. However, large numbers of farms will be required to produce statistically different results once there are several objectives measured. This problem is not peculiar to survey work.

6.7 Qualitative Surveys

Many surveys will need to obtain qualitative information as part of solving a problem. Often this information is quantified by asking a farmer to rank, or classify, the alternative answers possible. For example, in a survey to assess farmer's management style one statement the farmer is asked to comment on might be:

> For most things you seek the views of many people before making changes to your operations

where the farmer is required to assess the truth of the statement using a score of 1 to represent 'true' through to 5 for 'not true', with the values between representing different levels of truth.

Another question in the same survey might ask the farmers to indicate how much they had learned over their formative years. The respondent could be asked to provide, using a scale of 1 (a great deal) to 5 (not much), an answer to:

> Up to age 20 years, as far as you can remember, how much agricultural knowledge did you successfully learn over:
> 5 to 10 years of age?
> 11 to 15 years of age?
> 16 to 20 years of age?

In this same survey (Nuthall, 2009a), pure quantitative information was also obtained through questions such as:

> How many years have you lived on your current farm?
> What is the total area used including owned, leased and rented land?

Many surveys will have these mixed sets of questions, which in most cases will end up providing quantitative information. The examples given above were designed to discover the origins of managerial ability using a structural equation model. This model required numeric data to solve the proposed equations. For a useful discussion on combining qualitative and quantitative surveys, see Marsland *et al*. (2010). This publication covers the strengths of each approach.

In most quantitative surveys the researchers will have a hypothesis they are trying to prove. This is the classical research approach of having a concept, and then collecting the information needed to assess the hypothesis. Such surveys require prior knowledge, as otherwise a hypothesis could not be formulated.

In complete contrast, a researcher might have a general interest in a situation and, consequently, wish to explore the nature of potential problems and possible solutions. For example, if the thought process farmers go through when making decisions can be discovered it will be easier to devise methods of improving their skills. However, little detailed knowledge is available over the process, so a 'qualitative' approach can be taken.

In this method the researcher explores the situation with a completely open mind. While the literature might be consulted to obtain some initial ideas, this consultation is not for creating a defined hypothesis, and for this reason the approach is often referred to as *grounded theory*. The work is *grounded* in what the farmers' think, not what the previous string of researchers have concluded. It is only after the analyst has surveyed the farmers and assessed what the conclusions might be that the past research is consulted to compare and contrast the conclusions with what appears in the literature.

One of the advantages claimed for qualitative surveys is the depth of knowledge it is possible to obtain. With a quantitative survey a defined questionnaire is usually used so the information obtained is pre-determined to some extent. In contrast, a qualitative survey can be used to explore the depths of a situation where enumerators are trained to probe and explore. Thus, when surveying a farmer with a tape recorder running, an experienced interviewer can ask the farmer to explain any view he might bring up. The answer might then stimulate more unknowns, which can be followed up until all the questions that might relate to a topic are fully explored. The direction of the questioning is dependent on the answers and this might well be different for each farmer. This process, if professionally conducted, can provide information that would not emerge in a formal questionnaire process. The challenge is then to correctly interpret the information recorded.

Even in quantitative surveys it makes sense to explore the situation before creating a questionnaire. Such initial surveys might well be qualitative to discover 'how the ground lies', so to speak. Such an approach gives the researcher valuable background to the situation being explored.

Often, however, this next step of a quantitative survey is not carried out until much later, if at all, so the work is left at the qualitative conclusions. In these purely qualitative surveys often little is known about the population so sampling in the normal quantitative sense is not possible. The approach taken is referred to as *negative case sampling*. Various people in the target community are consulted to obtain farmers' names, which are then initially surveyed. These farmers might suggest further names, and so the process continues until further interviews fail to turn up any new ideas. The conclusion is then that the information collected covers the full range of possibilities. Of course, there is no knowledge of the prevalence of each attitude expressed by individual farmers, but at least there is a good chance that the full range has been determined. A full qualitative survey with statistical sampling would be necessary to conclude on the frequency of each attitude.

Thus, any situation about which prior knowledge is unavailable can be approached using a qualitative survey. The research needs to start with some ideas to provide a questioning schedule as a base for the discussion with the farmers being interviewed. Often a tape is used to record the interviews so it is possible to type up the responses verbatim for subsequent analysis. Where use of a tape recorder is not acceptable or possible, a full set of notes must be made.

Software is available to help analyse the responses once they have been stored in computer files. This software is designed to help isolate the key points, each of which may have sub-components, thus ending up with a tree-like structure of information and conclusions.

For example, in a defined farming area the objective might be to find out something of the production processes used on new crops. Talking to the farmers it might be concluded the fertilizer and irrigation systems are key to the production levels. Thus these two items might form the top nodes in a downward-looking tree. Sub-nodes might then be fertilizer type, quantity and application times, similarly for irrigation.

Other than the idea that the knowledge should stem from the farmers themselves (grounded theory), in contrast to a pre-structured approach in a quantitative survey, and that sampling involves adding cases until no new information is forthcoming, qualitative surveys must be set up and organized to suit each case. Accordingly, a search of the literature for examples can be beneficial to obtaining ideas. However, the nature of the approach means preconceived methods should not be entertained as this may restrict the investigation and findings.

To help obtain a full range of views in qualitative surveying it is helpful, where the information allows, to purposefully select farmers to cover the full range. Thus, for example, efforts might be made to cover the full range of farm sizes found in an area, or perhaps production types and so ensure representation in a similar way to stratified samples.

For a detailed explanation of the methods of qualitative surveying books such as Denzin and Lincoln (2005) are useful.

6.8 Concluding Comments on Research Method and Surveys

When advisers, analysts and researchers are developing improved farming systems there are only two basic approaches that can be used. Namely:

1. A survey to observe the better farming systems used in the population.
2. Using techniques that construct, from first principles, improved farming systems (prescriptive approaches) through modelling. For example, rather than use a survey to determine the better farming systems for, e.g. low fertility soil, a series of budgets on likely systems can be used to make a conclusion. Alternatively, more sophisticated techniques, such as linear programming, could be used to find optimal systems using information on the physical relationships between inputs and output. The point here is that current knowledge on input–output ratios is used, together with expected prices and costs, to devise what is regarded as an optimal system.

This second approach has *considerable advantage* as it does not limit its scope to the farming systems that are currently being used by the farming population. A survey, however, may be required to provide the input–output information necessary for such studies. Further, such studies attempt to estimate improved systems for the expected conditions (e.g. prices, costs, technology). For these reasons this second approach generally has more use in profitability studies than the survey approach, but this will depend on whether:

- it is expected there will be an appreciable group of extremely good decision makers in the population;
- conditions (prices, costs etc.) are expected to change considerably;
- there is currently available an extensive list of input–output information (a knowledge of the production functions);
- the costs involved in the alternative research approaches; and
- the knowledge and experience of the advisers/researchers.

In farming areas where there is little knowledge of the input–output information there may be little alternative to a survey, though this might be informal and designed to obtain from the farmers all the physical planning information required.

Where, however, the objective is to obtain farmers' opinions and views on an issue, there is no alternative to the survey technique.

6.9 Exercises

There are no fixed recipes for conducting successful survey research, though many principles apply as discussed in this chapter. As experience plays a major part in carrying out useful surveys it would be valuable for students of farm management to conduct a group survey exercise. Perhaps students can survey colleagues in other courses, or even a small farm survey, possibly using a postal survey to minimize cost. No doubt local farmers would be keen to help if they will benefit from the results. They may well have some pressing problems that can be solved using a survey.

Another useful exercise is to review a range of survey-based studies reported in the literature. This will provide ideas for possible surveys in the future and guide procedures and analysis.

References and Further Reading

Anon (2010a) Sheep and beef farm survey – revenue, expenditure, profit before tax. Available at: www.meatnz.co.nz (accessed April 2010).

Anon (2010b) Dairy economic survey. Available at: www.dairynz.co.nz (accessed April 2010).

Denzin, N. and Lincoln, Y. (2005) *The Handbook of Qualitative Research Methods*, 3rd edn. Sage Publications, NSW, Australia.

Dillon, J. and Jarrett, F. (1964) Response Patterns in some Australian Farm Economic Mail Surveys. *Australian*

Journal of Agricultural Economics 8, 81–84.

Freebairn, J. (1967) Mail Surveys and Non-Response Bias – Report on the NSW Mail Survey. *Australian Journal of Agricultural Economics* 11, 87–94.

Hair, J.F., Anderson, R.E., Tatham, R.L. and Black, W.C. (2005) *Multivariate Data Analysis*, 6th edn. Prentice Hall, New Jersey.

Keller, G. (2008) *Statistics for Management and Economics*, 13th edn. South-Western Cengage Learning, Ohio.

Marsland, N., Wilson, I., Abeyasekera, S. and Kleih, U. (2010) A methodological framework for combining quantitative and qualitative survey methods. Available at: www.reading.ac.uk/SSC/publications/guides/qqa.pdf (accessed March 2010).

Note: details of the questionnaire used are given in Nuthall, P.L. (2002) *Managerial Competencies in Primary Production. The View of a Sample of New Zealand Farmers*. Farm and Hort. Management Group Research Report, Lincoln University, No. 04/2002.

Nuthall, P. (2006a) Determining the important management skill competencies: The case of family farm business in New Zealand. *Agricultural Systems* 88(2–3), 429–450.

Nuthall, P. (2006b) *Development of Tests for Assessing Managerial Ability on New Zealand Farms*. Farm and Horticultural Management Group, Lincoln University, Research Report 01/2006.

Nuthall, P. (2009a) *Managerial factors in primary production: data from a sample of New Zealand farmers with an emphasis on experience as a factor in success*. Research Report No. 315. Agribusiness and Economics Research Unit, Lincoln University.

Nuthall, P.L. (2009b) Modeling the origins of managerial ability in agricultural production. *The Australian Journal of Agricultural and Resource Economics* 53, 413–436.

Yates F. (1981) *Sampling Methods for Censuses and Surveys* (Ch. 1–5). Griffin, London.

7 Improving Farming Systems Using Survey Data; and Information Systems

7.1 Introduction

This chapter covers the use of survey data in comparative analysis (benchmarking), cost accounting, monetary production functions, efficiency measurement and resource valuation, and comments on information systems.

The previous chapter lists problems and issues that might be solved using a farm survey, and how the data might be analysed. The appropriate method is largely dictated by the purpose of the survey. Benchmarking, or farm standards, was one of the survey uses mentioned. Farm standards act as guidelines for deciding on improved management systems. This chapter contains details of the methods that might be used in creating benchmarks, and provides a critique of their use as well as comments on the most beneficial use. A more sophisticated use of survey data, which has similar objectives, is the estimation of a whole farm production function which relates profit to inputs. This enables calculating input levels to maximize profit. This approach is also discussed. For students interested in the original description and debate on farm standards some of the original references are provided, and similarly for the remaining sections.

The next section covers the use of survey information for cost of production estimates, which is sometimes referred to as 'cost accounting'. The methods are discussed, and comments made on the efficacy of the approach. Thoughts on computerized data collection are also provided, for often a properly designed automatic collection system can provide relatively error free information for analysis (information systems for on-farm decision support). Some thoughts on objectives and measuring efficiency are offered, as any kind of comparative and statistical analyses requires a measureable criterion by which farms can be ranked. As an interesting aside, a discussion on resource valuation is provided as a postscript as efficiency measurement leads to a productive valuation for farm resources. Land, for example, can be given a value based on the profit it will earn – but this relies on correctly valuing all other inputs. Finally, some comments on decision information are offered.

7.2 Comparative Analysis – a Description

The idea behind comparative analysis for improving management systems is the comparison of an individual farm's statistics (e.g. stock units/ha; $ expenditure on labour/$ gross income; $ wheat return/ha etc.) with the statistics of comparable farms. This leads to suggestions on how the particular farms' systems might be improved. The idea of a farm standard, or benchmark as they are often called, is a simple extension of these comparisons. A *farm standard* is defined as a statistical value which is associated with a successful farming system on farms with defined characteristics (soil type, climate etc.). It is usual to average the statistics of the most profitable, say, 10% of farms and call these statistics farm standards (or *benchmarks*). An individual farmer can then compare his statistics with the farm standards and make conclusions (Blagburn, 1957; Burns, 1966; Cooper, 1995). A farm standard, however, does not necessarily have to be an average from actual farms; it could be a hypothetical value considered by an adviser to be optimal.

Information on which to base the standards can come from:

- farm surveys;
- farm accounts and tax returns; or
- recording schemes designed specifically to provide farm standards.

A discussion of the technique is included as it has popular appeal to farmers, their advisers and researchers. As defined here, however, it does have some fallacies.

7.3 Methods Used in Comparative Analysis

Potential uses of comparative analysis

Comparative statistics can be used to:

- show what economic success is being achieved as compared with farms of a similar type and size in the same environment;
- provide data for a diagnostic process that compares the statistics to indicate what might be wrong with the current farming system;
- suggest methods of achieving greater success.

The steps involved in a typical system

A number of methods have been proposed. The following is an example of one method.

Step One – Calculate the basic statistics

1. Calculate the level of input expenditure on, for example, a per hectare basis. This information is set out so that the particular farm's figures can be compared with similar farm's statistics. For example:

	Farm under study	Average farm	Top 10% of farms
Rent ($)	60	40	50.4
Seeds ($)	80.5	90.0	104.0
...	...	...	...

2. Calculate gross return per hectare for the products and compare these with similar farm statistics. For example:

	Farm under study	Average farm	Top 10% of farms
Cattle ($)	350	265	420
Sheep and wool ($)	280	255	355

3. Similarly, various other statistics based on simple input and output figures can be calculated and compared. The more recently developed systems usually include actual gross margin figures and a considerable amount of physical input–output information.

Step Two – Calculate summary statistics and make comparisons

Using the statistics from step one, the various aspects of the farm business are considered in turn using a range of statistics. Examples include the following.

NET INCOME This is usually converted to a percentage return on capital figure. Clearly, if it compares unfavourably with other farms this suggests considerable opportunities for improvement.

LEVEL OF PRODUCTION To ensure fixed resources are being fully utilized it is suggested the level of production, given the resource levels, be examined. This might be achieved through comparing:

(i) value of gross income/ha;
(ii) value of gross income – variable costs/ha.

The results will suggest intensification or otherwise. In order to decide whether **(i)** low yields and/or **(ii)** the products produced are the basic reasons for a poor level of production the following statistics might be calculated:

$$\textbf{(i)}\ \text{Yield index} = \frac{\text{Actual gross income from farm}}{\text{GI (best)}} \times \frac{100}{1}$$

where GI (best) = gross income from farm where *it is assumed* that yields achieved are those obtained by the best farms.

(ii) Systems index =

$$\frac{\text{Gross income / ha from farm assuming } \textit{standard} \text{ yields and prices}}{\text{Gross income / ha from best farms assuming } \textit{standard} \text{ yields and prices}} \times \frac{100}{1}$$

The yield index will indicate the yield efficiency compared with the best farms. The systems index compares the systems used and products produced through using *standard* yields and prices so effects due to variations in these factors are removed.

CROPPING EFFICIENCY The statistics given above relate to the whole farm and provide general conclusions. The next step is to examine individual aspects in order to suggest possible specific weaknesses. Thus, the following statistics might be calculated and compared:

- the cash crop system index;
- the cash crop yield index.

These are calculated in the same way as the yield and system indexes, except only cash crop gross incomes are considered.

LIVESTOCK ENTERPRISE/S EFFICIENCY A range of statistics can be calculated. For example:

1. stock units carried per stock hectare;
2. the stock system index;
3. the stock yield index;
4. feed units per stock unit on properties where purchased feed is important. It might also be important to examine the ratio of types of feed purchased. As a result of poor performance in this area it might be suggested that this could be due to:
(i) poor utilization of purchased feed;
(ii) understocking of available pasture, grown feed;
(iii) poor grazing management;
(iv) wrong type of purchased feed,
(v) a low level of feed crops and pasture production per hectare.

This list suggests further statistics might be examined. Similarly, a list of 'sub-statistics' could be compiled for each major statistic, thus indicating why the poor level is obtained. However, it must be noted advisers often informally carry out this process when examining a particular farm.

5. livestock gross income per stock hectare;
6. livestock gross income per stock unit. This is clearly an indication of productivity per animal and when combined with (**5**) above, may indicate whether a greater stocking rate may be profitable.

7. Where different groups of stock are run, the above figures can similarly be calculated on a stock type basis, e.g. one group might be beef cattle and another lambs.

COSTS The statistics suggested largely examine output levels. Thus, the other major area requiring examination is the input situation. Possible statistics in this respect are:

1. Gross income per $100 total costs. This indicates, when compared with other farms, the overall input–output efficiency. Note, however, that a low figure might mean greater profitability. The following figures demonstrate this:

Farm 1	Gross income	$310,000
	Less costs	155,000
	Profit	155,000
	Gross income/$100 costs	$200
Farm 2	Gross income	$450,000
	Less costs	270,000
	Profit	180,000
	Gross income/$100 costs	$167

2. Gross income per $100 expenditure on various major items of expenditure. Results of these comparisons suggest where poor input utilization is occurring. However, a favourable ratio level may mean, for example with respect to the labour cost ratio, that a lot of machinery is used. Thus, individual statistics can not be considered separately from other related factors.

A simplified example

The following basic information is calculated from accounts and records:

Farm size = 200 ha

Capital invested = $3,500,000

Costs/ha ($100 units)				Gross income/ha ($100 units)			
	Comparable farms				Comparable farms		
	Farm	Ave	Best		Farm	Ave	Best
Labour	11.3	12.5	10.9	Cattle	5.7	4.2	7.7
Feed	13.8	14.0	13.6	Milk	19.7	27.2	28.7
Seeds	3.0	2.0	2.0	Sheep and wool	1.2	0.5	–
Fertilizers	2.0	2.4	4.1	Pigs	9.2	3.4	3.3
Rent and rates	4.1	2.9	2.2	Total livestock	35.8	35.3	39.7
Machinery and power	7.0	8.6	7.8	Crops	11.1	17.3	22.3
Miscellaneous	2.1	3.7	4.1	Miscellaneous	1.0	2.2	1.9
Total costs	43.3	46.1	44.7	Total gross income			
Net farm income	4.6	8.7	19.2		47.9	54.8	63.9
% return on capital	1.2	4.0	10.0				
System index = 86%							
Yield index = 74%							

	Comparable farms		
	Farm	Ave	Best
Cash crop production			
Crop Gross Income ($100/cash crop hectare)	44	59	86
Cash Crop Area as % of Total Area (%)	25	34	32
Livestock production			
Stock hectares per livestock unit	0.4	0.8	0.7
Total feed units per livestock unit	2.4	2.7	1.4
Gross Income per livestock unit	131	132	138
Cattle/milk gross income per cattle livestock unit	177	185	201
Pig gross income per pig livestock unit	92	93	93
Costs			
Gross income per $100 costs	110	119	143
Gross income per $100 labour	424	438	586

Analysis

PROFIT Net farm income and percentage return on capital is very low – clearly room for improvement.

GROSS INCOME Gross income per hectare is low. May need to be improved.

CAUSE OF LOW GROSS INCOME System index is low – suggesting wrong mix of products. Yield index is also low – suggesting yields and stock output is low.

Stock hectares per livestock unit is low – suggesting stocking intensity is too high.

CONSIDER PRODUCT MIX Cash crops: percentage of farm in cash crops is low, suggesting cash cropping should be increased. (There is not, however, information here on which crops should be increased – this would normally be included.)

Also, cash crop gross income per cash crop hectare is low, suggesting yields need increasing.

Stock: Best farms have no ‘sheep and wool’, suggesting sheep should be dropped (from gross income/ha figures).

Livestock production figures suggest the cattle/milk combination is more profitable than pigs on a livestock unit basis (but does ignore costs).

CONSIDER COSTS (Would normally have many more figures.)

Gross income per \$100 costs is low, suggesting:

- some inefficiencies in input use;
- wrong input mix;
- wrong product mix.

The high gross income per \$100 labour costs suggests efficient use of labour (but it may pay to use labour less efficiently in terms of this ratio).

Conclusions

- Profit is low, but can be improved.
- Rearrange product mix – more cash crops and more cattle at expense of sheep.
- Improve yields (expenditure on fertilizer per hectare is low, so this may be a reason for low yields).
- Also, feed units fed per livestock unit is high, suggesting more efficient grazing and feeding management may improve stock yields given the same quantity of feed.

7.4 Comparative Analysis – Problems, Disadvantages and Advantages

Introduction

As comparative analysis is a specialized use of the general farm survey technique, it suffers from the same general disadvantages as listed for surveys. However, the method exhibits a number of other theoretical and practical inconsistencies. The discussion that follows lists and discusses the more important of these and, finally, outlines the advantages (Rasmussen, 1952; Tansey, 1957; Candler and Sargent, 1962).

Comparisons

The system is based on comparisons. This means that it is necessary to ensure that real differences exist between groups. That is, some kind of *statistical analysis is necessary* to ensure that significant differences exist between, e.g. the average profit levels of the top 10% of farms, the next 10% and so on (this of course is likely, as the groups have been formed on the basis of net profit). Given such differences are significant, comparing an individual farm's statistics with the top 10% is likely to have more meaning.

However, just because the groups do have significantly different profits does not necessarily mean that their average input and output statistics are going to be particularly meaningful. The averages may arise from individual figures that range over a wide distribution. Thus, *standard deviation and confidence limit information* should be provided when an individual farm's statistics are being compared with group statistics. If the confidence limits are wide it is doubtful whether it can be concluded that the particular average figure is indicative of a successful system as other farms in the group have used quite different inputs (thus the wide confidence limits) but have produced a similar profit. Thus, for example, the average gross income per $ labour expenditure for the top 10% and next 10% of farms should be tested for significant differences using the *t* test. Given that real differences in the averages occur, then it is more likely that a comparison between an individual farm statistic and the average of the top 10% will be meaningful *provided* the confidence limit is relatively narrow (it probably is if significant differences occur).

The reasons for these precautions is that profit is a random variable. Thus, a farmer operating a poor system may end up in the top 10% group due to good luck in a particular year and vice versa. Thus, the analysis must show that the input levels and systems used by different profitability groups are significantly different.

As with surveys, all these comparisons assume differences are due to farming system differences and not due to soil type and other resource differences. Thus, comparison must only be made between comparable farms.

The above discussion assumes that groups are formed on the basis of profitability. Logically, however, groups should be formed on the basis of the independent variables (the input levels), as these are the decision variables. Thus, farms should be placed into a group according to their particular farming system (fertilizer type and quantity, labour use etc.). Then, provided the groups had significantly different average profit figures, the group with the highest average profit could provide the farm standards. In this case the average input figures would be meaningful as the individual farm figures would all be much the same due to being the basis of the classification. However, a grouping system based on the independent variables may be impractical, as given a realistic list of input statistics, there may be insufficient farms with similar input levels in each group.

General inconsistencies and problems

The fallacy of physical efficiency measures (the technologists dilemma)

Where physical ratios are used as standards, there is often a problem of deciding which one, or ones, should be improved. Often if one is improved, others will decline. For example, on a dairy farm two physical standards might be: (i) milk production per cow; and (ii) milk production per hectare.

Clearly, if steps are taken to improve one the other will decline. Which one is the correct ratio to base decisions on? A knowledge of production economics (see Appendix 1) indicates, of course, that neither ratio should form the basis

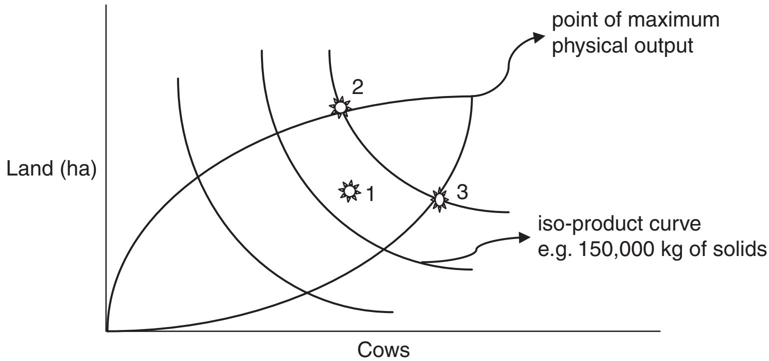


Fig. 7.1. The technologists dilemma.

of decision making (though at the optimal point, the farm will be producing a particular milk production per cow). What is important is the ratio of prices of inputs, outputs and the technical relationships. The above example is a factor–factor problem where the two factors are cows and land, therefore the problem can be graphed as in Fig. 7.1.

If a farmer is currently operating at point 1, per cow efficiency can be improved by moving to point 2, per hectare efficiency can be improved by moving to point 3. Which move is optimal? As the optimal point depends on the price ratios, a decision is not possible without the prices and costs to hand.

Thus, it must be concluded that physical efficiency measures should not form the basis of decisions.

The fallacy of monetary and physical factor–product ratios

Many of the statistics used are factor–product ratios. For example:

- $ output/$100 labour expenditure;
- wool production/ha;
- $ output/$100 capital investment.

In general, the value of the various factor–product ratios is given by the slope of a line drawn from the origin to the 'production function' as shown in Fig. 7.2.

Clearly, the maximization of such factor–product ratios is most unlikely to maximize profits. The optimal point should be determined through the use of the relevant price ratios (MC = MR), i.e. marginal costs equal marginal returns, whether monetary or physical production functions are used.

Yearly variations in standards

As prices, weather and other factors impacting on output can change from year to year, it is possible that farms in the most profitable group will change in different years, so recommended standards and benchmarks will change from year to year. Thus, for example, given the product–product opportunity curve shown in Fig. 7.3, a farm operating at A in year 1 will be more

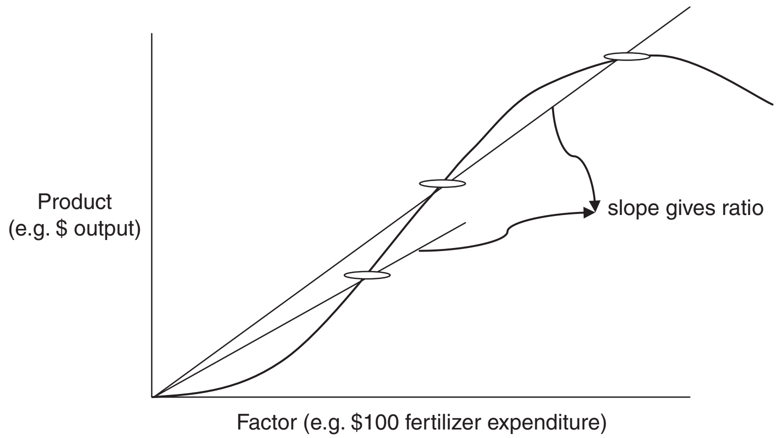


Fig. 7.2. The fallacy of monetary and physical factor–product ratios.

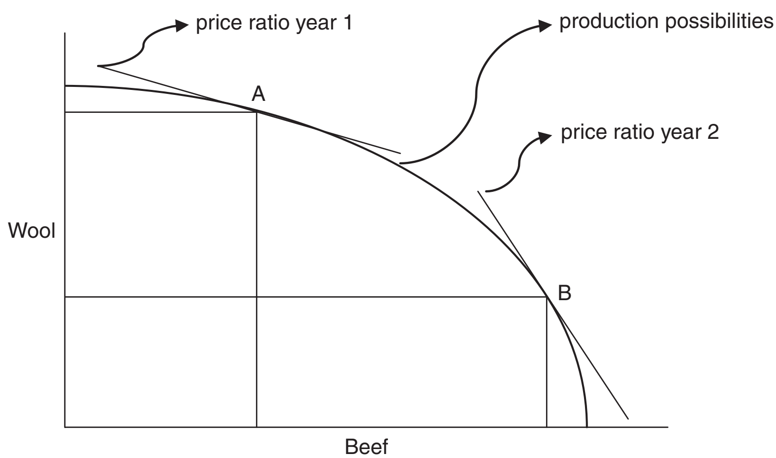


Fig. 7.3. Yearly variation in standards due to price variations between years.

profitable whereas in year 2 a farmer operating at B will be the best. As these farms have different systems the standards would change. Similarly, farmers operating at different points on factor–product and factor–factor curves will be the more profitable in some years and not in others.

The real problem here is that the standards are always a year behind. If past conditions are exactly repeated then suggestions for the current year will be useful. However, this is most unlikely because standards provide historical information. What the farmer wants to know is what should be done given the expected future prices and costs. In order to make this decision, information on the relevant physical production functions is required, not information on a mixture of physical and financial ratios.

Product and input aggregation

The members of the group of 'best farms' are usually calculated on the basis of net profit, for example using the Economic Farm Surplus (surplus less costs for unpaid resources such as the owner's input). This measure stems from aggregating different outputs and inputs to give the overall net profit figure. However, many so-called best farms are possibly producing some products efficiently while others inefficiently. Comparative analysis will not indicate this. If the analysis was based on a per product basis this possible source of further confusion would be eliminated. Some contemporary systems do in fact base their comparisons on individual product statistics but it is still usual to find the best farms selected on a whole farm net profit basis.

Somewhat similarly, conclusions made about the desired level of an aggregated input are not particularly informative. For example, if comparative analysis suggests, e.g. more fertilizer should be used per $100 gross income, this does not indicate what type of fertilizer should be purchased, on what crop it should be applied and at what time of year and other practical items of management information.

Is the best system optimal?

The set of standards obtained from the top group of farms can only, at best, improve other farms to their level. In most cases it is unlikely that the 'best farms' are operating optimal profit maximizing systems, so the 'best system' suggested by the calculated standards is only as good as the best systems in the population of farms used to obtain the standards.

The limited scope of the suggestions

Comparative analysis makes no reference to such problems as:

- ownership organization;
- taxation policy;
- financial organization.

Comparisons taking this kind of information into account could be made, given a sufficient number of farmers using alternative systems. However, as every individual farm has a, usually, somewhat different family and ownership setup, such comparisons are unlikely to be of extensive use.

Data comparability

The basic assumption in comparative analysis is that direct comparisons are valid as the farms have similar resource and opportunity situations as well as similar objectives. Clearly this is incorrect in most cases. This is the problem of uniqueness of farms and farmers.

Through appropriate grouping, comparisons would normally only be made for farms on similar soils, climatic regions and so on. Similarity in this respect is relatively easy to ensure, though whether there are a sufficient number in each group to obtain statistically significant differences is another matter. However, it is usually impossible to ensure comparability with respect to such factors as:

- managerial ability and labour quality;
- financial availability (working capital, availability of developmental finance etc.);
- stage of development.

Thus, what is an improved farming system for one farm is unlikely to be the best system for another, so that comparative analysis and farm standards cannot provide detailed suggestions. In some cases standards may even provide suggestions opposite to optimal suggestions.

Concluding comments on problems and disadvantages

The above discussion makes it clear that the use of comparative analysis and farm standards can provide incorrect decisions and in most cases probably will. The basic reason is that an improvement in any particular statistic is not necessarily associated with a high level of an individual farmer's objectives.

Rather than attempt to provide a mass of statistical goals, a successful decision-making system should rely on obtaining an *understanding of the basic production relationships* and then, combining this with *expected* price and cost information, make decisions based on these relationships for each particular farm, i.e. attempts should be made to determine the shape of the production economic concepts of the factor–product, factor–factor and product–product relationships.

Comparative analysis and the farm standards and benchmark systems attempt to bypass the need for an understanding of the basic technical relationships and of production theory. This is not possible. However, comparisons can have some advantages and it is for these reasons that comparative analysis can be useful in some situations.

The advantages of the comparative analysis approach to farm improvement

Ease of understanding

BY ADVISERS AND CONSULTANTS The method can be easily understood and operated by people with little training. However, as noted, a good understanding of production economics is necessary before farm standards can be used correctly. This 'ease of understanding' tends to encourage acceptance of the system by the uninformed and thus lead to incorrect advice.

BY FARMERS A farmer tends to find a simple comparison with a 'better' farm easy to understand and is usually quick to conclude his system should be adjusted to give the same statistical values as the 'best' farms. Of course, the managerial ability of the 'best' farms may be quite different, thus making it difficult for all farmers to achieve the same statistical values.

Create interest

Most farmers are very interested in comparing their 'achievements' with other farmers. For this reason comparative analysis tends to generate considerable interest in management systems, costs, returns and all the other statistics, and as such may lead to the seeking of advice. An improvement of the farming system may come through the farmer's own conclusions and other advice received, in contrast to the direct suggestions made by the comparative analysis. Comparative analysis does, therefore, tend to provide a stimulus to interest, to change and involvement.

Inexpensive advisory system

Comparative analysis and the farm standards/benchmark system is an inexpensive method of providing 'advice' to a farming community. Given a sufficient number of personnel to carry out a survey, a large number of farmers can be sent comparative analysis data at little cost. Thus, where funds are very limited, this technique can be used to cover many farms and thus indicate to farmers that their research investment is being used effectively.

Due to the inadequacies of the system, some suggest it should only be used as a diagnostic device, and this is often done. In this case it is desirable to provide advisers and farmers with information enabling management changes that are based on a logical production theory interpretation.

Records

If a farmer is involved in a comparative analysis system he is forced to keep adequate records to estimate the statistics. Thus, the system may lead to other benefits through this enforced work and thought. However, records can be kept without having to use them for comparative analysis purposes. (A major question here is, of course, what records are necessary for management purposes?)

Conclusion

The advantages have insufficient value to warrant using comparative analysis and farm standards as the sole farm management improvement system. This does not mean, however, that a very simple scheme could not be used to provide interest and a stimulus to examine current methods. For example, simple descriptive information on farm area, labour force, profit and physical output ratios and similar, could be provided without dividing the farms into success groups. Range figures would also be useful.

Further, in general, a number of simple standards can be of use to an adviser provided they correctly based. That is, given a knowledge of the underlying production relationships and information on the expected prices and costs, optimal (depending on the objectives and risk attitudes) output levels can be determined. For example, the use of production economics might suggest that an optimal stocking rate is two cows/ha and an optimal production per cow is 3,600 l milk. These figures are *not* based on simple observations of what farmers' *are* doing. Rather, they are normative guides. In using these 'correct' standards, it must be remembered, however, that they will not apply directly to any particular farm and will probably change as prices and costs change. Every adviser has in their head a whole range of benchmarks that they use to initially assess any farm. It is human nature to have these standards to enable judgements. These should come from a positive analysis of the farm situation and be intelligently used, in contrast to simply being applied with rigidity.

7.5 Automated Recording Systems (Decision Support, or Information, Systems)

Introduction

Recording systems have largely been computerized so that the tedious work involved is removed. The basic data must still be recorded but the software and computer does all the adding up, sorting and so on. While most systems are designed to provide the individual farmer with useful management information, the data collected can also be used to supply survey studies with information for not only research, but also for comparative analysis provided some basic comparability between farm data coding is maintained. Of course, where computer systems are not available, a reduced hand-operated system is essential for management if not for survey purposes.

The discussion will cover examples of the different types of farm records which might be put on to a computerized system, the operational procedures involved, and some of the advantages and problems of such systems. Some comments will also be made about the types of farm records that can be useful in farm planning. The topic does not introduce new decision-making techniques or theories, but merely points out that, increasingly, farmers are computerizing and consequently can provide electronic information as well as provide themselves with useful decision-making information (Rowe, 1971). Any system providing information to assist decision making, be it simple sorted data, or information that has been calculated from the raw data, is often referred to as a farm information system or, in some cases, a decision support system (DSS).

Uses of records

To orient the discussion on types of automated systems it is worth listing the major uses of farm records. These include the following.

- General planning purposes. For example, the recording of crop yields and crop input costs for each different crop enables the estimation of gross margins. Such records are also necessary for constructing budgets and cash flows and similar financial calculations. Feed and animal production records can also be useful in planning, particularly feed budgeting. In addition, soil nutrient application and use records, and nutrient budgeting, are increasingly becoming important.
- Use in *controlling* the farm business and in re-planning throughout a particular planning period. For example, the recording of monthly cash costs and returns enables estimates to be made of the current cash situation and thus whether re-planning is necessary, whether income tax might be a problem and so on. Similar comments apply to feed budgeting.
- Construction of taxation returns and assessments.
- Research work. Detailed records of the inputs and outputs of alternative farming systems enable comparisons to be made (surveys).
- Government policy work. Farm records, if they can be made available to governments, can be useful in assisting policy decisions. For example, indebtedness information would assist the government in deciding whether to make special loans available if they wanted to encourage farm development.

Advantages of computerized systems

Speed, ease and accuracy

A totally computerized system provides largely instantaneous output and sorting. After data entry, which can be accurate and simple if the system is well designed, obtaining suitable output only requires selection from a list of available reports. Given ease of input and output, accuracy is greatly enhanced as the system is not reliant on farmer sorting and calculations.

Farmers are notoriously bad at keeping records and putting them into a useable form. However, given the aid of a computer, many farmers find record keeping, and use, simple and are encouraged by automated systems to keep much more extensive records. Care must be taken, however, not to keep records for record's sake. Each must have a defined and valuable use.

Use of consultants' time

Many consultants and advisers are forced to assist farmers in their record keeping and analysis. When access to an efficient computerized system is available, the consultant no longer has to assist, leaving more time for either each farmer or more farmers to be advised.

Types of records and systems

General

Depending on computer costs and training in any country, farmers may use their own computer, or alternatively send information into a computer bureau. The information will include cash costs and returns that have occurred during that month, and associated physical quantities. This list of transactions is added and sorted to give a summary of the cash costs in each category (e.g. superphosphate, fence materials etc.). These data may be compared with budgeted estimates for each month and if the actual differs from the forecasted values by more than a pre-defined amount, the computer will print a row of stars, or some other eye-catching device, to draw the farmer's attention to the discrepancy. Cumulated totals for the year are also usually printed out, or displayed on the screen.

Where a farmer is involved in feed and stock management or nutrient budgeting then these records must be added to any system. Where farmers are using their own computers they can send in (or transfer through telephone lines/radio links) data disks to enable group statistics and comparisons for comparative analysis.

The following is an abbreviated example of typical financial system output.

Farmer's Name:	J Bloggs	**Address:**	RD 2016, Christchurch
Report for month ending:		September	

CASH INCOME

	Budget for Year	This month	Budget for this month	Difference	Total to date	Budget total to date	Difference
Wheat	122,000	0	0	0	0	0	0
Milk	135,000	10,680	12,000	1,320**	36,800	39,000	2,200
Calves	3,000	460	600	140	1,200	1,300	100
TOTALS	260,000	11,140	12,600	1,460**	38,000	40,300	2,300

CASH EXPENDITURE

Labour – casual	3,500	0	0	0	1,480	1,500	
– permanent	60,000	4,800	4,800	0	20,000	21,500	1,500
Contracting – haymaking	2,000	0	0	0	0	0	0
– hedgecutting etc.	2,500	3,600	2,500	1,110	3,600	2,500	1,110

Asset situation

Associated with cash reports (as above) a statement giving changes in the current asset situation is often provided. This would usually refer to stock numbers, cash balances and liabilities but could include such things as hay and barley on hand.

These statements would show the opening quantity/balance, the closing quantity/balance and information indicating why the difference occurred. For example:

STOCK SITUATION

	On hand at start	Purchases	Transfer	Deaths	Sales	Transfer	On hand at end
Ewes	2,536	–	500	3	–	–	3,033
Cows							
etc.							

Such information provides an immediate summary of the state of the farm. Note that this information is already available without the use of the computer; however, the computer enables data sorting and presentation in an easily usable form.

Summary accounts

The computer can be programmed to produce any kind of summary accounts required, provided the necessary data are available. These will usually be normal taxation accounts as well as adjusted accounts for managerial purposes. Provided the data are provided throughout a year, year-to-date accounts can be produced for taxation purposes. The type of accounts, and the detail provided, vary considerably. Some systems will go so far as to produce enterprise accounts. For example, a set of accounts concerning just the wheat enterprise might be produced and similarly for all other enterprises on the farm.

Gross margins

Provided data of sufficient detail are entered, some systems will use the data to produce the gross margin actually achieved for the last season for each enterprise on the farm. This would usually be compared with the estimated gross margin and any discrepancy clearly marked. This information is produced once per year. This data is of obvious use in future planning.

Physical information

Again, provided sufficient information is available, some systems reproduce, usually annually, considerable physical information that can be of use in future planning. For example, the following may be calculated:

- lambing percentage survival to sale;
- death percentage in various stock groups;
- wheat yield/ha;
- superphosphate applied/ha;
- tractor hours spent in cultivation work, etc.

Comparative analysis

Some systems provide a full comparative analysis. As the organizations providing the computerized systems usually have a large number of farms on their books, they can provide average data for different types of farms so that a comparative analysis can be carried out once per year. For farmers operating off their own computers their statistics need to be electronically sent to a central point for comparisons.

Advanced Services

Some computer bureau (organizations providing services to aid farmers and advisers based on computer packages), or 'service centres' as they are sometimes called, may provide a range of other services. Examples are linear programming studies, risk analysis and other operations research techniques. On the physical side, detailed individual animal records may be computerized and used in selecting animals for breeding and selection purposes.

Example

As an example, the annually provided output from one software package is given in Appendix 2.

Methods of operation

Data Collection

Various methods are possible. In most cases data entry software or printed forms are provided. These can be completed by:

- the farmer himself (or his spouse, or secretary on larger farms); or
- a travelling secretary who covers many farms; or
- a consultant or adviser.

The frequency of collection will depend on the type of service. Where a centre is used the completed forms are mailed, or the data sent electronically, when farm comparative information is required.

Before entering a scheme, farmers are sometimes requested to attend a 'school' on the use of the system. An example of an input sheet for a monthly cash balance service is given below. For online entry similar headings would be provided.

MONTHLY FARM RETURN Cash Payment

Farm Name: J Bloggs **Code No:** | 6 | 3 | 2 | 8 |

Date: | 0 | 4 | 1 | 8 | **Account Name:** Fertilizer **Act No:** | 3 | 6 | 8 |

DATE	DETAILS	CODE*	AMOUNT	QUANTITIES
3/4/18	Superphosphate for field 23 etc.	63298	$805.68	4000 kg

Coding

To enable sorting the information and the calculations, each specific type of information is given a numeric code. For example, the code numbers for the following data are taken from a popular coding system:

Transaction	Code No
Wages to permanent employee associated with cattle enterprise	240–1123–6
Wages to permanent employee associated with fat lambs enterprise	240–1123–5

When a farmer joins a bureau system he is usually given a booklet setting out the coding system. For on-farm systems a coding system is sometimes automatically provided when data are sent for central analysis. On the other hand, some systems allow the farmer to make up their own system, but where the data are to be sent centrally either a standard system is necessary or conversion definitions are provided. Other systems may even have word recognition, which displays a code for confirmation. The code is then used internally for ease of sorting and calculations.

Data return and interpretation

For on-farm systems most reports will be automatically produced. For central systems the output is usually returned to the farmer electronically or by mail, or alternatively the output is sent to an adviser who then delivers it to the farmer. The farmer can either be left to interpret the information himself, or someone will be available to discuss the data if a commercial consultant is not being used.

Problems

Improper use of information

With imagination a computer bureau or on-farm computer system can produce a vast array of output. There is always the danger that a farmer will misinterpret this mass of data, or that some of the data are somewhat irrelevant and even misleading. This may mean an adviser becomes an important intermediary in the successful use of the data (refer back to the discussion on the problems of comparative analysis).

Incorrect data

As many of the systems require farmer recording, coding mistakes are not uncommon. Farmers may find it difficult to easily understand coding systems, particularly after a hard day of physical activity on the farm, and make interpretational errors.

To prevent such errors, the data input systems and sheets clearly need to be as simple and clear as possible with clever checking software to detect possible mistakes and provide warning messages. Where professional secretaries and/or advisers do the coding and entry this problem will be reduced.

Acceptance by farmers

For systems to be accepted by farmers, they should generally satisfy the following criteria.

- Provided at a price relative to the benefits farmers perceive they obtain (some systems are government run and may therefore have no charge);
- Competent (provide useful data);
- Reliable (provide the output on time without fail);
- Timely (produce the output at times when it will be of most use);
- Easy to use and understand, with readily available help on hand.

Summary

A range of computerized record keeping and analysis systems is available. They provide no more information than is contained in the basic data. Their advantage lies largely in the convenience and speed offered due to the removal of tedious data finding, sorting and calculational work. Often banks and similar organizations can provide electronic copies of a farm's information, which can be automatically input to the farmer's database. Similarly, some suppliers can provide electronic data input. A farmer's system might also provide accountants and banks with summaries for their use, and then tax authorities can have the data they require sent electronically either from the accountant or by the farmer.

As far as survey-based systems are concerned, these automatic systems can provide accurate and timely information for comparative analysis, which means the farmer is provided management information automatically and with little effort. Similarly, researchers may be able to access some of this information for the various uses outlined in the first chapter on surveys.

7.6 Whole Farm Production Functions

Description

Given general farming system data have been collected for a population of farms, rather than use this to split the farms into profitability groups and so arrive at a conclusion regarding an improved system, these data can be used to construct a whole farm production function using multiple regression techniques. The data provide observations on output and the various inputs so that a general function of the form can be estimated:

$$Y = f(X_1, X_2, \ldots, X_n)$$

where Y is the output, e.g. \$ profit;

and the X values the levels of the various inputs, e.g. monetary or physical values.

The number and type of independent variables (the Xs or decision variables) that can be used will depend on the available data and will range from, for example:

Profit = f (value of land, value of machinery, value of stock, labour expenditure, other operating costs)

to much more detailed functions, such as:

Profit = f (area of land, no. of tractors, no. of ewes, no. of hoggets, kg of superphosphate, etc.)

Similarly, the dependent variable might be defined in physical terms, but given multi-product farms disaggregation of data to enable this is a problem.

The form of functions used must clearly vary from case to case. However, commonly used functions are the quadratic ($Y = a + bX + cX^2$), Cobb-Douglas ($Y = aX^b$) and Spillman ($Y = M - AR^x$) functions (a, b, M, A and R are constants to be determined and X represents the input levels).

Many examples of production functions can be found in the literature. One example of a Cobb-Douglas function is (Battese and Coelli, 1995):

$$\text{nl } Y = 2.86 + 0.37 \text{ nl } L + 0.38 \text{ nl } I + 0.85 \text{ nl } LBR - 0.33 \text{ nl } BL + 0.071 \text{ nl } C + 0.014 \text{ nl } Y$$

where nl refers to the natural log;
Y the value of output;
L the land area (ha);
I the proportion of land area irrigated;
LBR total hours of labour;
BL hours of labour driving bullocks;
C the total \$ costs; and
Y the year (the data used in this example to estimate the function covered many farms over several years, called panel data).

Note that this function covers both physical and monetary information.

An example of a simple physical production function for wheat output/ha comes from a study looking at yields for soils with a salinity problem (Datta *et al.*, 1998). The quadratic form estimated was:

$$\text{t wheat/ha} = 9.9679 + 0.0523Q - 0.005Q^2 - 0.3344C - 0.0002C^2 - 0.3823S + 0.0767S^2 + 0.0112QC + 0.1072QS - 0.0455CS$$

where Q was the quantity of water applied (cm);
C the salt in the irrigation water; and
S the initial root zone salinity.

A function such as this allows working out the optimal water application quantity to maximize return once the expected prices are known. The first function presented can similarly be used, but it does not provide detailed management-directing conclusions. For example, determining the optimal level of costs is interesting, but it does not provide operational information such as the amount to spend on each possible input.

Problems

The conceptual advantage of constructing a whole farm production function compared with making simple comparisons between groups is that the basic output–input relationship is being estimated so that optimal input combinations can be estimated through the use of calculus.

However, these functions suffer from much the same general disadvantages as surveys and comparative analysis, so they may have limited use in advisory and research work. Because it is difficult to obtain detailed data from the farms, the whole farm production functions have to be quite general, so their use is limited to general statements in contrast to detailed suggestions. For example, while a function of the general form:

Profit = *f* ($ fertiliser costs, $ feed costs, etc.)

may provide knowledge that, e.g. $1 spend on fertilizer returns $1.5, this does not indicate what type of fertilizer should be purchased, on what crops it should be used, nor at which time of year it should be applied. If, however, extremely detailed physical information could be collected from farms then input–output physical relationships could be estimated and used to give good advice when combined with price and cost information. Unfortunately, this is not possible in most cases. In practice, *considered estimates* of the physical relationships tend to be used, or in some cases detailed experimental data may be available enabling estimating a physical production function which will indicate the yield of e.g. wheat according to type, quantity and timing of fertilizer use, timing and quantity of any irrigation, and other detailed input decision variable information. Given a detailed function of this form it is then possible to use expected prices and costs to determine the profit maximizing levels of all inputs.

7.7 Cost Accounting

Description

Many of the early farm management research workers attempted to determine total cost of production per unit of output estimates for the alternative products. This was used to determine the most profitable. It was usually carried out on each farm using special records kept for the purpose. However, comparisons were also made between farms with the idea of indicating the technical production system that produced output at least cost. (Cost of production estimates are also required by governments for price fixing, and other policy and support systems.) The process of determining the cost of production is generally referred to as *cost accounting* (Giles, 1950; Mallyon, 1966).

Cost allocation methods

On single product-producing farms there is no cost allocation problem.

$$\text{Cost/unit} = \frac{\text{total costs}}{\text{no. of units of product produced}}$$

The problem arises on multi-product farms, as some method of allocating costs between products must be determined. In general, *variable costs* can be directly assigned to the various products as the input giving rise to the cost is completely used on one or several products in defined, recorded, proportions. *Fixed costs*, however, cannot be easily assigned as the assets giving rise to the fixed costs are used by many products.

Variable cost allocation

If two or more products are *supplementary*, or *competitive products*, variable costs incurred by each product can logically be assigned to each product. However, where some of the products produced are *complementary* or *joint products* there is no logical way of allocating variable costs. For example, the fertilizer put on a white clover crop increases the output of white clover seed and of the wheat crop following it due to the increased soil fertility. Thus, in attempting to allocate the fertilizer cost between the white clover and wheat what method should be used? There is no logical defensible answer. Similar comments apply to joint products. For example, given a lamb-producing property, should the animal health costs be assigned to the meat or the wool production? In this case the question is largely irrelevant as the joint products cannot be produced separately using a lamb system.

If the variable costs for complementary and joint products are to be allocated, some arbitrary assignment system will have to be used. For example, they might be apportioned on the basis of the ratio of the product values.

Fixed cost allocation

As fixed costs cannot be directly assigned to individual products, some arbitrary method must be developed. For example, the following systems might be used:

- On a per hectare time basis: e.g. wheat is grown on 1/3 of total farm area and occupies the ground 7/12 of a year, thus the proportion of the rent assigned to the wheat would be $1/3 \times 7/12$.
- On a percentage of gross income basis: e.g. if wheat produced \$50,000 and the whole farm gross income was \$225,000, then 50/225 of the fixed costs would be assigned to the wheat.

There is no logical economic reason why one of these examples should be used in preference to the other, thus indicating that a correct fixed cost allocation system does not exist.

Practical problems

Even for the costs that can logically be assigned there may be practical difficulties in doing this depending on the records that are available. For example, most farms would not have sufficient records to indicate how the electricity charge (similarly, the diesel fuel charge) should be divided between products even though it can be largely regarded as a variable cost. In such cases general knowledge on the inputs required for production would have to be used. For example, expected tractor hours/ha and tractor fuel consumption rates can be used to allocate fuel costs.

Usefulness of cost accounting information

FOR FARM MANAGEMENT PURPOSES In general cost accounting information is of little use as:

- It suffers from the same general disadvantages as the survey and comparative analysis systems, i.e. it is historical information.
- The cost of production estimates obtained rely on many arbitrary decisions.
- In the short run, fixed costs *are irrelevant* in decision making. The objective is to maximize returns *given* the available resources. In the long run, the real problem becomes one of organizing a fixed cost structure such that total returns (or whatever the objective is) are maximized.
- The cost of production estimates obtained are *average total cost* estimates. For decision making production economic theory proves that *marginal cost* information is the relevant decision-making criteria (marginal cost does not contain any fixed costs).

FOR GOVERNMENTAL USE Cost of production estimates can provide a general guide for price fixing and related purposes. However, as the government's aim is usually one of ensuring that certain producers obtain a reasonable standard of living and that certain quantities of products are produced, a more efficient approach must be one of 'playing the market' until the objectives are achieved. This may involve setting a price lower or higher than the cost of production estimate, depending on whether the producers are rational in a simple economic sense.

For general decision-making use by farmers it is clear that marginal cost information is generally more useful than total average cost of production estimates as defined above. To provide these estimates survey-type data must still be collected across many farms producing different output levels.

This can then be used to calculate a marginal cost function. Or better still, if physical output–input data are available these can be used to estimate a physical production function that can be combined with expected costs to provide a forecast marginal cost function.

7.8 The Measurement of Efficiency

The problem

Theoretically, a farm is efficient if no changes to the systems being used will increase the objective. In common usage, however, if it is said a farm is efficient this is usually taken to mean that the farm produces a greater net return than other farms. Thus, the measurement of efficiency refers to the problem of ranking farms; and of ranking alternative methods of operating a particular property; and of comparing a farm's return with some benchmark return figure.

From time to time various efficiency measures have been proposed. Four examples are shown below.

PERCENTAGE RETURN ON CAPITAL

$$= \frac{\text{Net profit} - \text{WOM}}{\text{TFC}} \times \frac{100}{1}$$

Where TFC = total farm capital; and

WOM = wages of management (an assessment of the opportunity cost of the manager's unpaid time input. The manager is often an owner/operator and consequently receives no salary, but this does depend on the ownership structure).

Net profit is the gross income minus all expenses other than any interest on the debt (and other than capital repayment) nor the managers salary, as this is allowed for as WOM.

Also note that rather than relate the surplus (net profit minus WOM and any other non-paid family labour) to the capital, it is sometimes called the Economic Farm Surplus and related to a productive unit such as per hectare. The return on capital, clearly, relates the surplus to the investment.

And further note that the net profit is sometimes referred to as Earnings Before Interest and Tax (EBIT), but again, for comparisons, it needs to be related to a measure of farm size such as the area.

RETURN TO THE LABOUR AND MANAGEMENT OF THE OWNER-OPERATOR

$$\text{Return} = \text{Net profit} - i \times \text{TFC}$$

where i = opportunity cost of capital expressed as a fraction.

RETURN TO LAND

$$\begin{aligned}\text{Return} = {} & \text{Net profit} - i \times \text{TFC} - \text{WOM} \\ & + \text{all expenses associated with the land}\end{aligned}$$

i.e. add back any interest on land value, rent, rates and any other expenses on the land that was deducted in the expenses.

PROFIT SURPLUS

$$\text{Profit surplus} = \text{Net profit} - \text{WOM} - (i \times \text{TFC})$$

It is assumed that in all the above examples the net profit has been made comparable by assuming the farm is debt free and freehold. To allow for debt structures, sometimes the interest charges are left as factual and the return is related to the net capital (or equity, i.e. TFC less debt).

The measures return to labour, return to land and profit surplus are not comparable due to property size variations. Thus, each must be divided by the hectares (assuming the area indicates size) to give comparable figures.

Each of the measures is what is referred to as a *residual measure*. All the expenses associated with the factors of production *except for the profit surplus measure* are deducted from the gross return and the *resulting residual* is said to be the return to the capital investment. In the case of the profit surplus measure, the residual is imputed to 'profit'.

Four examples were given above; each one is calculated in a different way so that if a group of farms were ranked using each measure it is probable that the ranking order would vary according to the measure used. However, all the measures cannot be correct and interchanged at will, therefore a decision is required on which is the correct measure.

The objective

The correct measure to use must depend on the objectives. Where alternative production systems are being ranked for a *particular farm*, the farmer's objective is the relevant gauge. But, it is unlikely that any of the measures defined reflect the farmer's full objective as it will include non-monetary factors. Where, however, *a group* of farms is being ranked it is usual to use a monetary measure as cash return is somewhat of a common denominator. In this case the correct measure must record the 'cash surplus' as most wish to maximize, on average, the total net cash balance. However, 'cash surplus' ignores that various levels of capital and the owners' labour and management that have been used and are, therefore, largely not comparable. These factors have an opportunity cost. Thus, the opportunity costs must be deducted from the cash surplus to give a correct measure (similarly other economically justifiable non-cash expenses should be deducted, i.e. depreciation and inventory changes). This estimate of 'profit' is not a cash figure. However, it can be shown that if, in choosing between alternative farming systems, profit, as defined, is maximized, then the cash return will also be maximized in the long run. In the short run, selling all assets would maximize cash returns.

Thus, the only measure of those suggested that correctly records the level of profit is 'profit surplus'. The other measures record other values and should not be used unless they are in fact the farmer's objectives. (In special cases percentage return on capital can be used as will be shown later.) For example, the percentage return on capital records the *ratio* interest surplus to capital investment. This measure does not indicate the level of the profit *but of* a ratio and should not in general be used where the farmer's objective is not the maximization of this ratio.

A comparison of some efficiency measures

To reinforce some of the arguments given above, consider the values given (and thus the ranking) by the various measures for three different farms, which are in the same environment and are operating according to a basic production relationship of the following form:

$$\text{Gross return} = 68.0\, X_1^{0.11} X_2^{0.09} X_3^{0.2} X_4^{0.51}$$

where X_1 = land area in hectares;

X_2 = labour input in 10 man h units including the input of the owner operator;

X_3 = capital investment in \$1000 units other than the unimproved value of the land;

X_4 = all other expenses not associated with X_1, X_2, X_3, in \$ units.

Further assume that:

the opportunity cost of capital is 6%;

all labour costs \$200/10 h unit;

unimproved value (UV) of land = \$100/ha;

cost/ha: rates, land tax, etc. = \$10.00;

other costs associated with capital investment are \$3/\$100 unit (repairs and maintenance, depreciation, insurance, etc.).

The three farms are operating the following systems:

	Farm 1	Farm 2	Farm 3
X_1	5,000 ha	5,000 ha	5,000 ha
X_2	350 (1.5 men)	750 (3 men)	1,000 (4 men)
X_3	1,000 (\$1,000,000)	1,500 (\$1,500,000)	2,000 (\$2,000,000)
X_4	90,000	135,000	180,000

Given these figures, the following table gives the relevant statistics for these farms, given the assumptions made above:

	Farm 1	Farm 2	Farm 3
1. \$ output	396,015	562,123	707,747
2. Expenses other than interest and WOM (\$)	210,000	345,000	455,000
3. TFC (\$)	1,500,000	2,000,000	2,500,000
4. WOM (\$) (opportunity cost)	53,000	53,000	53,000
5. Opportunity cost of capital (\$)	90,000	120,000	150,000
6. Profit surplus (\$)	43,015	44,123	49,747
7. Percentage return on capital	8.9	8.2	8.0
8. Return to labour and management (\$)	96,015	97,123	102,747

Note that WOM has been set at a standard figure – the WOM that should be used will be discussed below

The measure 'profit surplus' (and 'return to labour and management') ranks the farms *in the opposite order* to the measure 'percentage return on capital'. Where the farmer's objective is to maximize the difference between returns and costs then the best farming system of the three is Farm 3, the one with the *lowest* return on capital. This opposite ranking will not always occur. It has occurred in this case because the marginal return on additional investment is greater than the opportunity cost of capital, but less than the current *average* return on capital (decisions, of course, should be made on the basis of marginal returns). Thus, as additional investment occurs, as is the case as you go from Farm 1 to Farm 3, the marginal return is not sufficient to increase the average percentage return on capital, *but it is greater than the opportunity cost* so that the profit surplus increases. The farmer wanting to increase profit should continue to invest until the marginal return equals the marginal cost. At this point the average percentage return on capital will be quite low. Thus, efficient farms will be ranked well down on the list if percentage return on capital is used as the efficiency measure. The example given was designed so that 'profit surplus' and 'percentage return on capital' would be diametrically opposed. This will not always be the case – it will depend on the marginal relationships.

The comparability of the measure 'profit surplus'

In the example given above the farms could be directly compared as they were of the same size. Where this is not the case, the profit surplus must be divided by a measure of farm size. In some countries a reasonable measure of farm size is the physical area in hectares, as land is the critical input in terms of determining the potential output. In some countries this would not be the case as capital and labour are used more intensively so they may become the limiting factors. However, where economies, or diseconomies, of scale and size occur only farms *of a similar size* can be compared. If this restriction is not placed on comparisons the determination of an optimal system will be confounded by the effects of size. Thus, for example, a farm with a high profit surplus/ha might not be operating an efficient farming system compared with a smaller property with a low profit surplus/ha. The difference might be due to economies of size, not to an efficient farming system, e.g. the small farmer should increase his size, but not necessarily change his farming system.

The use of 'percentage return on capital'

Where the alternative farming systems, or the different farms being compared, have similar levels of capital investment, then the measure 'percentage return on capital' will rank correctly, as the farm with the greatest return must obviously have the greatest profit surplus. Thus, this measure can be used in such cases. In that, on an individual farm, most alternative systems will involve similar capital levels, the measure 'percentage return on capital' can have considerable use in simple budgeting work.

The opportunity cost of unpaid factors

When making comparisons of alternative systems on a particular farm, the opportunity cost of capital and the manager's 'labour and management' (WOM) should be charged at the level they could actually earn. When comparisons between farms, however, are being made these figures should be standardized as the opportunities are not the same for all farmers. Theoretically, the WOM figure should vary according to the manager's ability as this will affect his real opportunity cost. But this, in general, is difficult to assess. Note that the WOM calculated on the basis of a married man's wages plus 1% of TFC (this, or something similar, is a common approach) is *an erroneous technique* as it does not necessarily reflect the opportunity cost. Charging this 1% of TFC has the effect of increasing the opportunity cost of capital by 1%. (In general, larger farms do not necessarily have better managers than small farms. For a full discussion on managerial ability and its origins, see Nuthall, 2009.)

The need to measure efficiency

This discussion has implicitly assumed that there is a need to measure efficiency. This requirement is not, however, as essential as is commonly assumed. Consider the possible reasons for measuring efficiency:

- To indicate whether similar farms are more profitable and thus whether improvements can be made. However, it is fair to say that all farms can be improved – an efficiency measure is not required to indicate this.
- To enable farms to be ranked for comparative analysis and survey purposes. Where these techniques are being used there is no alternative to using an efficiency measure.
- To enable comparisons between alternative farming systems for a particular farm. Clearly, there is a need to record the relative merits of the alternatives. However, this does not have to be a comparable, formal measure. As most of the alternatives will involve the same level of the farmer's own capital and involve his own labour and management, these opportunity costs do not in general have to be considered (provided all alternatives are considered), so that a simple net profit ranking will be sufficient.
- General interest – perhaps for use in discussion groups and for comparing agricultural investments with off-farm possibilities.

7.9 Residual Imputation and Resource Valuation

It is sometimes suggested that efficiency measures that are residual measures give an indication of the value of the resource to which they are imputed. Thus, if a 12% return on capital is being achieved, it might be suggested that this is the value of capital so that if capital can be borrowed for less than 12% it would pay to do so. These assumptions, however, are in general incorrect. Residual values only give the true value of a resource (i.e. the marginal value product) under special circumstances, which are most unlikely to occur. Euler's theorem, which is discussed in most production economics texts, is concerned with this problem. Basically, the necessary conditions for a residual valuation to be correct are:

- The charge made for each input must equal its marginal value product (see Appendix 1 for a definition); and
- When all factors are charged a cost equal to their MVP, and the total is summed, there should be no unexplained residual.

Thus, while residual measures can be used to rank farms correctly in many cases, they cannot be used to value resources as these special conditions probably seldom occur. And, of course, if indeed the return on capital is 12%, as this is an average return, there is no guarantee that additional farm investment will return this level, and after all, it is the marginal return and cost that is important.

7.10 Concluding Comments

The use of survey-created farm standards and benchmarks is an appealing idea for helping farmers to improve their management. The discussion in this chapter, besides describing the processes used in creating standards, covers the advantages and problems with the system. The biggest advantage is that farmers can understand the system, it is easy to develop and it creates intense interest and debate between farmers and advisers. However, it has been pointed out that the results may well be misleading and at best must be treated with caution. This does not mean the idea of benchmarks should be discarded. Everyone needs some guidelines to work from in making decisions. Useful benchmarks should relate to ensuring farmers move towards investments and resource use that equate marginal returns and costs, and similarly equate marginal value products from alternative products. These benchmarks will differ from farm to farm, as each resource setup and financial background will differ. Furthermore, each farmer's and farm family's objective set will differ and should influence the appropriate benchmarks. In theory, it is possible to calculate correct benchmarks where production functions are known for each farm. In reality this does not occur, so benchmarks must rely on survey information and experience in adjusting these for each farm's circumstances.

With the wide availability of inexpensive computer systems decision support information systems (DSS) are increasingly becoming available to help farmers in their decision making. Computers also allow ready access to the World Wide Web and its vast array of information including price and cost information. This chapter has described some of the DSS available as they interconnect to survey-collected data. Decisions over which systems to use, and in some countries a wide array is on offer, must depend on a careful assessment of the costs and benefits. The costs are largely the time involved in data entry and software use, the benefits are the improved outcomes from better informed decisions. Any analyst involved in creating systems and helping farmers in their use must be very careful to assess these costs and benefits. They must also ensure farmers can observe the benefits in a simple manner, as if benefits are not obvious it will not be long before they discard a system. Farmers are very practical in this sense.

The chapter also contains questions about the value of whole farm production functions, pointing out that 'broad brush' functions may look sophisticated and elegant, but most do not allow conclusions on the detailed management systems that are required for practical decisions. This means it is better to work off detailed physical output production functions combined with expected costs and returns.

The chapter concludes with comments on efficiency measurement and residual valuation that should provide students with food for thought over conventional wisdom. Consequently, students should have had their thinking challenged, leading to a clearly understood conclusion on efficiency and valuations.

References and Further Reading

Battese, G.E. and Coelli, T.J. (1995) A model of technical inefficiency effects in a stochastic frontier production function for panel data. *Empirical Economics* 20, 325–332.

Blagburn, C.H. (1957) *A Simple System of Economic Analysis of a Farm Business*. University of Reading, Miscellaneous Studies No 8.

Burns, E.O. (1966) Comparative analysis of farm accounts. *Australian Journal of Agricultural Economics* 10(2), 169–182.

Candler, W. and Sargent (1962) Farm standards and the theory of production economics. *Journal of Agricultural Economics* 15(2), 282–290.

Cooper, I.M. (1995) *Comparative Analysis Revisited*. Farm Management Society, University of Adelaide, Roseworthy, South Australia.

Datta, K.K., Sharma, V.P. and Sharma, D.P. (1998) Estimation of a production function for wheat under saline conditions. *Agricultural Water Management* 36(1), 85–94.

Giles, B.D. (1950) Real costs and accounting costs in agriculture. *The Farm Economist* 6(8), 221.

Mallyon, C.A. (1966) *Principles and Practice of Farm Management Accounting* (Ch. 21, 22). Law Book Company, Sydney.

Nuthall, P.L. (2009) *Farm Business Management. The Human Factor*. CAB International, Wallingford, UK.

Rasmussen, K. (1952) The importance of variance in the interpretation of averages. *Journal of Agricultural Economics* 10(3), 226.

Rowe, A.H. (1971) Computerised Systems of Accounting and Control in Farm Business Management. In: *First International Congress on Farm Business Management*. British Farm Management Association, Warwick, UK.

Swanson, E.R. (1953) Profit maximisation and measures of farm success. *Journal of Farm Economics* 35(4), 628.

Tansey, A.W. (1957) Significance of variation in annual farm profits. *Journal of Agricultural Economics* 12(3), 326–336.

8 Constructing Improved Systems

8.1 Introduction

This and subsequent chapters are all about prescriptive analysis, that is, creating improved management systems from first principles. The first chapters in this book covered farmers' objectives, with particular emphasis on the importance of uncertainty and timing, and how longer term investments should be analysed. The determination of a farmer's (and family's) objective is clearly the first step in any analysis as it is necessary to know on what basis to choose between alternatives. The chapter contents then moved on to the recording and analysis of existing systems. This is the second step in any analysis as information must be available on which to base future decisions. This information can also be used directly for making recommendations about future courses of action (comparative analysis, whole farm production functions etc.).

There is, however, an alternative approach that can be used for making recommendations. This is the construction and analysis of *proposed systems* using basic technical information. A simple budgeting study on a farm is an example of this approach. That is, rather than simply compare systems that are currently in existence, all possible alternative methods of using the available resources are examined in order to select the best system. This approach has, in general, obvious advantages compared with the 'analysis of existing systems' approach. The remaining chapters cover discussions on this approach.

This chapter will consider *representative farms*. The reason is that when improved systems are developed it is important, in a general advisory/consultancy sense, for these systems to be developed for *typical situations* so that they are as generally applicable as possible.

The remaining topics and chapters will be concerned with methods of developing and analysing farming systems for particular farms. Such methods as budgeting, linear programming and general simulation will be discussed. Further, special attention will be given to analysing such part-farm problems as the optimal time of replacing machinery, livestock or other assets that require periodic renewal. It should also be noted that cost–benefit analysis (Chapter 4) can also be classed as prescriptive as it often involves budgeting out proposed investments, though it can also be used to evaluate actual and completed investments for which many records exist and can, therefore, provide data for an ex-post analysis. Results can then guide future investments.

8.2 The Study Unit – Representative Farms and Case Studies

When improved farming systems are developed this must be for some form of study unit (Carter, 1963). This may be a farm for which a consultant is developing a system or, particularly in research activities where general recommendations are required, a farm *typical* of many should be selected as the study unit. This is referred to as a *representative farm* that has characteristics similar to a large number of farms. Such farms can also be referred to as *case studies*, though any farm, whether representative or not, can similarly be referred to as a case study.

Using representative farms, improved farming systems can be developed using rigorous analytical techniques, then on each specific farm budgeting and other less expensive techniques can be used to adjust the general recommendations to the particular farm. The time involved in using the detailed analytical techniques means that it is often too expensive to apply them on individual farms.

Representative farms can be used for a wide range of types of study, as discussed later. The problem being addressed will affect the characteristics on which it should be selected. Thus, for example, if the farm is being used to estimate the effect of a new taxation policy, it should be selected on the basis of, for example, taxable income and family structure as these are the characteristics affecting income tax. If, as another example, the farm was being used to construct improved farming systems, then it should be selected so that it is typical of the characteristics affecting choice of system (e.g. mainly soil type, topography, rainfall, area).

A representative farm is usually an *actual property* selected from amongst a defined population of farms. The alternative farming systems are not usually physically experimented with on the property but rather developed on paper using symbolic models, e.g. a budget. In some cases a hypothetical farm is constructed on the basis of the *average statistics* of some population (e.g. average size, typical soil type, 20% non-cultivatable etc.) and referred to as a representative farm. This kind of 'representative' farm has limited use as it is unlikely to be *typical* of any group as few, if any, actual farms will exhibit the average statistic values for all statistics.

8.3 Uses of Representative Farms

Representative farms can be used to analyse and solve a wide range of problems. Many of these are the same as for surveys. Examples of important uses are:

- To determine improved farming systems.
- To evaluate new products and management systems and to provide the relevant input–output information for later use in budgeting and other analytical techniques. To be effective the alternatives should be physically experimented with on a property. This property may belong to a cooperating commercial farmer or to a research establishment. Products being tried on commercial properties would probably have shown promise in plot trials.
- To determine detailed input–output information, which is then used in planning similar farms. Recording schedules can be provided to cooperating farmers, perhaps with secretarial help.
- To evaluate the effect of government policy measures. A number of representative farms should be selected to determine the effect of the measures. To indicate the population effect, the results from each representative farm would be multiplied by the number of farms it represents.
- To estimate product supply functions (a product supply function indicates the quantity that would be produced given the farmers receive various prices; Sheehy and McAlexander, 1965). To determine this function, a range of representative farms should be used and a function estimated for each. These functions would then be aggregated to form a whole area function. For example, this could be done for wheat so that a government could determine the price necessary to obtain sufficient wheat for the national demand.
- To evaluate the effectiveness of various extension methods. A number of representative farms should be selected and each manager exposed to a different extension method. The effects would be recorded over time. The important characteristic on which to base the selection of the farms would be managerial ability and personality and also similar soils, area, available finance and other critical parameters, as otherwise the potential for change will differ between farms.
- As an extension device. Representative farms can be used to demonstrate various management aspects. Examples include:
  - Demonstrating, and showing, a particular farming system, or parts of a whole farming system (this might be a cooperating commercial unit or a research/advisory organisation's property);
  - Demonstrating planning methods and the resulting improved systems. The farm is used as a study unit on which to show farmers how to, e.g. construct budgets. The important 'characteristic' on which to base such a representative farm would be the type and complexity of budget necessary to plan properties in the district.

With a little imagination many more examples can be created. There is little to limit what is possible, except that the paper experiments will clearly not provide information on what farmers are actually achieving. Such information must rely on a survey of some kind. In the age of electronics a 'survey' can extend to, e.g. estimates of wheat output from satellite photographs and their analysis.

8.4 The Advantages of Representative Farms

The advantages largely relate to the differences between the survey and representative farm approaches. Thus:

- Realism. As a representative farm is a real world situation the advisor/researcher is more likely to develop realistic systems compared with the case where hypothetical situations are used as a basis for study. With hypothetical situations there is no farmer for the researcher to check the practicability of developed systems.
- Objectivity. As *all* alternative farming systems can be compared, the selection of an improved system is completely objective. In surveys you are limited to a comparison of the systems the farmers are actually using.
- Detail. The study carried out on the representative farm can be as detailed as required. In survey work the amount of detail is limited to the poorest set of records available, whereas with representative farms detailed recording systems can be initiated. In representative farm work the emphasis is placed on developing the basic technical production relationships and constructing improved systems from this information, not from simple observations of what appear to be successful systems.
- Variation in planning parameters. As the analysis is based on the basic technical relationships, the effect of variations in prices and costs can be readily estimated. Further, given probability information, the profit distributions of each alternative farming system can be determined. Similarly, it is a simple process to explore the effect of variations in input–output parameters, e.g. wheat yield, stock carrying capacity and so on. In survey-type work these kinds of analyses are not possible as the existing systems are simply recorded, though in some cases where sufficient physical information is collected limited analysis of this kind can be carried out.
- Advantages in extension work. Enables contact with many farmers at little expense through field days. Farmers are attracted to seeing what others are doing, and in this case, hearing about what are regarded as improved systems worked out for a real farm. Furthermore, it enables concentration on a defined group of farmers as only farmers with a similar farm will attend a field day. Methods such as radio talks must be more general as the audience is quite general.

Overall, as researchers are dealing with a real case study the results of analyses are more likely to be realistic and create a real interest from farmers due to the direct comparisons they can make to their own situations.

8.5 The Disadvantages and Problems of Representative Farms

- Large homogenous areas are desirable. For the representative farm approach to be useful there must be large groups of relatively similar farms. If this is not the case each representative farm will have limited application. Similarly, the same applies to the results of a survey for a defined population.

- The generality of results. The results obtained only apply in their entirety to the particular case study chosen as the representative farm, as each farm is unique (thus, the importance of selecting ‘good’ representative farms). Provided, however, a sufficient number of similar groups of farms are constructed and a representative farm selected from each, then some general interpolation should be possible, particularly where solutions are obtained for a range of resource combinations. Similarity is more likely where small groups are created (the same problem applies to any form of research work).
- Lack of statistical significance.
  - When representative farms are being used to actually experiment with farming systems, the lack of replication prevents giving the results a statistical backing so that, for example, chance weather effects may give misleading results. (Replication is seldom used due to the expense involved.) However, such trials can be carried on for a number of years, partially overcoming this problem.
  - In modelling work on representative farms, the systems developed are not actually physically evaluated before recommendations are made. This means that any errors in selecting the correct technical relationships will not become evident until real farm mistakes are made. Thus, recommended systems cannot be guaranteed on statistical grounds. However, recommendations can be tried on a few case farms before general promulgation.
  - As it is usually impossible to measure all factors or characteristics that determine whether a farm is representative or not, it is not possible to categorically state that a particular farm is representative.
- Disadvantages in extension.
  - A complete farming system may be too much for a farmer to fully understand and remember in one visit. Thus, the use of printed material in conjunction with field days may be necessary.
  - Unless the optimal farming system is always changing, a representative farm purchased and used as an extension device becomes of little use and interest. It may need to be sold and another purchased.

8.6 The Selection of Representative Farms

The steps involved in selecting representative farms include the following.

Determining the characteristics on which to base the selection

The important characteristics will be those affecting the problem under study. Thus, for example, if developing improved farming systems, profitability factors should form the basis of selection. However, this only relates to the 'fixed factors' that cannot be changed by the managers. For example, product types would not normally be a selection factor as they can be changed in an analysis. On the other hand, basic factors such as farm size, rainfall, topography, soil type and managerial ability are more likely to be selection candidates. Furthermore, while important 'fixed factors' such as, for example, available finance and milk production quota level are not directly controllable by the manager, they do not give rise to the basic technical production relationship and can easily be varied in any analysis on a representative farm.

Collecting the information on which to base the selection

This may be available from normal records kept by various organizations, e.g. a milk processing cooperative or local-body tax authorities. Alternatively, some kind of survey will be necessary to obtain the information.

Determining how many representative farms should be selected

Decide how wide a range of the selection factors a representative farm can adequately represent. For example, farms within some particular population may range in size from 200 ha to 600 ha. After examining some it may appear that if they are divided into groups with a range of around 150 ha, the results from one representative farm from each group can be expected to generally apply to all the farms within the particular group. Costs will clearly influence decisions as analysts will want the funds available to have as great a benefit as possible.

Select the farm, or farms

Divide the population into similar groups and select a *modal* farm from each. Where only a limited number of representative farms can be selected these should be selected from the groups with the largest number of farms. For example, a sample of 100 farms might fall into the following groups:

Soil	No. of farms	Area (ha)	No. of farms
		0–200	180
Heavy soil	320	201–300	120
		301+	20
		0–200	100
Light soil	680	201–300	150
		301–400	360
		401+	70

(Note, these groups might be further divided into smaller groups, according to some other factor.)

A representative farm could be selected from each group. Given limited resources, however, a farm would be selected from the 301–400 ha light soil group and the 0–200 ha heavy soil group depending on the resources available. These are the groups with the largest number of farms.

A special case where a factor that conceptually affects the profitability of farms should be ignored is where this factor never limits production. This is clearly most unusual. An example might be where available finance is non-limiting. Some kind of pilot analysis will be necessary to determine whether this situation exists.

8.7 Concluding Comments

A review of research and extension journals will show many studies rely on representative farms as the study unit. To have as much use as possible, logic suggests they should be selected on the basis of the factors that have the most influence on the problems being studied, and that actual farms are used in contrast to hypothetical creations based on average attributes of a population of farms. Such hypothetical farms may not be representative of any actual situation and consequently conclusions would be of little use. Selecting representative farms requires prior knowledge of the farm population, and if this is not available preliminary surveying may be necessary if organizations holding farm information do not exist, or the information is not publicly available. In the latter situation, sometimes it is possible to get the organization itself to carry out a selection process using instructions provided by the researcher.

The selection example provided uses a simple grouping process. In large studies it may be more appropriate to use the statistical techniques of factor and cluster analyses (see Kobrich *et al*., 2003). Intelligent factor analysis can suggest which factors are critical, and cluster analysis groups farms into cells based on these factors. Representative farms can then be selected from each cell.

References

Carter, H. (1963) Representative farms – guides for decision making. *Journal of Farm Economics* 45(5), 14–48.

Kobrich, C., Rehman, T. and Khan, M. (2003) Typification of farming systems for constructing representative farm models: two illustrations of the application of multi-variate analysis in Chile and Pakistan. *Agricultural Systems* 76(1), 141–157.

Sheehy, S. and McAlexander, R. (1965) Selection of representative benchmark farms for supply estimation. *Journal of Farm Economics* 47(3), 681.

9 Methods and Models of Income Variability Reducing Techniques

9.1 Introduction

While risk and uncertainty have been described in Chapters 2 and 3, the active steps a farmer can take to reduce income variability were not considered. As a farmer uses profit distributions (risk) to distinguish between farming systems it is important to consider the methods available to reduce risk when constructing plans. A risk averter will be particularly interested even though, in general, the use of the techniques decreases expected profit so the reduction in variance is achieved at a cost. The critical question is whether there is a net gain in utility.

The discussion considers the analysis of profit variance, and thus the calculation of expected profit/variance curves to allow an informed decision between alternatives.

The techniques available are described, and the analytical background and methods demonstrated. This leads to a quantitative discussion on diversification being one of the main risk-reducing techniques. Finally, an example of how to construct a quantitative model for another technique is presented. There is no point in giving an analytical model for all the techniques as the models largely follow the same principles. For students who would like a less mathematically oriented introduction to risk and uncertainty, see Nuthall (2010). Some of the descriptive material is similar, but tests for assessing a farmer's attitude to risk not included in this book are provided here. At the other extreme, for a more extensive treatise on risk, see Hardaker *et al*. (2004).

9.2 Methods of Reducing Income Variability

Introduction

The main possibilities are discussed below. A particular farmer may not, however, be able to use all of them depending on his particular farm and the conditions.

Two basic approaches are possible: (i) active measures to reduce income variability so that the profit distribution's shape will be altered; and (ii) a passive approach of accepting the shape of the profit distribution but setting aside funds in relatively liquid form so that in unfavourable years finance is available. In effect, the 'cash in pocket' distribution is altered through this passive approach.

The techniques

Selection of products and production processes that exhibit low variability

Where there are alternatives with similar expected income levels per hectare, clearly the ones with the lowest variability should be selected. Whether this approach can be used depends on:

- The number of alternative products and production techniques available. In many cases there is little product choice though on most farms there will be a range of possible production methods;
- Given several alternatives, whether they exhibit a range of income variability levels.

Where opportunities to control the effect of the weather are available there is control over profit variability. The use of irrigation is an example of a production technique. But note that for different products the variability may well be much the same if they are produced in the same period of the year. On the other hand, if products can be produced in different, less variable periods the opposite is likely.

This approach is different from diversification, and works by selecting systems with low variability. Diversification involves selecting products, or production processes, on the basis of the *relationships* between the alternatives in the hope that a low return from one will be made up by a high return from another. Diversification is discussed in detail later.

Discounting future prices

To negate high variability products and processes, the expected prices are set at a relatively low figure so they will be less likely to be selected. This is clearly a kind of 'head in the sand' approach. In some cases it may be justified where the cost of using correct analytical procedure is not warranted. It will tend to select the products and processes that would be selected under the first alternative above.

Formal insurance

The adverse outcomes of many random variables can be insured against. For example, buildings, machinery and crops can be insured for loss by fire and other causes. Thus, the 'whole farm cash return distribution' range can be narrowed by removing the chance of large losses through paying a constant insurance premium. Insurance converts an uncertain loss into a certain cost every year.

There is a limit to formal insurance depending on which random variables companies will insure against and the costs involved. (They are, in effect, guaranteeing a payout if a *poor* value of a variable occurs.) While many unfavourable outcomes can be insured against, the cost will restrict insurance for items such as, e.g. a major tractor breakdown and very low wheat yields.

Whether insurance should be used depends on the shape of a farmer's utility function and the costs involved. Where a farmer takes the risk himself it is likely he estimates the insurance costs are greater than the expected losses. Remembering that insurance companies must make a profit and cover their operating and administration costs, their charges will cover items that a farmer does not have. However, because the potential loss through major fire, theft and severe weather is so large, a farmer may not be able to survive such an outcome. Thus, insurance becomes a question of survival in many cases.

Forward contracts

A farmer can enter into a range of forward contracts that reduce variability and, in some cases, make planning easier as future prices (costs) are known with certainty. Possible types include the following.

CONTRACTS IN MONEY Contracts in money are where a farmer contracts to sell or buy a product or input at some point in the future at a *defined price*. Such contracts make planning easier. They may reduce year-to-year profit variation depending on whether the contracted price varies more than the free market price. Their effect on average profit depends on the relationship between the contracted and free market price, but in general the average or expected price will be slightly lower. There are planning and stability advantages to a firm if it knows what price it will have to pay a farmer – thus, firms may be prepared to offer quite reasonable forward contracts.

The planning advantages of a fixed price and/or cost are considerable. If the beef price, for example, for the animals on hand is known the farmer can plan to sow, say, 40 ha of new pasture and know that he will have the money to buy the stock to graze the extra production. If the open market was used, a price drop could mean the farmer was not able to buy stock so in hindsight he would have been better not to develop. With forward contracts such planning errors are less likely, though in the long run average profit may be lower as forward contract prices tend to be lower than the average of free market prices.

CONTRACTS IN QUANTITY These are sometimes an extension of contracts in money. An agreement to supply, or receive, a defined quantity of a product, or input, at a given time, usually at a defined price is entered into. Occasionally the contract is for quantity only. This technique may be important where there are marketing difficulties. Equally, losses may occur if the quantity can not be supplied. In these cases the farmer may buy on the open market to fulfil the contract, or accept a penalty clause in the agreement if one exists.

CONTRACTS IN KIND Contracts in kind involve a physical product. Share farming is a form of ‘contract in kind’. The farmer uses an owner’s land and provides a share of the output as payment. In such contracts the *land owner’s risk* is reduced, as instead of paying a fixed wage to a manager, his payout is a variable quantity depending on the season, i.e. his cost is the production from the farm that the share farmer keeps, and vice versa. Similarly, the share farmer’s risk is reduced as his rent, the proportion of the product provided, varies with the season so that in a poor season his rent is lower.

Diversification

Diversification involves producing a number of products and possibly using a range of production processes in the hope that when one product produces a low return another will compensate with a high return. On average, the yearly returns remain relatively stable. This stability is achieved at a cost, as the most profitable products are not being exclusively produced and any economies of size are lost with the whole farm (or large sections of it) not producing the most profitable, on average, product.

Whether diversification is useful depends on whether there is a range of products and production processes available that do not exhibit a high positive correlation. Furthermore, the farmer’s managerial ability must be reasonable to offset the added complexity of diversification. Yields and input costs tend to deteriorate with complicated systems.

Flexibility

A flexible system enables rapid changes as conditions change. Flexibility reduces income variability as changes allow for changing prices, costs and environmental conditions. Usually this flexibility is achieved at a cost as such systems tend to be less technically efficient. The reasons will become clear in the following discussion. There are a number of ways flexibility can be introduced into a farming system, including the following.

TIME FLEXIBILITY

- Select products that take a short time to give a return, e.g. finishing lambs as against breeding and selling cows. Due to the shorter production time there is less chance of the expected prices and costs changing, *and* if conditions do change the farming system can be changed quickly to suit these conditions.
- Select products that can be sold at different stages, e.g. cattle can be sold as weaners, yearlings and 18 months. Thus, the farmer can take advantage of changing conditions (price, feed, labour, etc.).

COST FLEXIBILITY

- Do not invest in assets that are designed for the production of a single product and cannot be used for other purposes should prices or conditions change. Or, if a specialized building or asset is necessary, construct it out

of semi-permanent materials so that its cost will be low (in the long run the cost might be high if the product continues to be profitable).
- Invest in sufficient assets and equipment so all tasks can be carried out *no matter* what kind of conditions eventuate. For example, employ sufficient labour to enable jobs to be completed under all conditions. Another example would be the use of two tractors where one tractor plus some contracting would be sufficient on average. This approach means, for example, that expensive delays are avoided, though at a cost.

PRODUCT FLEXIBILITY Produce items that can have several uses so that changing the plan as conditions change is possible. For example, a field of cocksfoot grass can be grazed, made into hay or silage, or saved for seed production. Some breeds of cows can be used for milk production or as beef mothers.

Liquid asset structure

If liquid funds can be readily obtained, such funds can be used to:

- Maintain consumption at a standard level despite a profit drop;
- Enable the farm to get into the production of currently profitable products, e.g. to buy up beef for finishing when beef prices rise.

These liquid funds may be in the form of stocks and shares, cash in bank, readily saleable land and a host of other possibilities. In some cases such investments may be economically rational in their own right relative to the farm return. However, if they themselves are to be safe readily available investments their return will probably be lower than farm investments on average, so this strategy has a net cost, although, depending on the shape of the farmer's utility function, it may provide a net gain in utility.

The use of tax options

Some countries have a number of options in the income tax system that can be used to level off disposable cash income. Some are profitable in themselves, others are not. Examples include depositing money with the tax department and withdrawing in a bad year at which time tax is paid; deferring tax deductible expenditure in a poor year and using the tax credit when income recovers, thus smoothing taxable income; and predicting a realistic income level when paying provisional tax. The cost or profit associated with each scheme will depend on each particular case. The overall principle is to even out the taxable income, so in a progressive tax system the total tax paid on average is less.

Concluding comments

In general the techniques outlined will be used by:

- Risk averters;
- Farmers who need to ensure that their cash returns will never fall below a certain level due to their high level of fixed costs, which is necessary for survival.

However, in some cases, some of the techniques outlined may be profitable in their own right. For example:

- In a period of falling prices it may be profitable to become involved in forward contracts depending on the prices being offered;
- In a rapidly changing farming environment it may be profitable (as well as reducing variability) to use a flexible farming system.

9.3 Analytical Background to Estimating the Profit (and Other Outputs) Variability of Farming Systems

Introduction

The discussion on utility (Chapter 3, sections 3.10–3.12) indicated if a farmer had a quadratic utility function, as many do, *only* two profit distribution parameters are important in distinguishing between systems (expected profit and profit variance). It will be recalled $E(P)$ (expected profit) and $V(P)$ (profit variance) can be estimated from the information contained in a payoff matrix, which results from the construction of a decision tree. However, creating a decision tree can be extremely time consuming. Fortunately, in many cases estimates can be made by using the relationships between the random variables that make up profit. This section shows how this can be done. No proofs will be given though many will be self-evident. All the proofs rely on the random variable 'profit' being a defined combination of other random variables, e.g. price, yield, animal health requirement etc. so that the $E(P)$ and $V(P)$ will be some combination of the expected values and variances of the constituent random variables.

The general comments about the practical use of payoff matrices and utility functions apply just as much to this particular topic. For a consultant it is worthwhile to do one or two analyses on case studies to determine the nature of the general results so that they can be extrapolated from for each individual farm. It is also important advisers have a good understanding of the principles so that research results can be interpreted and subjective judgements made about such problems as diversification on the basis of good knowledge.

Calculating expected values

The expected value of a random variable multiplied by a constant

Let V_j = the jth value of a random variable;

P_j = the probability of the jth value occurring;

A = a constant;

$R = A \times V$ where R is the random variable (perhaps profit) whose expected value is to be calculated.

Then,

$$E(R) = \sum P_j A V_j = A \sum P_j V_j = A \times E(V)$$

i.e. the expected value is simply multiplied by the constant.

Thus, for example, if V represents the gross margin per hectare for wheat, then the expected total gross margin as a result of growing 200 ha of wheat would be:

$$E(\text{total gross margin}) = 200\ E(V)$$

Calculating the expected value of a random variable formed from several other weighted random variables (important in estimating the expected profit from the whole farm)

Let R_i = the ith value of a random variable;
W_j = the weight associated with the ith random variable.

$$S = W_1R_1 + W_2R_2 + \ldots.. = \sum W_jR_i$$

Where S is the random variable for which the expected value is to be calculated. Then:

$$E(S) = E(W_1R_1) + E(W_2R_2) + \ldots$$
$$= W_1E(R_1) + W_2\ E(R_2) + \ldots$$

That is, the expected value of the new random variable S is simply the sum of the weighted constituent expected values. As an example, consider a mixed cropping farm growing 100 ha of wheat, 50 ha of barley and 20 ha of peas, then the expected total gross margin would be:

$$E(\text{TGM}) = 100\ E\ (\text{wheat gross margin/h}^+\text{a}) + 50\ E\ (\text{barley gross margin/ha}) + 20\ E\ (\text{peas gross margin/ha})$$

Calculating variances

Variance of a random variable which is multiplied by a constant or weight (important in calculating the whole farm profit variance)

Let A = a constant;
V = a random variable;
$W = A \times V$, where W is the random variable whose variance is to be calculated.
Then:

$$\text{Variance }(W) = A^2 \times \text{Var}(V)$$

where Var(V) is a symbol to represent the variance of the random variable.

For example, if the variance of the wheat gross margin/hectare is \$260, and a farmer grows 100 ha of wheat, the variance of the total gross margin from the 100 ha would be:

$$\text{Var (TGM)} = 100^2 \times 260$$

The definition of covariance and the correlation coefficient

In order to estimate the variance of a combination of several random variables, covariances and correlation coefficients must be known.

Covariance

Covariance is defined as the expected value of the product of the deviations from the means of two random variables. That is:
where X and Y are two random variables and

X_i and Y_i represent the ith possible values of these variables;
P_i = the probability of the ith value set occurring;
$\bar{X}$ and $\bar{Y}$ = the expected values (or means) of the variables.

Then,

$$\text{Cov}(X,Y) = \sum P_i(X_i - \bar{X})(Y_i - \bar{Y})$$

Note: to estimate the covariance we need to know the probability of each of the possible value *combinations* of X and Y occurring. Covariance can also, of course, be estimated from a set of *paired observations*. In this case,

$$\text{Cov}(X,Y) = \frac{\sum \left(X_j - \bar{X}\right)\left(Y_j - \bar{Y}\right)}{n}$$

where X_j and Y_j is the jth observation;
n = the number of observations.

Correlation Coefficient

The correlation coefficient between two random variables X and Y is defined as:

$$\text{correlation coefficient} = \rho = \frac{\text{Cov}(X,Y)}{\sigma_x \times \sigma_y}$$

where σ_x and σ_y = standard deviations.

Thus:

$$\text{Cov}\ (X,Y) = \rho \times \sigma_x \times \sigma_y$$

The correlation coefficient measures, of course, the relationship between two random variables.

The possible extreme values of the correlation coefficient are:
$\rho = 1$, there is a perfect positive correlation;
$\rho = 0$, there is no correlation, i.e. variables are *independent*;
$\rho = -1$, there is a perfect negative correlation (i.e. if one variable is at a high value the other will have a low value).

For most random variables ρ will lie *between* +1 and −1, rather than be at one extreme or the other.

Farm examples

Some practical farm examples are:

- The random variables wheat yield/ha and barley yield/ha are probably positively correlated at quite a high level as a good season will give a high yield for both crops;
- The random variables price/lamb at a market *and* the number of lambs being put into the market are likely to be negatively correlated.

The variance of two random variables

Let X and Y be two random variables.

$$S = X + Y$$

Then

$$\mathrm{Var}(S) = \sigma_x^2 + \sigma_y^2 \times 2\mathrm{Cov}(X, Y)$$

$$= \sigma_x^2 + \sigma_y^2 + 2\rho\sigma_x \times \sigma_y$$

Where

$\sigma_X^2 = \mathrm{Var}(X)$;

$\sigma_Y^2 = \mathrm{Var}(Y)$.

As an example, the variance of the total gross margin from a farm, where half the farm is in sheep and the other is in wheat, would be:

$$\mathrm{Var(TGM)} = \sigma^2_{\text{sheep}} + \sigma^2_{\text{wheat}} + 2\rho\, \sigma_{\text{sheep}} + \sigma_{\text{wheat}}$$

Note that the sheep and wheat gross margin variance figures must be for the half farm and not on a per hectare basis, i.e.

σ^2_{sheep} = the ½ farm *total* gross margin variance from the half of the farm which is in sheep

A formula given below will show how to estimate the variance using per hectare variance figures.

The variance of a weighted sum of two random variables

Let X and Y be two random variables, and A and B be two constants.

Then the variance of a new random variable which is

$$S = AX + BY$$

is given by

$$\mathrm{Var}(S) = A^2\, \sigma_x^2 + B^2\, \sigma_y^2 + 2AB\rho\sigma_x\sigma_y$$

As an example, the variance of total gross margin for a 300 ha farm which has 200 ha devoted to sheep and 100 ha of wheat, where the *per hectare* variances of gross margin are given by σ^2_{sheep} and σ^2_{wheat}, will be:

$$\text{Var(TGM)} = (200^2\ \sigma^2_{\text{sheep}}) + (100^2\ \sigma^2_{\text{wheat}}) + (2 \times 200 \times 100 \times \rho\sigma_{\text{sheep}}\ \sigma_{\text{wheat}})$$

Note that whole farm variance figures will be extremely large and are also difficult to interpret. A figure that is easier to work with is the standard deviation. If it is assumed that the shape of the profit distribution is approximately a normal distribution (and this will not always be the case), then it can be assumed that 95% of all outcomes will lie within the range:

$$E(\text{Profit}) \pm 1.96\sigma$$

As the normal distribution is asymptotic to the origin and most farm variable distributions have finite ranges, a more realistically useful figure to use is the 70% probability range. Approximately 70% of all outcomes lie within E(Profit) $\pm$ 1.0σ, given a normal distribution.

These relationships make a variance figure easier to interpret once it has been converted to the standard deviation:

$$\text{Std. Dev.} = \sigma = \sqrt{\sigma^2}$$

The variance of a weighted sum of *n* random variables

This is an extension of the above formula for many random variables.

Let W_i = the weight associated with the ith random variable;
X_i = the ith random variable;
Q_{ii} = variance of X_i;
Q_{ij} = covariance of X_i and X_j, $i \neq j$
(recall that Cov(X_iX_j) = $\rho\sigma_{xi}\sigma_{xj}$).
Then, a random variable S which is given by:

$$S = W_1X_1 + W_2X_2 + \ldots + W_nX_n = \sum_{i=1}^{n} W_iX_i$$

has a variance given by:

$$\begin{aligned}\text{Var}(S) = {} & W_1X_1Q_{11} + W_1X_2Q_{12} + \ldots + W_1W_nQ_{1n} \\ & + W_2X_1Q_{21} + W_2X_2Q_{22} + \ldots + W_2W_nQ_{2n} \ldots \\ & + W_nW_1Q_{n1} + W_nW_2Q_{n2} + \ldots + W_nW_nQ_{nn}\end{aligned}$$

Note that:

(i) the diagonal in the above table of numbers is in fact $\mathrm{W}_\mathrm{i}^2\ \sigma_\mathrm{i}^2$

(ii) the term $W_iW_jQ_{ij}$ ($i \neq j$) appears above *and* below the diagonal because $W_iW_jQ_{ij} = W_jW_iQ_{ij}$ (i.e. the subscript ordering is not important). Thus, all the terms, say, above the diagonal can be removed and those below multiplied by 2 to give:

$$\text{Var}(S) = W_i^2\,\sigma_i^2 + W_2^2\,\sigma_2^2 + \ldots + W_n^2\,\sigma_n^2$$
$$+\, 2W_2W_1Q_{21} + \ldots + 2W_{n-1}\,W_nQ_{n-1n}$$

The Q_{ij} terms can, of course, be replaced with $\rho\sigma_{xi}\,\sigma_{xj}$

As an example of the use of this formula, consider a farm of 300 ha which has the following land utilization:

70 ha wheat
30 ha barley
30 ha peas
170 ha devoted to sheep

and for which the variance and covariances of the gross margins per hectare are given by:

Variances = $\sigma^2{}_{\text{wheat}}\ \sigma^2{}_{\text{barley}}$ etc.
Covariance = $Q_{\text{wheat sheep}}\ Q_{\text{wheat, barley}}$ etc.

Then,

$$\text{Var(Total gross margin)} = 70^2\,\sigma^2_{\text{wheat}} + 30^2\,\sigma^2_{\text{barley}} + 30^2\,\sigma^2_{\text{peas}} + 170^2\,\sigma^2_{\text{sheep}} + (2\times70\times30Q_{\text{wheat, barley}}) + (2\times70\times30Q_{\text{wheat, peas}}) + (2\times70\times170Q_{\text{wheat, sheep}}) + (2\times30\times30Q_{\text{barley, peas}}) + (2\times30\times170Q_{\text{barley, sheep}}) + (2\times30\times170Q_{\text{peas, sheep}})$$

Note that fixed costs have been left out of all the examples so that the variances calculated in the above way do not, on the surface anyway, appear to be the whole farm profit variance. However, where fixed costs are *constant* then they will not affect the variance, i.e.

Var(total gross margin) = Var(farm profit)

provided fixed costs are constant. This is a reasonable assumption in most cases. It must be remembered, however, that E(farm profit) = E(total gross margin) – fixed costs.

9.4 Diversification

Introduction

Under certain conditions diversification can be used to reduce total farm profit variance. This section contains a discussion of these conditions and how to determine the land allocation necessary to reduce profit variance to a minimum using diversification. The resultant loss in expected profit is also discussed relative to whether the reduced income variability is worthwhile.

The conditions under which diversification will reduce profit variance

To obtain a general understanding a number of simple cases are examined.

Assume constant variance and zero covariance

Given a number of products with the same profit variance per hectare *and* where the profit covariance from the products is *zero*, then the greater the number of products produced, the smaller will be the total farm profit variance.

To see this,

Let K = farm area in hectares;

σ^2 = the constant profit variance per hectare (i.e. σ^2 is the same for all products);

x = the number of products produced.

Then, the area of the farm in each product = $\frac{K}{x}$ where an equal area of each is produced.

Thus, the total farm profit variance is:

$$\mathrm{Var}(P) = \left(\frac{K}{x}\right)^2 \sigma^2 + \left(\frac{K}{x}\right)^2 \sigma^2 + \ldots + \left(\frac{K}{x}\right)^2 \sigma^2$$

and where there are x such terms (the covariance terms are equal to 0 as assumed covariance = 0)

$$= x\left(\frac{K}{x}\right)^2 \sigma^2$$

But,

$$1\left(\frac{K}{1}\right)^2 \sigma^2 \rangle\ 2\left(\frac{K}{2}\right)^2 \sigma^2 \rangle\ 3\left(\frac{K}{3}\right)^2 \sigma^2 \rangle \ldots$$

as

$$(K)^2 \rangle\ 2\left(\frac{K}{2}\right)^2 \rangle\ 3\left(\frac{K}{3}\right)^2 \rangle \ldots$$

Note: this can be proved by induction, i.e. trying out a few simple cases.

Thus, the greater the value of x, the smaller will be the total variance. This reduction is greatest as x increases from 1, but as x gets larger the variance drop rapidly declines. This is shown in Fig. 9.1.

This case of zero covariances and an equal profit variance for all products is unrealistic. However, it indicates the effect of increasing the number of products produced without a confounding effect brought about by the other parameters. Further, the above discussion assumes that the same area of each product is produced. This may be unrealistic.

Assume constant variance and unity correlation coefficient

Assume that all the products available have equal profit variance per hectare and *all* combinations have a profit/hectare correlation coefficient equal to *one*.

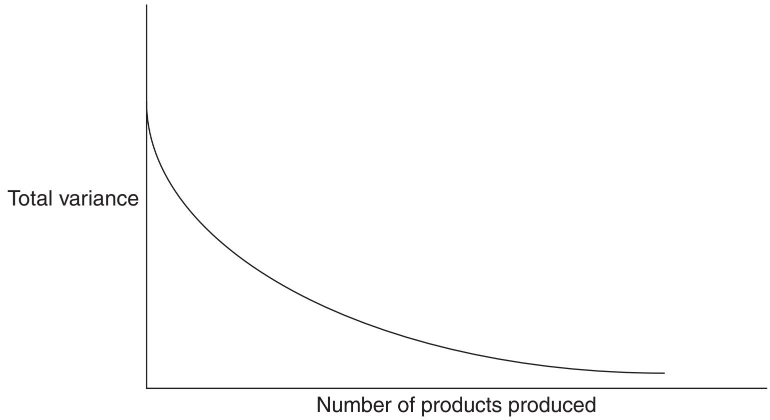


Fig. 9.1. Variance reduction with an increasing number of products produced.

Then, as the number of products produced increases, the total farm variance remains constant. To see this, assume

K = farm area in ha

Let σ^2 = the constant variance, and consider the case where two or three products are produced (assume equal area of each).

Now, as σ^2 is the same for each product and $\rho = 1.0$, then

$\rho\sigma_1\ \sigma_2 = \sigma^2$

Thus, *to show* that the number of products produced does not affect the variance, *it must be shown that:*

$$\left(\frac{K}{2}\right)^2\sigma^2 + \left(\frac{K}{2}\right)^2\sigma^2 + 2\left(\frac{K}{2}\right)\left(\frac{K}{2}\right)\sigma^2$$
$$= \left(\frac{K}{3}\right)^2\sigma^2 + \left(\frac{K}{3}\right)^2\sigma^2 + \left(\frac{K}{3}\right)^2\sigma^2 + 2\left(\frac{K}{3}\right)\left(\frac{K}{3}\right)\sigma^2$$
$$+ 2\left(\frac{K}{3}\right)\left(\frac{K}{3}\right)\sigma^2 + 2\left(\frac{K}{3}\right)\left(\frac{K}{3}\right)\sigma^2$$

where the left-hand side of the equation is the total variance where two products are produced and the right-hand side gives the total variance where three products are produced.

This equation can be simplified by dividing both sides by σ^2 and by collecting terms to give:

$$4\left(\frac{K}{2}\right)^2 = 9\left(\frac{K}{3}\right)^2$$

$\therefore\ 4 \times 0.5K \times 0.5K = 9 \times 0.3333K \times 0.3333K$

$$\therefore\ 4 \times 0.25K^2 = 9 \times 0.1111K^2$$

$$\therefore K^2 = K^2$$

i.e. the equality does hold.

Thus, if a farmer can choose between a number of products, each of which has the same profit variance per hectare, and the correlation coefficients are equal to 1, diversification *will not* reduce income variability. Again, it must be noted that these conditions are most unlikely to occur. However, this conclusion helps to indicate the nature of the problem. These last two cases primarily indicate the importance of the correlation coefficient and of the number of products produced. Diversification does not reduce income variability given a perfect positive correlation between the profit per hectare on each product. The other extreme, which is seldom, if ever, found in reality, of a perfect negative correlation does, however, markedly reduce total farm profit variability.

Assume there are two alternative products and the correlation coefficient is zero

In order to obtain a rather more specific relationship, this example will be limited to considering the case where the farmer has available only two alternative products. It will be assumed that the profit per hectare for the two products is *not* correlated. The above examples assumed that an equal area of each product was produced. This example will indicate *the importance of selecting the correct area* for each product for the profit variance to be reduced. To see this, consider the following situation:

Let R = area of the farm;

σ_A^2 and σ_B^2 = profit variances/ha of each product.

Thus, if only product A is produced, total farm variance will be given by:

$$\text{Var}(P)_A = R^2\sigma_A^2$$

But, if Q ha of product B are produced, then

$$\text{Var}(P) = (R-Q)^2\ \sigma_A^2 + Q^2\sigma_B^2$$

(Note: as $\rho = 0$, all the covariance terms = 0.)

$$\therefore\ V(P)_{A\&B} = R^2\sigma_A^2 + Q^2\sigma_A^2 - 2RQ\sigma_A^2 + Q^2\sigma_B^2$$

Thus,

$$\text{Var}(P)_A - \text{Var}(P)_{A\&B} = -\ Q^2\ \sigma_A^2 + 2RQ\sigma^2_A - Q^2\sigma_B^2$$

Thus, for the production of both products to reduce profit variance compared with producing only product A, the right-hand side of the above equation must be positive. That is,

$$-\ Q\sigma_A^2 + 2RQ\sigma_A^2 - Q^2\ \sigma_B^2 > 0$$

$$\therefore\ 2RQ\sigma_A^2 > Q^2\sigma_A^2 + Q^2\sigma_B^2$$

$$\div\, Q\ \therefore\ 2R\sigma_A^2 > Q\sigma_A^2 + Q\sigma_B^2$$

$$\div \sigma^2_A \quad \therefore 2R > Q + Q\frac{\sigma^2_B}{\sigma^2_A}$$

$$\therefore 2R > Q\left(1 + \frac{\sigma^2_B}{\sigma^2_A}\right)$$

That is, for a total farm profit variance reduction to occur, Q (the area of product B) must be selected such that the above relationship holds. Clearly, Q must also depend on the relationship between σ^2_B and σ^2_A.

For example, let $\sigma^2_B = 2\sigma^2_A$
then, substituting in the above relationship,

$$2R > Q\left(1 + \frac{2\sigma^2_A}{\sigma^2_A}\right)$$

$$\therefore\ 2R > 3Q$$

$$\therefore Q < \frac{2}{3}R$$

i.e. area of product B must be less than two-thirds of the total farm area for the profit variance to be reduced.

Summary

The previous examples show that:

- the value of the correlation coefficient(s) affects whether diversification will reduce profit variability. All other variables being equal, if $\rho < 1.0$, diversification will reduce profit variance;
- the greater the number of products produced, the smaller will be the profit variance, given $\rho < 1.0$;
- all other variables being equal, the product variances will affect the areas of the products that need to be produced in order to minimize profit variance (as will the correlation coefficients also).

Optimal land allocation

The examples above show the important diversification relationships. In some cases, however, it may be necessary to determine a minimum profit variance farming system using diversification. Given the formula for the total farm variance, calculus can be used to determine this farming system.

Consider the case where there are two alternative products:
Let q = proportion of farm in alternative A
$\therefore\ 1 - q$ = proportion of farm in alternative B
Let σ^2_A and σ^2_B = total farm variance assuming the farm produced *all* A *or all* B respectively.
Then, the total farm variance is given by:

$$\text{Var}(P) = q^2\sigma^2_A + (1 - q)^2\sigma^2_B + 2q(1 - q)\,\rho\sigma_A\sigma_B$$

Thus, to determine the combination of A and B giving minimum variance the equation giving the slope of this Var(P) equation must be set equal to 0 and the result solved for q (the slope of the Var(P) equation will equal 0 when it is at a minimum).

Thus, slope of Var(P) equation is given by:

$$\frac{d(\mathrm{Var}(P))}{dq} = 2q\sigma^2_{\mathrm{A}} - 2(1-q)\sigma^2_{\mathrm{B}} + 2(1-2q)\rho\sigma_{\mathrm{A}}\sigma_{\mathrm{B}}$$

Setting this equation equal to 0 and solving for q gives:

$$q = \frac{\sigma^2_{\mathrm{B}} - \rho\sigma_{\mathrm{A}}\sigma_{\mathrm{B}}}{\sigma^2_{\mathrm{A}} + \sigma^2_{\mathrm{B}} - 2\rho\sigma_{\mathrm{A}}\sigma_{\mathrm{B}}}$$

For example:
Let σ^2_{A} = \$1,000,000
$\therefore \sigma_{\mathrm{A}}$ = \$1,000
σ^2_{B} = \$640,000
$\therefore \sigma_{\mathrm{B}}$ = \$800
$\rho = 0.2$

$$\therefore q = \frac{640{,}000 - 160{,}000}{1{,}000{,}000 + 640{,}000 - 320{,}000}$$

$= 0.36$
i.e. 36% of the farm should be in alternative A and 64% in alternative B.

Sacrifice in expected profit

When a property is diversified expected profit will usually decrease, assuming the property was using a profit maximizing farming system before diversification occurred. A major decision is how far to diversify. In many cases the plan giving minimum profit variance will not be optimal. The shape of the farmer's utility function may mean the income sacrifice will drop utility more than the increase from lower risk. Thus, a decision must be made regarding how far to diversify. This will depend on:

- the attitude of the farmer to risk situations;
- the variances and covariances of the alternative products available.

To estimate an optimal solution the expected utility for a number of alternatives can be determined and compared.

In some practical situations, diversification does not markedly reduce variability. Consider the example given where the optimal combination was 36% of the farm area in alternative A. Further, assume:
Area of property = 500 ha
Expected profit per hectare of alternative A = \$160
Expected profit per hectare of alternative B = \$120
First, consider an optimal plan:
Thus, maximum E(profit) = 500 × 160 = \$80,000
(i.e. in order to maximize profit the farmer should produce all A)
Recall that σ^2_{A} = 1,000,000

$\sigma^2_B = 640{,}000$

where these are whole farm variances.

Thus, the optimal farming system (all A) has a variance of:

$\text{Var}(P) = 1{,}000{,}000$

Thus, assuming the profit distribution is normal, 95% of years will have a profit level within

$80{,}000 \pm 1.96\ \sigma\ (\sigma = 1{,}000)$

i.e. profit will be within the range

$78,040–81,960 in 95% of cases (range = $3,920).

Second, consider the variance minimizing plan:

$$E(\text{profit}) = (0.36 \times 500)160 + (0.64 \times 500)120$$
$$= \$67{,}200$$

Thus, the sacrifice in income compared with the optimal plan is:

$$80{,}000 - 67{,}200 = \$12{,}800$$

$$\text{Var}(P) = 0.36^2\ \sigma^2_A + 0.64^2\ \sigma^2_B + 2(0.36 \times 0.64 \times \rho\sigma_A\ \sigma_B)$$
$$= 465{,}472$$

$$\therefore \sqrt{\text{Var}(P)} = \ \$682 \text{ approximately}$$

Thus, again assuming a normal profit distribution, 95% of all years will produce a profit within the range:

$$\$67200 \pm 1.96 \times 682 = \$65{,}863–68{,}537$$

This gives a range of $2673.

Thus, it must be decided whether the $12,800 sacrifice in average income would be worth reducing the absolute profit range from $3,920 to $2,673. Clearly, in many cases it will not be worthwhile to diversify to the extent of minimum variance. In this case the variance difference between the alternatives is not great.

Other effects of diversification

Varying returns to the level of production

The discussions above assume:

- The profit variance per hectare does not increase or decrease as the number of units of a product produced increases; and
- The expected profit per hectare does not vary as the number of units of a product produced increases.

In many cases these are invalid assumptions. As the area of, e.g. wheat increases it is likely that management inefficiencies will give rise to a *greater* profit variance and a *lower* expected profit. In such cases diversification is worthwhile from a simple profit maximization view as returns per hectare will be greater given only a small area of each product is produced.

Where *increasing* returns to size occur, diversification will decrease the average expected profit per hectare. The additional returns cannot be achieved where many products are produced, as only small areas of each can be produced. It is most unlikely, however, for the profit variance per hectare to decline with an increasing area.

Product relationships

Where two or more products are complementary, diversification into these products may be profitable because it allows taking advantage of such complementary relationships. If, however, *many* products are produced and some are competitive, then the complementary advantages of a small number will be partially lost with the reduction in quantity produced.

Managerial ability

Farmers may not be able to maintain high returns per hectare for each product where many are produced, due to the complexity of the combined management task.

9.5 Models of Other Techniques

Introduction

Some time has been spent on analysing diversification. By now it should be clear how to analyse various other techniques. A symbolic model giving the total farm profit variance from using the various measures is required. Given such models, the various techniques, and combinations of them, can be evaluated to see which is worthwhile.

Some examples are developed. The discussion on diversification used the general formula for variance. This section will take a different approach. The models will be based on a process of determining the possible outcomes and associated probabilities, and from this estimating the variances.

Formal insurance

Assume a farmer is not using any insurance, so the possible resultant net return outcomes and probabilities are:

$ outcomes = b_i, $i = 1, 2 \ldots n$

Associated probabilities = p_i $i = 1, 2 \ldots n$

Thus, the expected profit and variance will be:

$$E(P) = \sum_{i=1}^{n} b_i p_i$$

$$V(P) = \sum_{i=1}^{n} \left(b_i - E(P)\right)^2 p_i$$

Some of the b_i will allow for possible fire losses, accident losses and others, i.e. all the unfavourable outcomes that would normally be insured against. Some of the b_i will probably be negative to allow for the fact that if, e.g. the dairy shed is burnt there will be a considerable outlay to replace it. The probability attached to such outcomes will, however, be extremely low.

Now, consider the case where the farmer insures against loss by fire of all his buildings. The b_i values will need changing to account for:

- The various fire losses than can occur will now be met by the insurance company. Thus, those b_i values that include an expense for rebuilding a fire-damaged or destroyed building will require adjusting. Let f_i represent the building costs and other costs associated with the ith outcome (many of these f_i values would equal 0).
- The insurance premiums paid each year (let c = the insurance premium per year. Assume this is known with certainty, though this could also be assumed to be a random variable if premium rates were not known).

Thus, the possible outcomes given formal insurance is used will be:

ith possible outcome = $s_i = b_i + f_i - c$

and the expected profit and profit variance will be given by:

$$E(P) = \sum_{i=1}^{n} s_i p_i = \sum_{i=1}^{n} (b_i + f_i) p_i - c$$

$$V(P) = \sum_{i=1}^{n} (s_i - E(P))^2 p_i$$

To decide whether formal insurances should be used, the expected profit and profit variance figures for each need comparing (note that these two systems may well give two points on an Expected Profit/Profit Variance graph). Then, depending on the farmer's attitude to risk, a decision can be made.

For many farmers, however, a large fire loss may prove disastrous due to insufficient reserves or an inability to borrow sufficient funds. Thus not insuring may not be a feasible alternative. A farmer can insure against many different losses, so there is a wide range of alternative insurance plans. Some may be infeasible due to the large losses, but others will be feasible and can be analysed in the way suggested. A major problem would be the estimation of the probabilities of the various degrees of fire damage. At least this kind of analysis does not have to be repeated on every farm, as the conditions will not change much.

Forward contracts in money

Similarly, a symbolic model can be developed in order to determine whether a monetary forward contract will be worthwhile. Assume a farmer is operating a system involving, say, maize.

Let b_i = the ith possible net return outcome from the farm, excluding maize gross returns but including the maize production costs, where $i = 1, 2 \dots n$.

Let p_i = the probability of the ith b_i occurring;

m_j = the jth possible maize output (in kg);

s_j = the probability of the jth maize output occurring;

q_k = the kth possible price that can occur in the free market for maize (per kg);

c_k = the chance of the kth maize price occurring.

Assume i, j and k vary through $1, 2 \dots n$ in all cases and

the random variables b_i, m_j and q_k are independent.

Thus, given the maize is sold on the free market, the expected profit and profit variance will be:

$$\mathrm{E}(P) = \sum_{i=1}^{n} b_i p_i + \sum_{j=1}^{n} m_j s_j + \sum_{k=1}^{n} q_k c_k$$

To determine V(P) all the possible net return outcomes and their associated probabilities must be calculated.

Thus, let B_i = ith possible net return outcome:

$$B_i = b_i + m_j q_k$$

As the variables are all independent, for each b_i that can occur, n different maize yields can occur. Further, for each maize production level there can be n different maize prices. Thus, the number of possible net return outcomes is:

$n \times n \times n = n^3$

The probability of each of these outcomes will be given by:

$$P_i = p_i \times s_j \times q_k$$

where $i = 1, \dots n$,

$j = 1, \dots n$,

$k = 1, \dots n$

Thus,

$$\mathrm{V}(P) = \sum_{i=1}^{n^3} (B_i - E(P)^2) P_i$$

Given the alternative case where the farmer contracts to sell the maize at a given price per kg, then the E(P) and V(P) estimates are given as follows:

Let q = the price/unit of maize (known with certainty)

Then:

$$E(P) = \sum_{i=1}^{n} b_i p_i + \sum_{j=1}^{n} m_j q s_j$$

$$= \sum b_i p_i + q \sum m_j s_j$$

Then:

$$B_i = b_i + m_j q$$

But, instead of n^3 outcomes there will only be n^2 possible values of B_i, as price is no longer a random variable.
Thus,

$$V(P) = \sum_{i=1}^{n^2} \left(B_i - E(P)\right)^2 R_i$$

where $R_i = p_i \times s_i$
where there will be n^2 possible R_i values.

Given that E(P) and V(P) have been calculated for both systems, a choice is then possible.

9.6 Concluding Comments

Similarly, symbolic models could be constructed for the other income variability reducing techniques allowing decisions on the best combinations. Conceptually, an optimizing model could be developed and used to determine the optimal combination maximizing expected utility. However, such a model would be extremely complex and is beyond the scope of this text. Given the methods suggested, to determine an optimal system *various combinations* of the techniques would have to be evaluated in order to estimate E(P) and V(P) for each. Given this information the best combination can be selected. It will have been noted the critical factor in all these analyses is the estimates of the possible outcomes and their associated probability. In many cases subjective estimates will be necessary using the available evidence as a base. Further, a lot of work is necessary in the calculations, so developing computer models is likely to be beneficial. However, where very limited data are available decisions must rely on well considered subjective conclusions.

References

Hardaker, J.B., Huirne, R.B.M., Anderson, J.R. and Lien, G. (2004) *Coping with Risk in Agriculture*, 2nd edn. CAB International, Wallingford, UK.

Nuthall, P.L. (2010) *Farm Business Management. The Core Skills*. CAB International, Wallingford, UK.

10 Budgeting – The Simplest Form of Farm Systems Analysis

10.1 Introduction

Budgeting is the simplest analytical technique for developing improved farming systems. Furthermore, in some cases it is also the most useful even though it does not actively allow for risk. Once promising systems are developed they may need to be assessed for risk along the lines already discussed. However, risk-reducing approaches can be included in a budget right from the outset, depending on the farmer's attitude and available options.

While budgeting is the cheapest of all techniques, it can be inefficient where there are many systems to be compared, though for some farms the number of basically different systems available can be quite small. For large farms the potential gains from good planning will enable the use of more sophisticated techniques. But again, this must also depend on the existence of considerable choice, and large farms still need to have a basic budget for estimates of profit, cash flow and all the physical factors associated with a plan.

Budgeting involves developing the physical details of a farming system and estimating the costs and returns that are expected, assuming the particular farming system is put into operation. Through comparing alternatives a decision is made. Successful budgeting requires experience, as a set of rules indicating how to estimate whether, for example, there will be sufficient labour to carry out the programme proposed does not exist. Budgeting relies on the experience of the budgeter to ensure that *feasibility* will occur. (A feasible system is where, for example, labour supply equals demand, feed supply equals demand, working capital supply equals demand, harvesting capacity equals demand and so on, for all resources.) This chapter contains a discussion of the types of budgets than can be useful. Details of budgeting procedures are assumed to be known. Readers interested in these details should consult Nuthall (2010).

Budgeting can be used under most real world situations. No simplifying assumptions are made about the nature of the real world *except*, as noted, for risk. Budgeting assumes certainty with the input–output coefficients, prices and costs all assumed to have fixed values. This is clearly unrealistic. Analysts, when using budgeting techniques, attempt to overcome this problem by using conservative estimates and by using a range of parameter values to indicate the likely profit range. Such methods are largely attempts to ensure that over-optimistic estimates are not made rather than attempts to correctly and positively account for risk. Besides this one important limitation (and in many cases it is a limitation overcome *by extending* the budgeting process into a decision tree, payoff matrix type approach), any kind of relationship can be incorporated into a budgeting study. For example, there is no need to assume only *linear relationships*, though in many cases this is done to simplify the analysis. A specific example might be the estimation of the yield of wheat per hectare to use in a series of budgets as the number of hectares produced increases. Commonly, a standard figure is used no matter what the area, but there is nothing to prevent the budgeter from decreasing the yield as the area increases and so assuming a non-linear relationship. The same applies to all the other input–output coefficients and the prices and costs used.

There are a number of different types of budgets, which are all discussed. However, gross margins analysis and parametric budgeting will be discussed more fully as they can be very useful. The chapter finishes by outlining 'programme planning', which is not a practical concept in this day of fast affordable computers, but it does demonstrate some important principles.

10.2 Types of Budgets

Simple forecast budgets

Budgeting is used to predict the expected cash surplus and other parameters for a defined farming system. This is in contrast to using budgets to choose between alternative systems. This simple forecast budget has many planning advantages such as enabling a prediction of taxation (and thus, maybe, the use of legal tax-reducing techniques), a prediction of cash surplus and therefore what personal consumption is feasible and so on. A budget also acts as a blueprint for the farmer to follow in daily operations. An analyst, or farmer, might believe mental analysis indicates the best farm system and creates a budget for the reasons just outlined in contrast to calculating several budgets to choose the best system.

Simple forecast budgets may be constructed for a single period or for many periods (a typical period would be for 1 year).

Comparative Budgets

A major use of budgeting *in planning* is of course for the comparison of systems. Once the best system has been chosen the budget becomes a simple forecast budget as discussed above.

Comparative budgets can take several forms (Flinn *et al.*, 1991). These include a budget for:

- part of the farm (partial budgets):
- one, or many, future time periods (comparative 'development' budgets).

Partial budgets in the general sense of the word can also take one of two possible forms. That is:

- Budgets for part of a farm where both variable and fixed costs are included; these are 'conventional' partial budgets. Also, such budgets might involve a change in products produced;
- Gross margins. A gross margin budget is for a technical unit (e.g. 1 ha) for a particular product and only includes variable costs. A separate section below will be devoted to discussing gross margins in more detail.

The use of all forms of comparative budgets can be, using production economics terminology, for determining an optimal factor–product, factor–factor and product–product combination. Each budget determines the profitability of a point on the graphs representing the possibility curves as shown in Fig. 10.1. If *all* points on the graphs were compared, an optimal system would be determined.

However, a budget for every point is clearly impractical due to the work involved and, furthermore, is usually unnecessary as general experience and a few simple calculations will indicate which systems should *not* be budgeted as they are obviously not as good as alternative systems. (Production economics principles indicate which systems are likely to be the best.) Figure 10.1 indicates, through an 'x', which points might be explored in a general sense.

10.3 Parametric Budgets

Introduction

A further form of budgeting is *parametric budgeting* (Byrne, 1964). This is designed to explore the effect on profitability of variations in critical parameters such as the price of wool, the yield/ha of grass seed and the many other non-predictable factors. A parametric budget can be developed for any of the types of budgets discussed above.

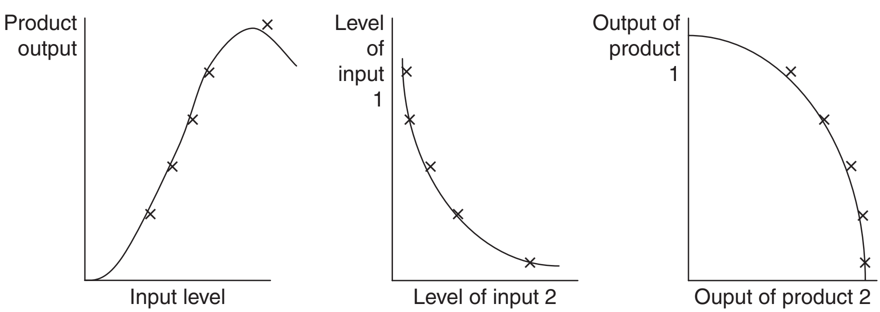


Fig. 10.1. Budgeting and production economics.

A parametric budget is expressed as an equation in which the factors to be explored are expressed as symbols.

Thus, for example, the parametric budget for a rangeland property producing fine wool from Merino wethers might be:

$$\text{Profit} = (Wp - 1.2W - 20)S - 70{,}000$$

Where

W = wool cut/head in kg;
p = price of wool/kg;
S = number of wethers carried;
1.2 = selling costs per kg of wool;
20 = the variable costs of running one wether;
70,000 = the fixed costs.

A parametric budget is constructed by first estimating a normal budget for the system and, as a result, determining the values of the constants, i.e. the costs and returns that are *not* going to be varied in the analysis. Secondly, the equation is constructed using these constants and symbols for the parameters.

Most parametric budgets will include some constants as well as the parameters. However, there is no reason why an equation could not be developed that includes only parameters. In effect, this is what a normal budget is, except that it is not formally written down in equation form before the calculations are carried out.

Using a parametric budget

Once the equation has been developed the profit can be determined for various levels of the parameters by inserting the relevant values in the equation and evaluating it. The example given above was for a whole farm profit. The equation could be designed to estimate any required figure, e.g. gross margin estimates. To help interpretation the results can be graphed. For example, given values for W and S in the wether example, profit can be graphed against wool prices as shown in Fig. 10.2. Graphs of this form can be constructed for many W and S combinations.

Uses of parametric budgets

They will be of use where profit estimates or gross margins are required for a large number of parameter combinations (Candler and Cartwright, 1969). Graphs clearly provide a quick synopsis of the impact of parameter variations. However, as a parametric budget is just an ordinary budget expressed in another way, where only a few parameter variations are required it will be easier to repeat a few normal budgets than develop an equation. Also note in the example provided it is assumed linear relationships hold, thus the straight line. There is nothing to stop the analyst introducing non-linear equations (e.g. the wool yield per animal may decline as animal numbers increase).

Fig. 10.2. Parametric budget graph.

The specific uses include:

- As budgeting assumes certainty, it is useful to explore the possible effect of different prices, costs, technical coefficients on profit, and any other objective, to explore the sensitivity of the results to changes. A farming system that is relatively stable to expected change will be preferred. An examination of a budget, however, will often provide this information without the need to formally prepare a parametric budget. Graphs, however, enable farmers to see easily the effect of variations.
- To compare the costs and benefits of improved techniques. For example, the effect of increasing the lambing percentage by *x*% can be seen, where lambing percentage is a parameter, and thus compared with the cost of achieving such an increase (e.g. by extra feeding).
- Somewhat similarly, a parametric budget can be used to determine the *conditions* (prices, yields) under which a particular farming system will be better than some other system. For example, a parametric budget could be developed for a particular farm assuming: (i) it is run as a simple lamb finishing property; and (ii) it is run as a mixed lamb-cattle finishing property. Given the prices of lamb, cattle, wool, growth rates and similar, were put into the equations as parameters, the cattle price and growth rate necessary for system (ii) to be the best can be determined for given lamb, wool etc. prices.
- For budgetary control purposes. Given that a parametric budget has been developed for a farm, the effect of changing conditions (prices, yields etc.) on profit can be rapidly determined. This can also be done using a normal budget, though it will not be quite as simple a task.

Concluding comments on parametric budgets

Parametric budgets do not offer analytical advantages over a normal budget – they are simply expressed differently. Their advantage lies in the ease of calculation where the effect of many different prices, yields etc. must be evaluated.

Most parametric budgets use linear equations. However, in some situations a non-linear equation will be necessary, particularly where the farm under study is large enough to affect a market, or perhaps to experience economies of size or scale. For example, a large property producing Friesian/Holstein replacement heifers for sale in a small market would impact on the price received per beast depending on the numbers giving a non-linear price/quantity relationship.

Thus, instead of using a constant, a function such as:

$$\text{Price} = 600 - 0.1\,H^2$$

might have to be used, where H represents the number of heifers sold in units of ten.

10.4. Gross Margins

Introduction

The *gross margin* for a particular product is defined as the gross revenue minus the variable costs per technical unit.

This technical unit is usually a unit of area (1 ha) in the case of cash crops and a single animal or a stock unit (SU) in the case of animals. That is:

Gross margin/ha = output/ha × price/unit − variable costs/ha

GM/SU = ((output/animal × price/unit − variable costs/animal) ÷ no. of SU)

An example of an SU would be the feed necessary to support a 50 kg sheep. Different sizes and types of animals can be compared by calculating the SU each represents. Thus, a ewe might be 1 SU and a beef animal 6 SU. Alternatively, a unit such as the number of kiloJoules (kJ) required for maintenance and growth/production can be used.

The gross margin is commonly used to indicate which products should be produced. The discussion that follows reviews their underpinning theory and lists possible uses.

The theory behind gross margins

All farm costs can be divided into fixed (or overhead) and variable (or working) costs, where these costs are defined as:

- Fixed costs – costs that remain constant with varying levels of output and with changes in the pattern of production (farming system). Examples are rent, rates, permanent labour, fencing repairs and maintenance.

- Variable costs – costs that vary as the level of output is varied and as the farming system is changed within certain limits. Examples are fertilizer, fuel, cartage, seeds and animal health.

Figure 10.3 shows variations in costs with level of output and differing farming systems.

This distinction between variable and fixed costs relies on the extent of cost changes which occur as the pattern of production and level of output is changed. In the long run land can be purchased and sold, similarly machinery and other resources, so that all costs can become variable. This is, however, somewhat of a circular discussion as the 'long run' is defined as a *period of time over which the entrepreneur (farmer) is prepared to vary all resource levels*. Such a time period may be 1 day or perhaps several years. Thus, whether the distinction between fixed and variable costs can be made depends on whether a farmer wishes to make basic structural changes to his resource structure. Given a farmer has a certain area of land, set of machinery and buildings, and a permanent labour force *which he does not wish to vary*, the distinction between fixed costs and variable costs can be made as the fixed resources give rise to the fixed costs. In this case, *fixed costs can be ignored* when planning as they will not vary no matter what farming system is used. Qualifications, however, are made to this general statement later on.

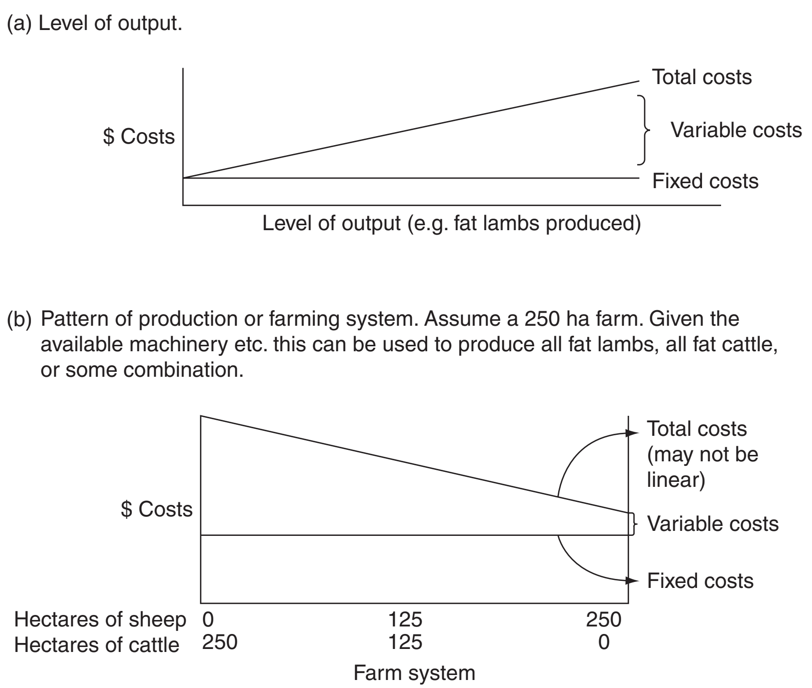


Fig. 10.3. Graphical representation of variable costs, showing variation with output and the production pattern.

Uses of gross margins

Given a fixed set of resources, the farmer's decision problem, assuming his objective is to maximize expected profit, is to select a farming system which maximizes the expected total gross margin where the total gross margin is defined as:

$$\text{TGM} = \sum_{i=1}^{n} x_i g_i = x_1 g_1 + x_2 g_2 + \ldots + x_n g_n$$

where:

x_i = the number of units of product i produced;
g_i = the gross margin/production unit of product i.
This TGM is the income available to 'pay':

- fixed costs;
- taxation;
- consumable surplus.

Given this basic objective, gross margin estimates can be used in the following ways:

- *Developing a farming system* for a newly purchased property by selecting the products and production methods giving the highest gross margins. In doing this, however, it is not simply a matter of selecting the product with the highest gross margin as there is probably a fixed resource that limits the quantity produced (e.g. insufficient labour, machinery capacity) to less than the whole farm. Further, soil fertility considerations may limit the area produced. Thus, a combination of products must be selected such that the *resource requirements must not exceed* the resources available. That is, the system *must be feasible* and *also sustainable*. Further, risk considerations must be also be taken into account.
- *Making marginal changes* to an existing farm system. This is the most common use of gross margins. Due to resource levels, the reluctance of farmers to change rapidly, the availability of new breeding stock and other constraints, it is difficult to make major changes to an existing farm system within a short period. Thus, an improved system is developed step by step. Gross margins indicate the direction each change should take.
- *Indicating profitable changes to the fixed cost structure*. It was noted earlier that gross margins are essentially a short-run planning tool. They can, however, be used to indicate changes to the long-run fixed cost structure. If a particular product has a high gross margin but the necessary resources for its production are not currently available, it may pay to purchase these resources. Simply estimate the net change to TGM that would occur if the product was produced (there will be an increase from the new product but this may be partially offset due to a drop in production of other products with resources being released) and compare this with the change in fixed costs that would occur. Similarly, changing the level of existing products may be worthwhile where the fixed costs are changed beneficially, e.g. decreasing or increasing the size of the permanent labour force.

- Gross margins can also be used in sophisticated planning techniques, such as programme planning and linear programming, as will be discussed later.
- *Indicating profitable production processes* and to compare alternative factor–product and factor–factor combinations. Gross margins can used to compare alternative production processes for the production of the same product. To do this a gross margin would be calculated for each production method. An example might be the gross margins of wheat on lighter land for different levels of irrigation.

A 'comparative analysis' may include actual gross margins achieved for the various products produced as one of the statistics used for comparisons.

Difficulties in calculating gross margins

The major problem in calculating gross margins is to ensure that:

- The parameter values used relate to the particular farm and the ability of the manager (the same applies to any budget); and
- The 'products' for which the gross margins are estimated are *independent* activities. That is, the output and input levels assumed do not rely on this product being produced in some fixed proportion with some other product. For example, a gross margin for a wheat crop may be estimated assuming the crop follows a white clover (legume) crop. In this case the unit 'white clover–wheat' (a rotation) would have to be considered as the 'product', for one confers benefits on the other (N supply regime). Thus, a gross margin for a whole rotation can be calculated and compared as it is independent as a combination.

10.5 Programme Planning

Introduction

If no other information besides the gross margins is available it is difficult to develop a feasible farming system other than through experience. No direct account is taken of the *fixed resource requirements* by each product or production process for which a gross margin has been calculated. To overcome this problem the planning technique known as *programme planning* was developed. This determines a farming system that maximizes the total gross margin, i.e. maximizes short run profit. Unfortunately, it is not foolproof and requires considerable judgement. An extension of this technique is linear programming, which is relatively foolproof but requires many more calculations. Fortunately, most modern computers have routines that automate linear programming (Candler and Musgrave, 1960) solutions. The researchers who developed programme planning tried to incorporate the advantages of linear programming without the calculational disadvantages. While this is no longer relevant, the principles used in programme planning are well worth learning as they provide a basis for intuitive decision making, and an introduction to linear programming.

General method

The basic principle involves determining which resource is the most limiting and then selecting products on the basis of giving the greatest return per unit of the most limiting resource. For example, assume a farmer has 450 ha of good cropping land and only $5,000 of working capital without access to any more borrowed finance. In this case the fact that he has 450 ha is probably irrelevant as he will have no opportunity of using all this land given only $5,000 to spend on fertilizer, fuel, seed, stock and all the other inputs necessary before any income is received. Thus, the farmer's level of working capital is the limiting resource. He must attempt to use this working capital as efficiently as possible to maximize the return/unit of capital. It does not matter if the return/unit of land is not as great as it could be as the land is not limiting.

For example, consider the case where the farmer must choose between the following alternative products:

	Sheep	Cattle
Gross margin/ha (return/unit of land)	$80	$140
Working capital requirement/ha	$12	$30
Return/unit of working capital	$^{80}/_{12}$ = 6.67	$^{140}/_{30}$ = 4.67

It is clear that working capital limits production rather than land. Thus, to maximize the total gross margin the farmer should produce sheep as this alternative gives the greatest return per unit of the limiting resource: the return/unit of land is less than what it could be as cattle have a greater return/ha.

A system of producing only sheep will give the following statistics:

Number of hectares devoted to sheep = 5000/12 = 417
Number of hectares left idle = 450 − 417 = 33
Total Gross Margin = $80 × 417 = $33,360

In contrast to this system, if cattle were produced, the relevant statistics would be:

Number of hectares devoted to cattle = 5000/30 = 167
Number of hectares left idle = 450 − 167 = 283
Total Gross Margin = $140 × 167 = $23,380

This example is very simple and somewhat unrealistic. It indicates, however, quite clearly the principle that must be followed. Practical problems will contain many more alternative products and there will be many more resources constraining or limiting production. Further, constraints may be imposed for other reasons besides resource constraints. For example, a farmer might decide that he should not produce more than 10 ha of potatoes for market uncertainty reasons (to decide this you would need to know something about the shape of his utility function). Another example would be the limit on the number of litres of milk imposed by a nationally set quota to prevent oversupply.

Where there are many constraints and alternative products or activities (each alternative production possibility is often referred to as an *activity*), the selection process is more complicated. After selecting the activity with the greatest return/unit of the most limiting resource at the maximum level, there are usually resources left over, which can be used for other production. Thus, the next step in the process is to select an activity that gives the greatest return per unit of what now appears to be the most limiting resource of those remaining. This process is continued until all of one of the resources required by all activities is completely used so no further activities can be introduced into the plan. Having reached this point, attempts are made to adjust the plan to see if any idle resources can be *used profitably*. Thus, planning does not consist of a well-defined set of rules enabling the unique optimal farming system to be determined, but rather a basic principle (maximizing return/unit of the most limiting resource) which is implemented in a somewhat trial and error method.

Steps involved

The steps can be summarized as follows.

1. List the possible alternative activities and calculate their gross margins.
2. List the limiting resources and other constraints on production.
3. Calculate the per unit requirement by each activity for each resource and constraint.
4. Estimate the return/unit of each resource earned by each activity.
5. Determine what appears to be the most limiting resource.
6. Develop a farm plan by firstly selecting the maximum level possible of the activity that has the greatest return per unit of the most limiting resource.
7. If other constraints (e.g. risk constraint) prevent the most profitable activity from using all the most limiting resource, select to the maximum level possible the activity with the next highest return/unit of the resource. Repeat this process until the most limiting resource is completely utilized.
8. Attempt to improve the developed farm system by making marginal changes. These will involve either:
- **(i)** Removing some of the activities in the plan so that with the resources made available another activity which makes better use of some of the idle resources can be introduced, or
- **(ii)** Altering the levels of the activities in the plan so that some increase at the expense of others.

In making these marginal changes the emphasis must be on re-estimating what appears to be the most limiting resource and attempting to maximize returns to this resource. In many cases the resource that was initially selected as being the limiting turns out to be non-limiting.

Finally, examine whether or not changing the fixed cost structure (and thus the resource levels) will improve profit. This may involve buying or hiring extra resources thus enabling a change in the activities and their levels in the developed farming system.

It is clear that this process involves judgement and considerable trial and error.

An example

The only way to obtain a clear understanding of the rules is to follow through an example. Consider a farm with the following:

1. Resources and other constraints.
 - **(i)** Land 200 ha.
 - **(ii)** Working capital $6,000.
 - **(iii)** Maximum potatoes 6 ha – a limit on casual labour prevents any more than this quantity.
 - **(iv)** Maximum wheat and/or barley 70 ha – soil fertility constraint.

2. Alternative activities and their resource requirements

	Gross margin/ha ($)	Capital requirement
Potatoes	320	200
Wheat	120	224
Barley	100	20
Sheep (12/ha)	80	40
Cattle (2/ha)	68	6

3. Return per unit of capital
Potatoes $^{320}/_{200} = 1.6$
Wheat $^{120}/_{24} = 5.0$
Barley $^{100}/_{20} = 5.0$
Sheep $^{80}/_{40} = 2.0$
Cattle $^{68}/_{6} = 11.3$

4. The most limiting resource appears to be land, as:
 - **(i)** Potatoes are limited to 6 ha of land. Capital is sufficient to produce this level ($6 \times 200 = 1200$).
 - **(ii)** Wheat and/or barley are limited to 70 ha and there is sufficient capital to produce this level ($70 \times 24 = 1680$ for wheat; $70 \times 20 = 1400$ for barley).

(iii) Cattle are limited to 200 ha by the land and there is sufficient capital to produce this quantity ($200 \times 6 = 1200$).

(iv) The only activity limited by capital rather than land is sheep. The limit imposed by capital is 6000/40 =150 ha.

Given the above information, consider the selection of a farming system.

1. Select activities on the basis of their return to land:

	Cumulative land use (ha)	Cumulative capital use ($)	Cumulative total gross margin ($)
Ist Selection			
Potatoes	6 (constraint limit)	1,200	1,920
2nd Selection			
Wheat	+70 (constraint limit)	+1,680	+8,400
Totals	76	2,880	10,320

Resources left: 124 ha, $3,120 capital.
Continue to select on basis of return to land.

3rd Selection – max allowed by capital Sheep		$\frac{3{,}120}{40} = 78$	
	+78	+3,120	+6,240
	154	6,000	16,560

No further activities can be selected as capital is fully utilized.
Thus, Plan **1** is:

	Land (ha)	Capital ($)	Total gross margin ($)
Potatoes	6	1,200	1,920
Wheat	70	1,680	8,400
Sheep	78	3,120	6,240
Total	154	6,000	16,560

2. Examine the plan to see if marginal changes can be made so that total gross margin is increased. As capital has turned out to be the most limiting resource the next logical step is to examine the return per unit of capital achieved by each activity to see if more efficient use can be made of this resource. Such an examination indicates cattle has the greatest return/$ and it is not in the current plan. Thus, introduce cattle at the expense of some other activity. As potatoes have the lowest return/unit of capital reduce the quantity to release capital for cattle production. Trial and error calculations indicate that if potatoes are reduced by 1.4 ha $280 of working capital is available for cattle production. This is sufficient for cattle to use all the idle land.

Thus, an improved plan is:

	Land (ha)	Capital ($)	Total gross margin ($)
Potatoes	4.6	920	1,472
Wheat	70	1,680	8,400
Sheep	78	3,120	6,240
Cattle	47.4	264.4	3,221
		5,984	
		(near enough	
Totals	200.0	to $6,000)	19,333

Therefore the total gross margin has been improved by almost $3,000 and all resources are used.

Continuing the search for profitable marginal changes, the only other activity that can be introduced is barley. Wheat is preferable to barley, both on a per hectare and per $ of working capital basis.

Thus the only other change that might be profitable is an alteration to the levels of activities in the current plan. To examine this aspect see if the activities with a greater return/unit of resource can be increased at the expense of activities with a lower return/unit of resource. It does not appear that this can be done in this case. The only way to test this is to carry out a few changes and estimate the change in total gross margin. For example, increase cattle at the expense of sheep, as cattle have a higher return/unit of capital:

To increase cattle by 1 unit must decrease sheep by 1 unit as all the land is currently utilized.

Net effect on TGM = +68 − 80 = −12

Thus this change is not profitable.

Therefore it appears as though the plan developed is optimal, although a plan consisting of potatoes 6 ha, wheat 70 ha, sheep 70 ha and cattle 54 ha has a slightly greater TGM. This indicates that the programme planning technique is by no means foolproof.

10.6 Relationship Between Programme Planning and Production Economics

Background

Programme planning attempts to determine exactly the same optimal point as production economics would indicate as being optimal. It does this through defining a separate activity for each point on the factor–product and factor–factor curves and proceeds to select the combination of these activities that maximizes profit. In practice, not every point on the curves can be represented by an activity as there are an infinite number of points.

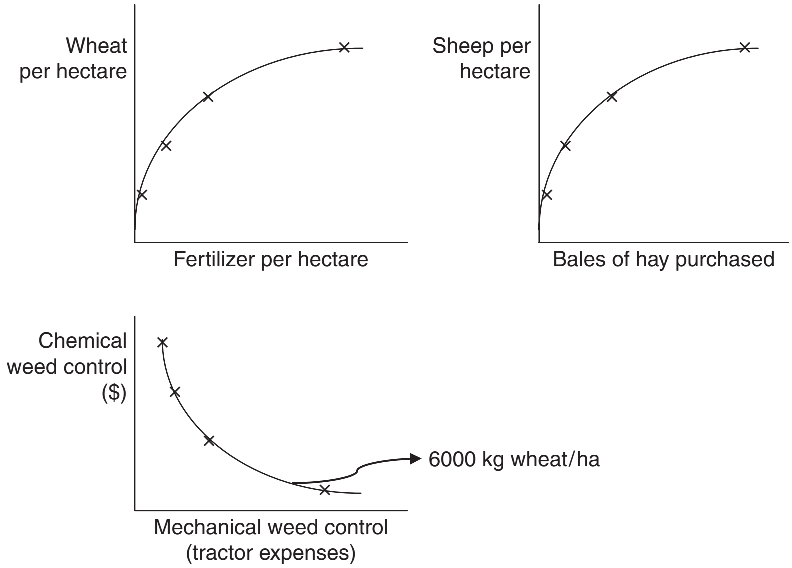


Fig. 10.4. Representing alternative production systems as points on opportunity curves.

Graphically, a number of activities are defined to represent a number of points, as shown in Fig. 10.4.

An obvious question is why use programme planning instead of production economics? The reason is that in realistic problems, where there are several products *and* many inputs *and* resource constraints, the maths involved in directly applying production economic theory becomes too complex so that the optimal point cannot in fact be determined.

The advantages and disadvantages of programme planning

Advantages

- It complies with production economic theory through constructing a profit maximizing programme on the basis of diminishing marginal return to the limiting resource, i.e. it attempts to equate the marginal value product for each activity.
- It enables an estimate of the effect on profit of varying the resource levels and thus the fixed cost structure of the property.

Disadvantages

- It assumes linear relationships, i.e. if 1 ha of wheat requires 5 h labour, it is assumed l00 ha will require 100 × 5 h. In reality, the requirement/unit of the activity might decline or increase as the case may be.

- It is not foolproof and requires considerable judgement. This comment becomes more important as the number of alternative activities and constraints increases.
- Each activity must be independent. Thus, activities such as ‘haymaking’ cannot be considered as an independent activity providing feed for various stock activities. These activities must be combined in a defined way with stock activities to form an independent separate activity. (An activity which produces a product that is used by other activities is referred to as an *intermediate* activity.)
- It does not actively take account of risk and uncertainty and the farmer’s attitude to risk.

10.7 Concluding Comments

Budgeting is the most important planning and general managerial aid for farm advisory and consultancy work. It is also used in limited research work. The basic principle involves *making comparisons* between alternative systems. Successful budgeting, however, requires considerable experience, particularly as it is impractical to compare all alternatives. Thus, a knowledge of production theory and systems analysis becomes important in suggesting which systems are likely to be the most profitable for analysis.

To develop useful budgeting skills an analyst will benefit from an understanding of programme planning as this leads to insights into the structure of typical, constrained, farm decision-making problems, and demonstrates the importance of the principle of maximizing returns to the most limiting resource. Furthermore, a good understanding enables an intelligent use of simple gross margins analysis. A mentally based version of programme planning for developing improved farming systems with gross margins can lead to successful systems.

Given complex problems, such as an intensive mixed cropping farm, the need for large numbers of budgets makes a purely budget-based study difficult, and mistakes can be easily made. Given *all the necessary data has been calculated* it will become clear, after discussing linear programming, that the data would be better used through linear programming.

References

Byrne, P.F. (1964) Parametric budgeting using a model of the sheep enterprise. *Review of Marketing and Agricultural Economics* 32(3), 95–136.

Candler, W. and Musgrave, W. (1960) A practical approach to the profit maximisation problems in farm management. *Journal of Agricultural Economics* 14(2), 208–222.

Candler, W. and Cartwright, W. (1969) Estimation of performance functions for budgeting and simulation studies. *American Journal of Agricultural Economics* 51(1), 159–169.

Clarke, G. and Simpson, I. (1959) A theoretical approach to the profit maximization problems in farm management. *Journal of Agricultural Economics* 13(3), 250.

Flinn, J.C., Jayasuriya, S.K. and Maranan, C. (1991) Enterprise, partial and parametric budgets. In: *Basic Procedures for Agroeconomic Research*. IRRI, Philippines, pp. 31–46.

Nuthall, P.L. (2010) *Farm Business Management. The Core Skills*. CAB International, Wallingford, UK.

11 Linear Programming – The Farm Model and Finding an Optimal Solution

11.1 Introduction

A major component of farm business management is, of course, deciding on the allocation of resources and the use of management techniques that maximize the farmer's objective. A study of production economics provides a framework within which resource allocation problems can be studied, but for most real-world farming situations production economic theory cannot be formally applied. This is largely due to the complexity of the maths involved and thus the planning costs. Further, classical production economics does not take into account uncertainty. Thus, *operational* planning techniques have been developed. Two examples of simple techniques discussed in the last chapter are budgeting and programme planning, but both have limitations. A technique with fewer limitations is linear programming (LP). As LP planning costs are greater compared with e.g. budgeting, its major use is in developing farming systems on representative and large farms.

A study of LP does not provide new theories compared with advanced production economics. However, students will find that LP provides a clear understanding of the nature of the real-world farm decision problem and reinforces production economics. Furthermore, this understanding facilitates the correct use of the simpler planning techniques such as budgeting and gross margins analysis.

The study of LP is divided into eight topics over two chapters (topics **1–3** and **5** are in this chapter, and topics **6–8** in Chapter 12) and Appendix 3 (topic **4**). The topics include:

1. The LP model of a farm (sections 11.2–4).
An introduction to the structure of the farm decision-making problem assumed by the LP model.
2. The graphical solution of the linear model (sections 11.5–9).

An insight into the nature of an optimal farming system, which enables relating the LP model to production economics.

3. Obtaining solutions to sets of simultaneous linear equations (section 11.10).
As the LP model is a set of simultaneous equations, a technique that creates solutions to linear equations is used to determine an optimal farm system. This section provides a general discussion of the method.

4. The simplex method of solving LPs (Appendix 3).
The technique discussed in (**3**) is not computationally efficient. Modern computers use matrix algebra to find solutions and, in its simplest form, the process is called the 'simplex method'. While analysts do not need to be familiar with the technique, some will find an understanding useful. Appendix 3 contains a description of the simplex method using the example problem presented in the previous sections for demonstration purposes.

5. The assumptions in the LP model (section 11.11).
This section contains a discussion and interpretation of the economic and computational assumptions made in the LP model.

6. The LP solution (sections 12.2–6).
The solving technique used and the solution obtained is given an economic interpretation. The solution provides a wide range of useful information for farm decisions.

7. LP model building (sections 12.7–20).
To obtain realistic farming systems the LP model must comply with the real world. This extensive section contains a discussion of how to fit real-world problems into an LP format. This section provides the greatest insight of all the sections into the nature of the decision problems faced by a manager.

8. The application of LP (sections 12.21–24).
These sections cover the practical use of LP and outline some of the applications to decision problems related to farming (determining least cost feed mixes, and a least cost transport system for collection and delivery of farm output and inputs).

11.2 The LP Model of a Farm

Constraints in a typical farm problem

To demonstrate the LP model, consider a typical farm problem. Assume 200 ha of homogenous soil run by an owner-operator and part of a labour unit that is shared between three farms. The farmer has estimated that labour is not a problem *no matter* what farming system is used other than in September. In this month the farmer has estimated 200 man h are available. Thus, any farm system developed must not, for feasibility reasons, use more than 200 man h in September. Further, due to the nature of the soil, the farmer has estimated that no more than 70 ha of grain crops can be grown in any one year for soil fertility reasons. Potatoes are a possible crop, but due to the nature of the potato market the farmer, being a risk averter, will not grow more than 20 ha.

This is a typical farm management problem: a farming system to be developed that maximizes an objective subject to satisfying constraints. Summarizing these constraints:

- Land use cannot exceed 200 ha.
- Labour use cannot exceed the supply. But it has been estimated that any farming system on the 200 ha cannot use more than is available other than in September. September labour use cannot exceed 200 man h. The other periods can be ignored.
- Area of grain crops cannot exceed 70 ha.
- Area of potatoes cannot exceed 20 ha.

The available products

The farmer can produce a number of different products using various forms of production. Thus, he must select some combination of the alternatives to form an operational farming system. For this example, assume that the alternatives are:

	Gross margin
Wheat	\$100/ha
Barley	\$80/ha
Potatoes	\$240/ha
Sheep	\$5/ewe (includes cost of pasture, etc.)

In order to ensure that any farming system is *feasible*, i.e. does not violate the constraints, information on the requirement of the various products or activities for the resources must be known.

Assume these requirements are:

Input–output coefficient table

	Requirements for resources/unit of the following activities			
	Wheat	Barley	Potatoes	Sheep
Activity unit	1 ha	1 ha	1 ha	1 sheep
Resource or constraint				
Land	1	1	1	0.1
September labour	0.5	3	4	0.2
Max. grain crop	1	1	0	0
Max. potatoes	0	0	1	0

Note that if the activities had been defined in different units the input–output, or requirement, coefficients would be different; e.g. potatoes could have been defined in terms of 1 t, thus the GM would be for 1 t and if the yield was 20 t/ha, the land requirement would be 1/20th. What is important is the units used across a row of the table must be the same.

A feasible farming system

Given the information it is possible to define relationships that must be satisfied if the farming system selected is to be feasible. First define a term:

Let x_j = the number of units of the jth activity produced.

Thus, a farming system will consist of values for *every* x_j.

For example, letting:

x_1 = ha of wheat
x_2 = ha of barley
x_3 = ha of potatoes
x_4 = number of sheep

then a particular farming system might be:

$x_1 = 60$
$x_2 = 10$
$x_3 = 15$
$x_4 = 2000$

Is this system feasible? For any farming system to be feasible, the following relationships must be satisfied:

1. Land: land use must not exceed supply. Thus,

$$200 \geq 1x_1 + 1x_2 + 1x_3 + 0.1x_4$$

where the coefficients are the requirements per unit of the activities for land. These are taken directly from the input–output coefficient table. Thus, the right-hand side of the relationship represents the land *use* by any particular farming system.

2. Labour: September labour use must not exceed supply. Thus:

$$200 \geq 0.5x_1 + 3x_2 + 4x_3 + 0.2x_4$$

3. Grain crops: the quantity grown must not exceed 70 ha. Thus:

$$700 \geq 1x_1 + 1x_2 + 0x_3 + 0x_4$$

4. Potatoes: the quantity grown must not exceed 20 ha. Thus:

$$20 \geq 0x_1 + 0x_2 + 1x_3 + 0x_4$$

Each of the relationships **1–4** must be satisfied. Thus, a feasible farming system is a set of values for the x_j such that the relationships are simultaneously satisfied.

The objective function

Given a feasible farming system the total gross margin from such a plan will be given by:

$$Z = 100x_1 + 80x_2 + 240x_3 + 5x_4$$

where Z = total gross margin.

The planning objective

The planning objective is to determine values for the x_j (i.e. find a farming system) that maximizes the value of Z but at the same time simultaneously satisfies the resource use and supply relationships (the constraint relationships); i.e. find values for the x_j which maximize

$$Z = 100x_1 + 80x_2 + 240x_3 + 5x_4$$

subject to:

(i) $200 \geq 1x_1 + 1x_2 + 1x_3 + 0.1x_4$ (land)
$200 \geq 0.5x_1 + 3x_2 + 4x_3 + 0.2x_4$ (September labour)
$70 \geq 1x_1 + 1x_2$ (max. grain crop)
$20 \geq 1x_3$ (max. potatoes)
(ii) $x_1 \geq 0, x_2 \geq 0, x_3 \geq 0, x_4 \geq 0$

This last requirement states the obvious. It is economic nonsense to consider negative production of a product.

As the relationships are linear the typical farm problem has now been framed as a *linear* programming problem.

This statement has used specific values for various coefficients. To convert this specific problem to the general case, define the following terms:
Let C_j = the net revenue (or gross margin) per unit of the jth activity ($j = 1, 2, \dots n$);
b_j = availability (or level) of the ith resource or constraint ($i = 1, 2, \dots m$);
r_{ij} = the requirement by one unit of the jth activity for the ith resource (i.e. the input–output coefficients).
Thus, the LP problem is to find a set of x_j values that maximize

$$Z = \sum_{j=1}^{n} x_j C_j$$

and satisfy:

(i) $b_i \geq \sum_{j=1}^{n} x_j r_{ij}$ (for each $i = 1, 2 \dots m$)
(ii) $x_j \geq 0$ (for each $j = 1, 2 \dots n$)

Gross margins analysis, programme planning and LP

The relationship between these planning tools is now clear. They all attempt to achieve the same thing, to find the farming system that maximizes the total gross margin given a defined set of resources and thus a defined fixed cost structure. Further, they can be used to explore the effect of varying the fixed cost structure by allowing resources to be sold and purchased. The difference between the methods lies in the degree of accuracy with which they estimate an improved system. Gross margins analysis requires considerable judgement but little detailed computational effort. In general, the opposite holds for LP. Thus, a study and understanding of LP assists in the efficient application of gross-margins analysis.

11.3 Converting Inequalities to Equalities

The method

The problem has been expressed using inequalities. To solve the problem it is necessary to convert the resource and constraint relationship inequalities to *equalities*. One way of doing this would be to simply state, for example, that land used must equal land supply, i.e.

$$200 = 1x_1 + 1x_2 + 1x_3 + 0.1x_4$$

However, it is by no means sure that there will be sufficient of the other resources to enable all the land to be used, thus this method is unsatisfactory (in a few cases it may not *pay* to use all the land. Any ideas why?). A satisfactory method is *to define a new activity* for each resource and other constraints. The level of this new activity associated with each constraint represents the number of units of the resource which is *not used* by a particular farming system. Such an activity is referred to as a *disposal*, or *slack*, *activity* as it represents the quantity of a resource that is not used (put into 'disposal').

Consider the land constraint. Let the level of the new activity associated with land, i.e. the land disposal activity, be represented by x_5. Thus, the land relationship becomes:

$$200 = 1x_1 + 1x_2 + 1x_3 + 0.1x_4 + 1x_5$$

Note that the r_{ij} associated with x_5 is set at 1. Thus, if $x_5 = 10$, this indicates that 10 ha of land are not used.

It should be clear why adding this activity enables converting the inequality to an equality. This disposal activity accounts for the difference between use and supply of land so that:

use of land ≤ supply of land

is converted into

use of land + non-use of land = supply of land

As this activity is designed to record only the non-use of land, it does not appear in any of the other constraints. In other words, the coefficient of r_{ij} associated with x_5 in the *other* constraints is 0.

Similarly, disposal activities are defined for the other constraints. Thus:

x_6 = non-use of September labour
x_7 = non-use of the maximum grain crop restriction
x_8 = non-use of the maximum potatoes restriction

Thus, the set of relationships becomes:

$$200 = 1x_1 + 1x_2 + 1x_3 + 0.1x_4 + 1x_5 + 0x_6 + 0x_7 + 0x_8$$
$$200 = 0.5x_1 + 3x_2 + 4x_3 + 0.2x_4 + 0x_5 + 1x_6 + 0x_7 + 0x_8$$
$$70 = 1x_1 + 1x_2 + 0x_3 + 0x_4 + 0x_5 + 0x_6 + 1x_7 + 0x_8$$
$$20 = 0x_1 + 0x_2 + 1x_3 + 0x_4 + 0x_5 + 0x_6 + 0x_7 + 1x_8$$

Writing out the disposal activity coefficients by themselves in the form of a table (matrix) shows that the coefficients associated with the disposal activities are all zeros except for a diagonal line of ones:

$$\begin{matrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{matrix}$$

This arrangement is referred to as an *identity matrix*

Having added these disposal activities (sometimes called disposal, or slack, variables), these new variables now appear in the objective function. As non-use of a resource has, usually, neither a cost nor a return, the net revenue (gross margin) of these activities is set equal to 0. That is:

$$C_5 = C_6 = C_7 = C_8 = 0$$

The objective function is now:

$$Z = 100x_1 + 80x_2 + 240x_3 + 5x_4 + 0x_5 + 0x_6 + 0x_7 + 0x_8$$

Given that disposal activities have been added to the problem, one for each constraint, the general constraint relationship can now be expressed as:

$$b_i = \sum_{j=1}^{n+m} x_j r_{ij} (\text{for each } i = 1,\ 2 \ldots m)$$

where it is assumed there are m constraints, and thus m disposal activities, and n possible activities.

The effect of adding disposal activities on the original problem

Adding disposal activities makes no difference to the optimal farming system (a set of x_j) and its total gross margin (or total net revenue). An optimal solution to the original problem (i.e. the problem in inequality form) will also be an optimal solution to the problem in equality form (augmented problem) and vice versa. This is because the disposal activities are given a zero net revenue. Consider:

1. The profit equation
Let Z^* = optimal value of Z
x_j^* = optimal value of x_j
Then,

$$\begin{aligned} Z^* &= 100x_1^* + 80x_2^* + 240x_3^* + 5x_4^* \\ &= 100x_1^* + 80x_2^* + 240x_3^* + 5x_4^* + 0x_5^* + 0x_6^* + 0x_7^* + 0x_8^* \end{aligned}$$

i.e. the set of x_j^* maximizing both equations are identical.

2. Feasible solutions
Given the solution consisting of the x_j^* ($j = 1, \ldots n$), then

$$200 \geq 1x_1 + 1x_2 + 1x_3 + 0.1x_4$$

is satisfied no matter what the values of the disposal activities satisfying the equality (provided they are positive), and similarly the other constraints are satisfied, i.e. the solution is feasible. (If this were not the case the x_j values would not be optimal.)

Thus, the value of x_5 in order to convert the inequality to an equality can be easily determined by noting the difference between use and supply of land. Therefore a set of x_5–x_8 can be determined such that x_j (j = 1–8) *will also be a feasible solution*. Thus a set of x_j satisfying the original problem also satisfies the augmented problem and vice versa. *Therefore*, if the optimal farming system for the augmented problem is determined, this will be the optimal system to the original problem.

11.4 A Formal Statement of the LP Problem

By way of summarizing the preceding discussion, the following formal statement can be made:

1. A farm firm employs a number of factors of production (resources and other limitations on production) of which m are in limited supply (if a resource is non-limiting it can be ignored).

Let b_i = the available level of the ith resource ($i = 1, 2 \ldots m$) or constraint.

2. The firm can use up to n different production activities.

Let x_j = the number of units of the jth production activity ($j = 1, 2, \ldots n$).

A particular set of x_j represents a particular operational system (farm system).

Let P_j be a code given to the jth activity for identification purposes (e.g. potatoes = P_3).

Let the net revenue, or gross margin, earned per unit of activity P_j be C_j ($j = 1, \ldots n$).

3. Denote the requirement by the jth activity for the ith resource or constraint by r_{ij} per unit of P_j. Thus, the activity P_j can be described by a column of r_{ij} figures (referred to as a column vector).

$$P_j = \begin{pmatrix} r_{1j} \\ r_{2j} \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ r_{mj} \end{pmatrix}$$

4. Thus, the total production technology of the firm can be described by a matrix of coefficients. This matrix will have n column vectors and m row vectors.

Thus:

$$\begin{pmatrix} r_{11} & r_{12} & \cdots\cdots\cdots & r_{1n} \\ r_{21} & r_{22} & \cdots\cdots\cdots & r_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ r_{m1} & \cdots & \cdots\cdots\cdots & r_{mn} \end{pmatrix}$$

5. Adding disposal activities, this matrix can be expanded to form the following matrix having m rows and $n + m$ columns:

$$\begin{pmatrix} r_{11} & \cdots\cdots & r_{1n} & 1\;0\;0 & \cdots\cdots & 0 \\ r_{21} & \cdots\cdots & r_{2n} & 0\;1\;0 & \cdots\cdots & 0 \\ . & & . & .\;\;\;1 & \cdots\cdots & 0 \\ \vdots & & \vdots & \vdots & \ddots & \vdots \\ r_{m1} & \cdots\cdots & r_{mn} & 0 & \cdots\cdots & 1 \end{pmatrix}$$

6. An operational system for the firm is defined by a set of values for the x_j ($j = 1, 2 \ldots n$) (or if disposals are added, $j = 1, 2 \ldots n, n{+}1, \ldots n{+}m$). For such a system to be feasible, the use of resources must not exceed the supply. Thus:

$$b_i \geq \sum_{j=1}^{n} x_j r_{ij} \quad \text{(for all } i = 1, \ldots m)$$

If disposal activities are added:

$$b_i = \sum_{j=1}^{n+m} x_j r_{ij} \quad \text{(for all } i = 1,\ 2,\ \ldots m)$$

Also,

$x_j \geq 0$ for all $j = 1, 2, \ldots n + m$

7. Denote the contribution of the jth activity to the value of the objective by C_j per unit of P_j. Thus, the value of the objective for a set of x_j values is given by:

$$Z = \sum_{j=1}^{n} x_j C_j \quad (j = 1,\ 2,\ \ldots n)$$

8. Thus, an optimal operating system is a set of x_j (j = 1, 2, … n+m) which maximizes the objective function

$$Z = \sum_{j=1}^{n} x_j C_j$$

and satisfies:

(i) $b_i = \sum_{j=1}^{n+m} x_j r_{ij} \quad (i = 1,\ 2,\ \ldots\ m)$

(ii) $x_j \geq (j = 1,\ 2,\ \ldots\ n + m)$

11.5 Graphical Presentation and Solution

Introduction

To obtain a clear understanding of the LP framework for a farm problem, and to see the form an optimal solution will take, it is necessary to consider the problem in graphical form. As it is only possible to draw graphs for a problem with two activities, consider the following simplified and slightly adjusted version of the problem introduced before:

$$\text{Max } Z = 100x_1 + 200x_2$$

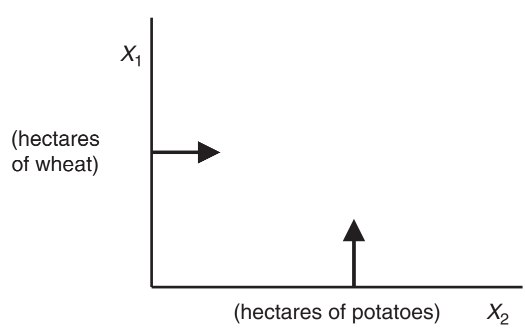
subject to:

$$200 \geq 1x_1 + 1x_2$$
$$200 \geq 0.5x_1 + 4x_2$$
$$45 \geq 0x_1 + 1x_2$$

Thus, x_1 and x_2 represent the hectares of wheat and potatoes.

The feasible values of x_1 and x_2

There will be many pairs of values for x_1 and x_2 that will satisfy the constraints. The problem is to find the pair that maximizes Z. First, however, draw a graph delineating the feasible values of x_1 and x_2. Set up a graph with values of x_1 and x_2 on the axes:



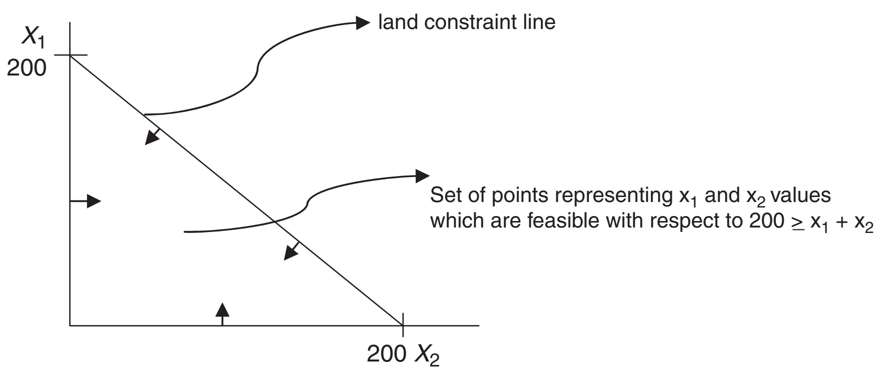
The first requirement for feasibility is that $x_j \geq 0$ for all j. Thus, to satisfy this requirement we can only consider positive values of $x_1$and x_2. That is, only points on the graph which are to the right of the x_1 axis and above the x_2 axis are feasible. All such points are not, however, necessarily feasible as they may not satisfy the other constraints.

The second requirement for feasibility is that $b_i \geq \Sigma\, x_j r_{ij}$ all i

1. *Consider the first constraint*. For feasibility the set of (x_1, x_2) values must satisfy

$$200 \geq 1x_1 + 1x_2$$

i.e. in terms of the graph, any point which lies on or below a line drawn between the points $x_1 = 200$, $x_2 = 0$, and $x_1 = 0$, $x_2 = 200$ will be feasible (satisfy the constraint).



To find where the boundary line lies it is only necessary to change the inequality to an equality and to express the level of one activity as a function of the other (recall that this is how the product/product curve in production economics theory is defined):

$$x_1 = 200 - 1x_2$$

Then, setting x_2 at various levels the value of x_1 can be determined and the boundary points plotted. As the relationship is linear only two points are required to draw the boundary line. Note that the slope of the line is:

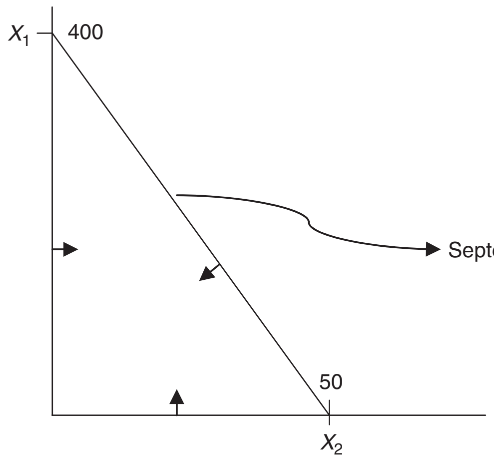
$$\frac{dx_1}{dx_2} = -1$$

(is this the marginal rate of substitution between wheat and potatoes?)

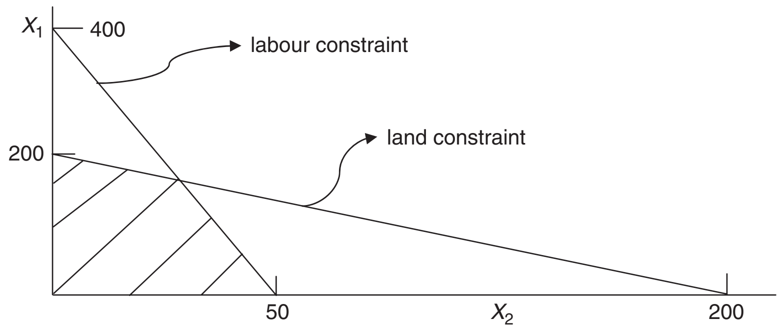
However, all points satisfying the first constraint may not simultaneously satisfy the second.

2. Consider the points satisfying the second constraint

$$200 \geq 0.5x_1 + 4x_2$$

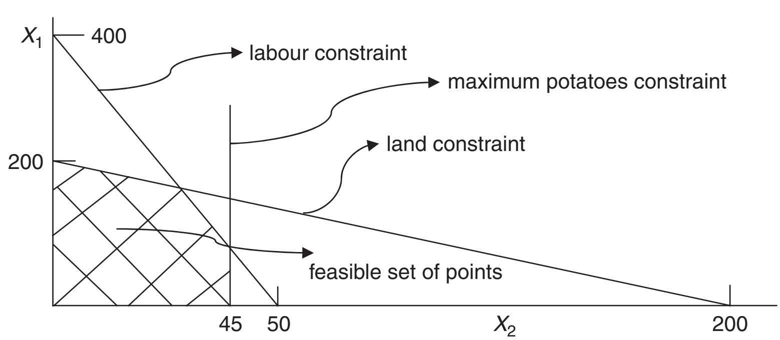


Clearly, not all points in the area above fall in the area matching the first constraint. Thus, to determine the area representing points that satisfy both *constraints simultaneously*, superimpose the two constraint lines on the one graph:

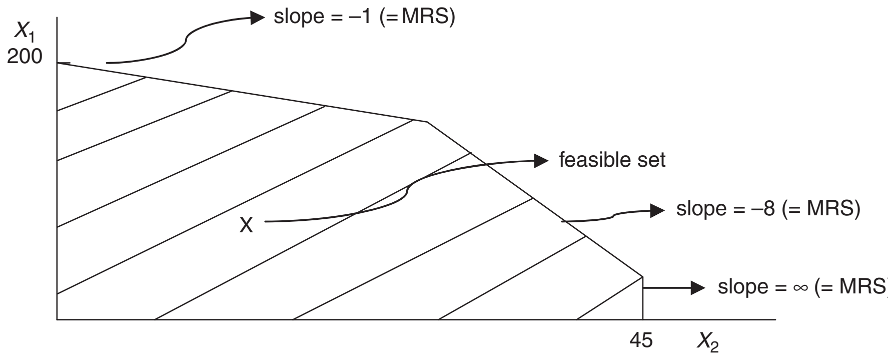


The shaded area represents the points (sets of x_1, x_2 values) that satisfy both constraints simultaneously. Points outside this area may satisfy one constraint but not both together.

Similarly, the area or set of points which satisfies all the constraints simultaneously is:



Simplifying, by removing the non-wanted segments of the graph, gives:



The boundary of the area is the production possibility frontier, which looks similar to the product–product relationship of production economics theory.

Having defined the feasible set the next problem is to find the points (a pair of x_1, x_2 values) from within this set which maximizes the value of the objective function.

The profit-maximizing solution

Given that the gross margins per unit of P_j will not decline as x_j increases, it is clear that production should never occur at a point *within* the feasible area (at an interior point of the feasible set). The reason is that greater production is possible given the same quantity of resources so that it will always pay (provided $C_j > 0$) to move on to a boundary point. The problem is to decide *where on the boundary* to produce.

To determine the optimal boundary point, superimpose on the graph an iso-profit curve. Given that:

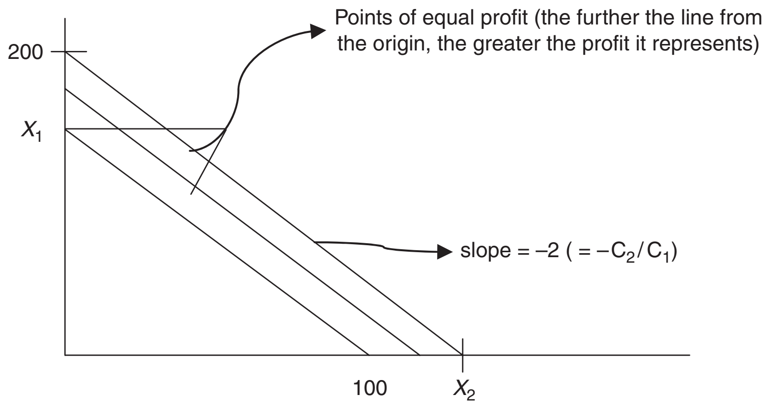
$$Z = 100x_1 + 200x_2$$

the slope of an iso-profit curve will be:

$$x_1 = \frac{Z}{100} - \frac{200x_2}{100}$$

$$\therefore \frac{dx_1}{dx_2} = -2$$

Thus:

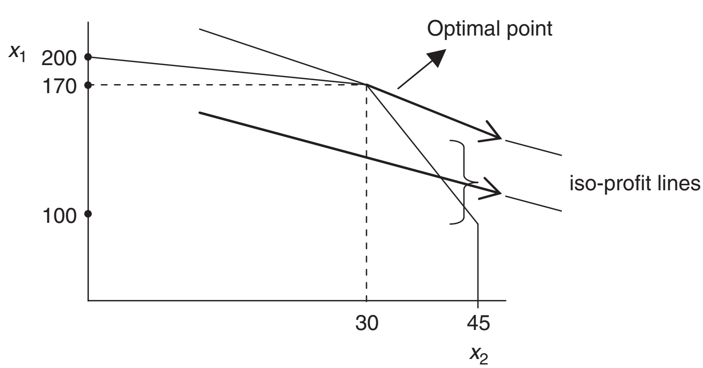


Given that C_j *does not vary* as x_j varies, the slope of lines representing higher and higher levels of profit will not change.

To plot a particular iso-profit line, set Z equal to the desired profit level, set x_1 at an arbitrary (but reasonable) level and solve for x_2 (but remember x_2 cannot be negative). Repeat with x_1 at various other levels and plot the x_1, x_2 points (only two points are necessary as the function is linear). As Z increases, the curve representing points of equal, but higher, profit will be further and further from the origin. Furthermore, they will be parallel (as slope does not change as x_j varies).

Given that profit is to be maximized, *the optimal farm system* will be represented by the point on the production possibility boundary that is also on the iso-profit line furthest from the origin. No other point will have a higher profit and still be feasible. Thus, to determine the optimal point, superimpose an iso-profit curve on the production possibility curve and move it parallel to itself until it cannot be moved further from the origin and still have a point in common with the production possibility curve.

That is:



The optimal point appears to be at $x_1 = 170$, $x_2 = 30$, i.e. 170 ha of wheat and 30 ha of potatoes forms the optimal farming system. In order to check this, note that the optimal point is at the intersection of the land and labour constraint lines, i.e. these resources are fully utilized. Thus, the x_1, x_2 pair of values must simultaneously satisfy these equations, and therefore can be derived from simultaneously solving the equations:

$$200 = 1x_1 + 1x_2$$
$$200 = 0.5x_1 + 4x_2$$

If this is done, $x_1 = 171.43$, $x_2 = 28.57$. Profit at this point is:

$$Z = 171.43\ C_1 + 28.57\ C_2 = \$22{,}857.$$

(In order to obtain the true surplus, fixed costs must be subtracted from this figure.)

To check that the solution is feasible, the values of x_j can be substituted into the original constraint equations and evaluated.

Thus:

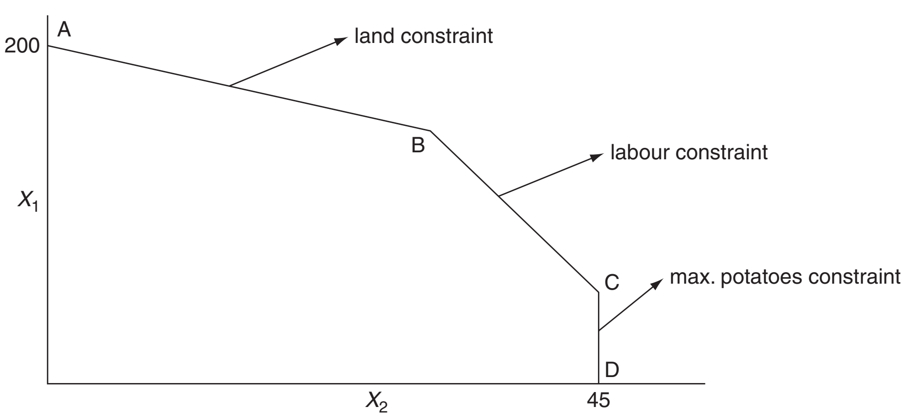
1. Use of land = (1 × 171.43) + (1 × 28.57) = 200
2. Use of labour = (0.5 × 171.43) + (4 × 28.57) = 200
3. Use of max. potatoes constraint = (0 × 171.43) + (1 × 28.57) = 28.57

All constraints are satisfied (in this case this was obvious. In a realistic problem this is seldom obvious).

Solving this problem using the graphical method is particularly easy. With realistic problems it is clear, however, the graphical method cannot be used as the number of possible activities is greater than two or three.

11.6 The Need for Disposal Activities

Disposal activities are also known as ‘slack’ activities. The graphical presentation of the problem makes it clear why disposal activities must be included. Consider the example problem:



As the optimal solution will lie somewhere on the production boundary, there is no point along the boundary at which all *three* constraints hold simultaneously as equalities without including disposal activities. Thus, in order to have equality relationships holding *at all points that might be optimal*, the disposal activities must be introduced.

Consider which disposal activities would be required at a positive level to make the constraints hold as equalities along each segment and point on the possibility curve:

(i) A→B: labour and maximum potatoes disposals as these two resources are not fully used.
(ii) At B: maximum potatoes disposal.
(iii) B→C: land and maximum potatoes disposals.
(iv) C: land disposal.
(v) C→D: land and labour disposals.

11.7 The Important Features of the Optimal Point

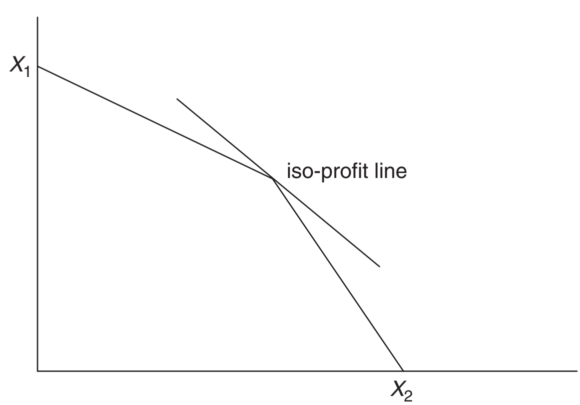
Introduction

The optimal point exhibits special features that enable conclusions to be drawn about the nature of an optimal solution. These lead to the development of an efficient computational method for solving the problem.

Corner points and the optimal point

The optimal point will always be a corner point of the production possibility frontier, i.e. a point lying at the intersection of two (or more) constraint lines.

Consider the following production possibility graph:

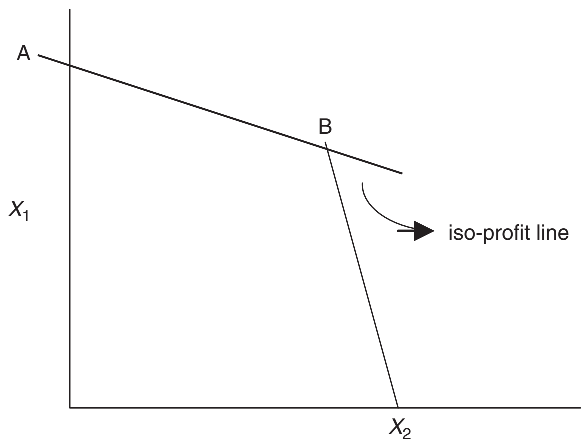


The optimal iso-profit line, the slope of which is given by the ratio of the prices of the activities

$$\left(-\frac{C_2}{C_1}\right)$$

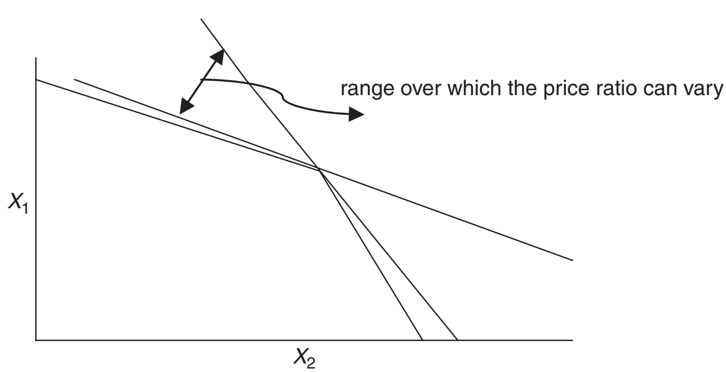
can only be in one of *two* general positions:

(i) With the *only* point in common between the optimum iso-profit line (or price ratio line) and the production boundary at a corner point; or
(ii) Where the optimum iso-profit line is parallel to a linear segment of the production possibility curve. That is:



Consider case (**ii**) above. In this case there are many points (an infinite number) giving an optimal value of Z, all along segment AB. These are all alternative optimal solutions. However, *there are still two corner points (A & B) which are in the set of alternative optimal solutions*. Thus, there will be *at least* one corner point which represents a solution having an optimal Z value (opt. profit) in *all cases*. Thus, in determining an optimal solution, *only corner point solutions need be examined*.

(Note that an optimal solution often exhibits stability as the ratio of activity C_js can vary over a defined range without the optimal point changing.)

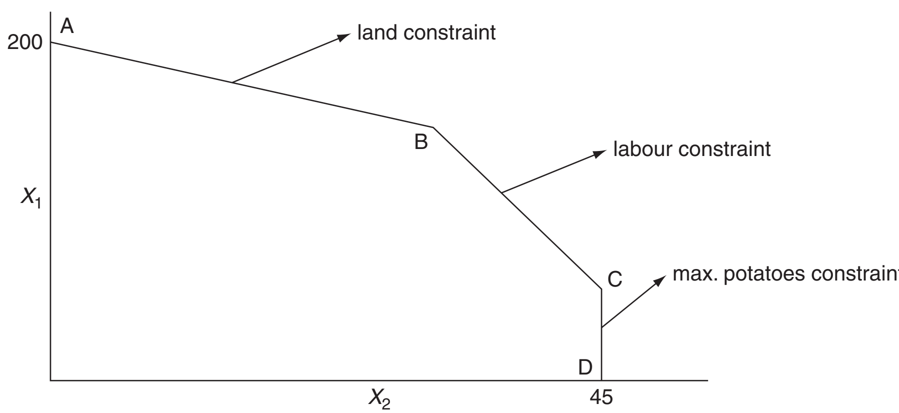


The nature of corner points (extreme points)

Given that disposal activities have been included, any corner point of the production boundary represents a solution to the problem *in which* m *of the* x_j *are at a positive level, and all other* x_j *are equal to 0 (where* m = *number of constraints).* Thus, if the number of activities (real and disposal) equals $m+n$ and the number of constraints equals m, then an optimal solution will have:

- m of the x_j $(j = 1, 2, \ldots m+n) > 0$
- n of the x_j $(j = 1, 2, \ldots m+n) = 0$

To see this, consider the graph of the example problem and examine each corner point and determine the number of variables that are at a positive level for each point:



Let x_3 = hectares of land disposal,
x_4 = hours of labour disposal,
x_5 = hectares of max. potatoes disposal.

(a) Point A

$$x_1 > 0,\ x_2 = 0,\ x_3 = 0,\ x_4 > 0,\ x_5 > 0$$

i.e. all land is used, some labour is not used and all of the max. potato constraint remains non-used. Total variables at positive level = 3.

(b) Point B

$$x_1 > 0,\ x_2 > 0,\ x_3 = 0,\ x_4 = 0,\ x_5 > 0$$

Total variables at positive level = 3.

(c) Point C

$$x_1 > 0,\ x_2 > 0,\ x_3 > 0,\ x_4 = 0,\ x_5 = 0$$

Total variables at positive level = 3.

(d) Point D

$x_1 = 0, x_2 > 0, x_3 > 0, x_4 > 0, x_5 = 0$

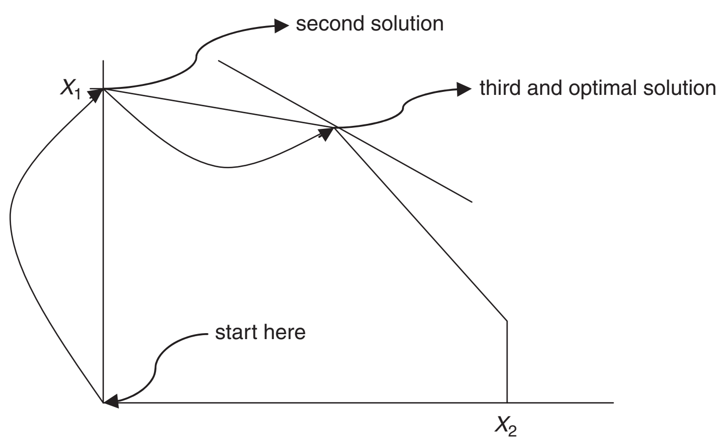
Total variables at positive level = 3.
Thus, the number of variables at positive level at all comer points is three. This number is equal to the number of constraints in the problem (in general, this is m).

Conclusion

In looking for an optimal solution, only solutions containing m (the number of constraints) variables at a positive level need be examined. Furthermore, it is clear that there will only be a finite number of corner points and thus a finite number of solutions with only m variables at a positive level. Thus, a solving process that looks at the corner points will eventually find the optimal solution.

11.8 An Outline of the Solving Process

The practical solving process uses an iterative procedure. It initially finds a solution representing a corner point and examines whether or not moving to corner points immediately *adjacent* to it will improve profit. If not, an optimal solution has been reached. If profit can be improved, the solution representing the adjacent corner point is determined. This process is repeated until the optimal corner point is found. Graphically:



Note that if at each step, or iteration, the profit improves, then all corner points do not have to be examined when searching for the optimal point.

Given many activities and many constraints, a multi-dimensional production possibility frontier is obtained. The principles discussed for the two activity problem (two dimension) apply in exactly the same way for this multi-dimensional problem.

11.9 Production Economics and LP

Production economics shows that the profit maximizing point for the constrained product–product problem is where:

Marginal rate of substitution between products = the negative of the inverse of the product price ratio

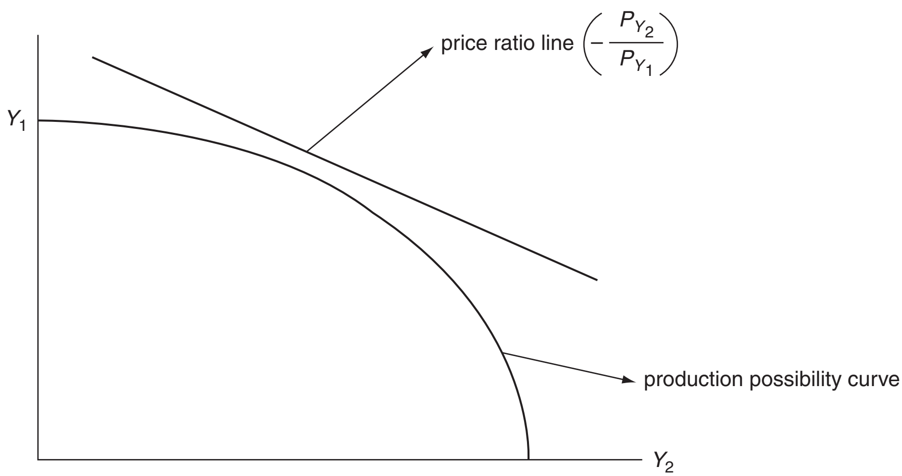
i.e. $\frac{\Delta Y_1}{\Delta Y_2} = \frac{P_{Y1}}{P_{Y2}}$

i.e. $MVP_1 = MVP_2$
Where MVP is the marginal value product;

Y_1 and Y_2 = output of products 1 and 2;

and P_{Y_1} and P_{Y_2} = product prices per unit of output.

In other words, production must be organized so that the above condition is met. Graphically:

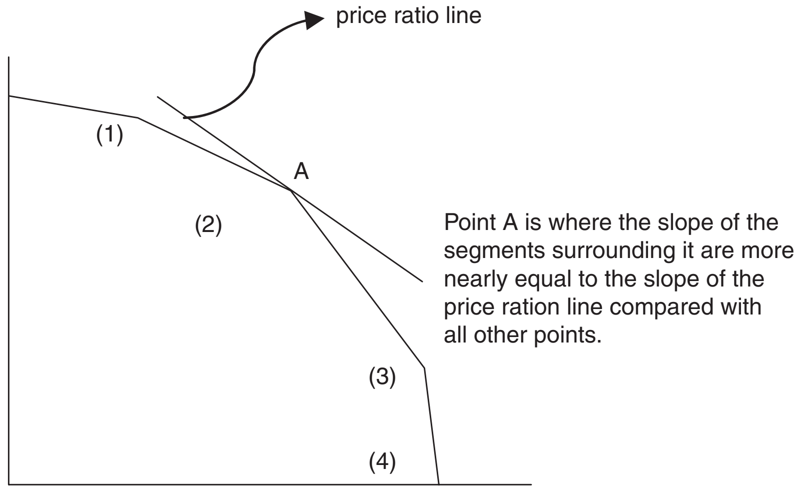


Recall that the production possibility curve must represent the maximum possible output given the resources available, i.e. represents technically efficient points.

The above situation is very similar to the LP problem. Consider the example problem. If x_1 is adjusted to represent the output of wheat (kg) and x_2 kg of potatoes, the problem becomes almost identical. In the LP problem the production possibility curve is segmented so that the marginal rate of substitution does not continuously change. Thus, in general it is impossible to exactly equate the marginal rate of substitution with the price ratio. Therefore, the optimality condition in the LP problem is:

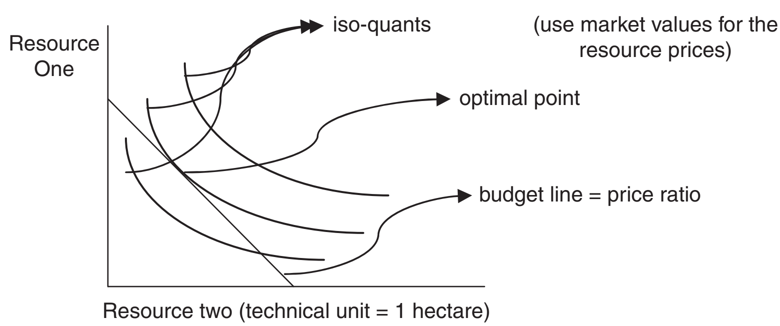
> A particular point is optimal if there is no other point with the marginal rate of substitution (MRS) of the segments surrounding it as nearly equal to the inverse of the product price ratios at this point (though occasionally MRS = Price Ratio).

i.e.

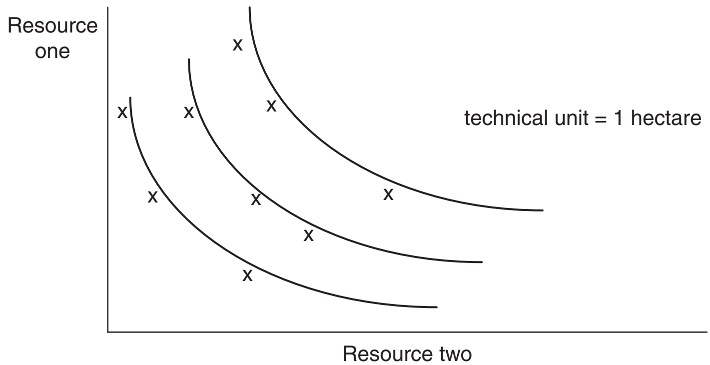


The classical production economics optimality condition can only hold given a continuously curving possibility curve. If in practice this does not hold, then the LP optimality condition becomes the important condition.

There is a direct correspondence between LP and production economics only if the production activities in LP are technically and financially efficient producers of the product. In defining the wheat activity for example, there is no assurance that this is the case. That is, in production economics terms, given the resources available, there is no assurance that the factor–factor optimality condition is satisfied. Graphically, for a particular product:



However, if for each product a number of activities are defined, each one of which uses the resources in a different proportion, then it is likely that one of these will approach the optimal combination. The LP routine will select the particular activity which maximizes profit. Thus, an activity could be defined for each of the following points, each would have different r_{ij} coefficients and a different gross margin.



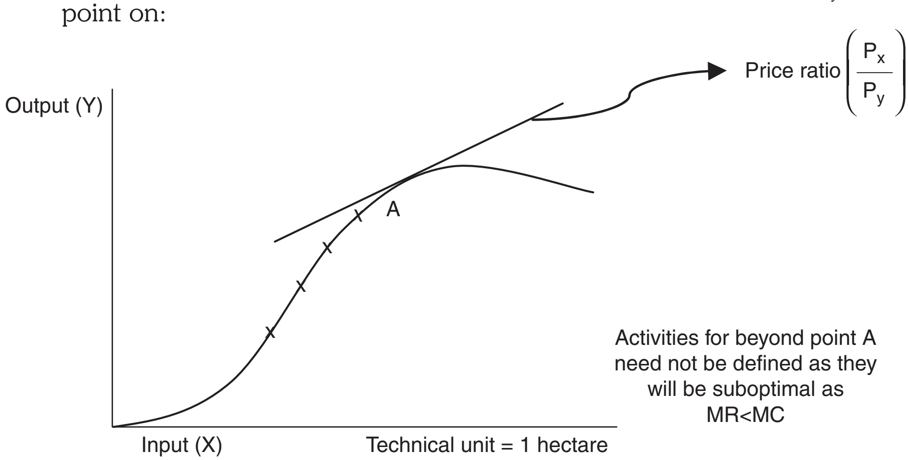
There is one further point to consider in the comparison. Production economics requires that, at the optimal point for each resource and inputs:

Marginal return ≥ Marginal cost

i.e. $\dfrac{\text{MR}}{\text{MC}} \geq 1$

because MR will be greater than MC at the optimal point if there are insufficient resources to enable the MR = MC point to be reached. In defining the LP activities there is no assurance that MR/MC will equal the optimal ratio. However, if a sufficient number of activities for each product, each one with a different combination of variable factors, is defined then the LP routine will ensure that one with MR/MC at approximately the same ratio will be chosen.

That is, for each product that uses the fixed resources in a given proportion, define a different activity (each of which will have a different C_j) for each point on:



Thus, provided a sufficient number of different activities are defined for each alternative product (each one will have either different r_{ij} and/or C_j coefficients), there is a one-to-one correspondence between production economics and LP where linear relationships can be used to represent the production possibility curve. Note that a study of production economics can be directly related to practical problems through LP as it is an operational method for solving practical problems. Similarly, the relationships between gross margins analysis and LP has been made clear. Correspondingly, the relationship between production economics and gross margin analysis should also now be clearer.

11.10 Details of the Solving Process

Introduction

Given a set of *m* simultaneous equations that represent the constraints of an LP problem, there will be many different solutions (that is, sets of x_j values) that satisfy the equations. Of these solutions, it has been shown the only ones of interest have *m* variables (*m* = number of constraints) at a positive level, and *n* variables at zero level (*n*+*m* = the number of disposals and real activities). Thus, to solve a LP problem a method that provides solutions having this property is required. Given a '*m*' variable solution it can be checked to see whether any changes will increase *Z* (the total gross margin). If a change improves *Z*, this change is made. This *iterative* process is continued until an optimal solution is obtained. Optimality is identified by checking that any change to this solution will only reduce *Z*.

This section outlines a method for obtaining an '*m*' variable solution, including how to develop a new solution with a greater Z value. The method developed is not an efficient operational technique but it does provide an understanding of the principles involved, which are required for efficiently using LP. The details of an operational system (Simplex method) are provided in Appendix 3 for students who might wish to create a computer program or better understand readily available packages. The basic principles used in both systems are identical.

Equations in canonical form

Take any set of simultaneous equations. If this set has the following general form it is said to be in *canonical form*:

$$
\begin{aligned}
b_1 &= 1x_1 + 0x_2 + 0x_3 + \ldots\ldots + 0x_q + r_{1,q+1}x_{q+1} + r_{1,q+2}x_{q+2} + \ldots\ldots \\
b_2 &= 0x_1 + 1x_2 + 0x_3 + \ldots\ldots + 0x_q + r_{2,q+1}x_{q+1} + r_{2,q+2}x_{q+2} + \ldots\ldots \\
b_3 &= 0x_1 + 0x_2 + 1x_3 + \ldots\ldots + 0x_q + r_{3,q+1}x_{q+1} + r_{3,q+2}x_{q+2} + \ldots\ldots \\
&\quad\vdots \\
b_q &= 0x_1 + 0x_2 + 0x_3 + \ldots\ldots + 1x_q + r_{q,q+1}x_{q+1} + r_{q,q+2}x_{q+2} + \ldots\ldots
\end{aligned}
$$

The general nature of the form is easier to see if the x terms are left out:

$$
\begin{array}{lcccccccccccccccc}
b_1 & = & 1 & + & 0 & + & 0 & + & \ldots\ldots & + & 0 & + & r_{1,q+1} & + & r_{1,q+2} & + & \ldots\ldots \\
b_2 & = & 0 & + & 1 & + & 0 & + & \ldots\ldots & + & 0 & + & r_{2,q+1} & + & r_{2,q+2} & + & \ldots\ldots \\
b_3 & = & 0 & + & 0 & + & 1 & + & \ldots\ldots & + & 0 & + & r_{3,q+1} & + & r_{3,q+2} & + & \ldots\ldots \\
. & & . & & . & & . & & 1 & & . & & & & & & \\
. & & . & & . & & . & & \quad 1 & & . & & & & & & \\
. & & . & & . & & . & & . & & . & & & & & & \\
b_q & = & 0 & + & 0 & + & 0 & + & \ldots\ldots & + & 1 & + & r_{q,q+1} & + & r_{q,q+2} & + & \ldots\ldots
\end{array}
$$

An identity matrix (a diagonal set of 1's, 0 elsewhere) appears within the set of coefficients. In any particular case, the order of the variables may need to be re-arranged to make the identity matrix coefficients appear side by side.

The above definition is in algebraic terms. Using words, a set of equations in canonical form is defined as:

> For every row (equation) there is a variable (x_j) which has a coefficient (r_{ij}) equal to 1 in that particular row and a 0 coefficient in all other rows.

Note that the constraint equations for an LP problem, given that disposal activities have been added, are in this form. For example, consider the example problem used before. The constraints with disposals added were (x_1 = hectares of wheat, x_2 = hectares of potatoes).

$$200 = 1x_1 + 1x_2 + 1x_3 + 0x_4 + 0x_5$$
$$200 = 0.5x_1 + 4x_2 + 0x_3 + 1x_4 + 0x_5$$
$$45 = 0x_1 + 1x_2 + 0x_3 + 0x_4 + 1x_5$$

The disposal activities coded P_3, P_4 and P_5 provide variables that satisfy the above definitional requirements.

A solution to a set of equations in canonical form

A solution to a set of equations in canonical form which *has only* m *variables* at a positive level is immediately available by inspection. This solution is given by the following:

Set the *variables* which have a unit coefficient in a particular row, and zero coefficients in other rows, equal to the left-hand-side constant of the row in which the unit coefficient appears. Set *all other variables* equal to 0. Note that as there are m rows, there will be m variables at a positive level.

For example, consider the example problem:

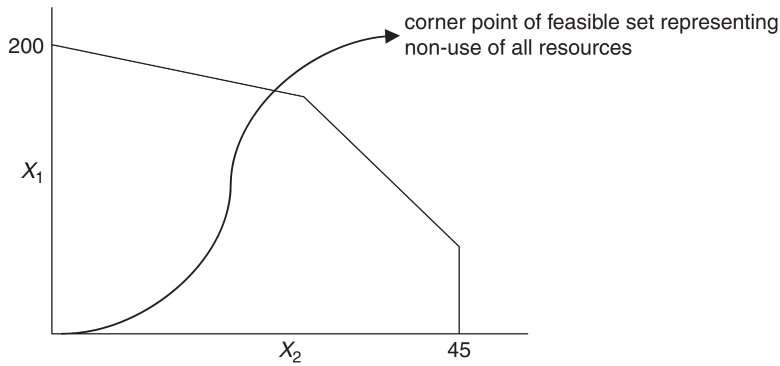
$$200 = 1x_1 + 1x_2 + 1x_3 + 0x_4 + 0x_5$$
$$200 = 0.5x_1 + 4x_2 + 0x_3 + 1x_4 + 0x_5$$
$$45 = 0x_1 + 1x_2 + 0x_3 + 0x_4 + 1x_5$$

An obvious solution is:

$x_3 = 200$
$x_4 = 200$ (check to make sure this is a solution)
$x_5 = 45$
$x_1 = x_2 = 0$

This solution for this particular example is one in which no production occurs. All resources are put to 'non-use'. This is the starting solution used in the iterative LP solving process as it is an easy solution to obtain in order to start the process. (The associated $Z = 0$.)

Graphically, this solution is the origin of the feasible set graph:



In obtaining this general kind of solution the equations and variables do not have to be arranged such that the variables with a unit coefficient in one row and zeros elsewhere are side by side. They can appear in any order provided there is in fact one such variable for every row.

Some terminology

A solution to a set of m simultaneous equations in which only m variables are at positive levels is referred to as a *basic solution*. (Recall that such a solution represents a corner point on the production possibility boundary, and that an optimal solution will be a basic solution.)

The m variables at positive levels are referred to as *basic variables*. The other n variables at zero level are referred to as *non-basic variables* (where the total number of variables, both real and disposal, = $n+m$).

Obtaining new basic solutions

Assume there is a set of equations in canonical form and, therefore, a basic solution. Further, assume that if:

(a) One of the variables at a positive level is set equal to 0 in a *new* solution, and
(b) One of the variables *previously* at zero level is set at a positive level in this new solution,

the profit from the new solution is greater than the profit from the old (i.e. old $Z <$ new Z). Later it is shown how to ensure this is the case.

To determine the variable values in this new solution it is necessary to determine a new canonical form (a new basic solution).

This is achieved by adjusting the equations so that:

(a) The m variables to be positive in the *new* solution have a unit coefficient in one row and zeros elsewhere, *and*
(b) The unit coefficients *do not appear* in the same rows (i.e. the coefficients must form an identity matrix).

$$\begin{array}{ccccc} 1 & 0 & 0 & \dots & 0 \\ 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ . & & & & . \\ . & & & & . \\ 0 & \dots & \dots & \dots & 1 \end{array}$$

By *adjusting* the equations is meant using simple calculations such as multiplication, division and subtraction of the equations using constants as required. Using these simple operations to both sides of any equation does not alter the *basic relationships* the equations express. The *adjustment* processes must be selected so that the equations end up in the desired canonical form. Once this has been done a new basic solution is self evident.

For example, take the first two equations of the example problem:

$$200 = 1x_1 + 1x_2 + 1x_3 + 0x_4$$

$$200 = 0.5x_1 + 4x_2 + 0x_3 + 1x_4$$

The current basic solution is:

$x_3 = 200$ (200 ha of land in disposal)
$x_4 = 200$ (200 h of September labour in disposal)

Now, assume for the moment that a new basic solution with variables x_1 and x_4 positive is required. Thus, the equations must be adjusted so that:

(a) The variable x_1 has a unit coefficient in one row (equation) and a zero coefficient in the other, and
(b) The variable x_4 has a unit coefficient in the row (equation) in which x_1 has a zero coefficient, and has a zero coefficient in the other row (equation).

Having done this, the basic solution becomes self evident, i.e.

$x_1 =$ left-hand-side constant of the row in which the unit coefficient attached to x_1 appears

x_4 = left-hand-side constant of the row in which the unit coefficient attached to x_4 appears.

Note that in adjusting the equations the left-hand-side 'constants' will change.

Now, examine how the required canonical form might be obtained:

(a) As x_1 has a unit coefficient in row one and x_4 has a zero coefficient in the same row, equation one is already in the desired form.
(b) Thus, adjust equation two (row two) such that x_4 has a unit coefficient.

Using trial and error, it is found that the following steps give the desired form:

(i) Multiply equation two by 2.

$\therefore\ 400 = 1x_1 + 8x_2 + 0x_3 + 2x_4$

(ii) Subtract equation one (row 1) from the product, i.e.

$400 - 200 = (1x_1 - 1x_1) + (8x_2 - 1x_2) + (0x_3 - 1x_3) + (2x_4 - 0x_4)$
$\therefore\ 200 = 0x_1 + 7x_2 - 1x_3 + 2x_4$

(iii) Divide the result by 2:

$\therefore\ 100 = 0x_1 + 3.5x_2 - 0.5x_3 + 1x_4$

Thus, the equation is in the desired form.

The basic solution is now evident. Putting the two equations together:

$200 = 1x_1 + 1x_2 + 1x_3 + 0x_4$

$100 = 0x_1 + 3.5x_2 - 0.5x_3 + 1x_4$

the solution is clearly:

$x_1 = 200$ (200 ha of wheat)
$x_4 = 100$ (100 h September labour non-use)
$x_2 = x_3 = 0$

Similarly, the other basic (corner point) solutions could be determined. (These are the following combinations at a positive level: x_1 and x_2, x_1 and x_3, x_2 and x_3, x_2 and x_4.) It is clear, however, that given many equations the trial and error method of adjusting the equations could become difficult. Fortunately, there is a set of rules that enable the equations to be adjusted into the desired canonical form with a minimum of calculations, as explained in Appendix 3.

Determining the change in profit that would occur if a variable is made positive

The basis of the method

Given a basic solution, *to decide* the variables to include in a new basic solution that has a greater profit, determine the effect on profit of making one of the variables at zero level become positive. (In moving from one basic solution to another, *only one* variable is increased above zero and another dropped to zero. All other variables in the basic solution remain at a positive level, though the levels will probably change as the equations are adjusted.)

Recall that given a basic solution, all resources are 'used' by the variables or activities at a positive level (this 'use' may of course consist of non-use if a disposal activity or variable is at a positive level). Thus, if a variable (activity) which is at zero level is increased above zero the levels of the other basic activities must decrease to free up resources to allow its production (in a few cases where a basic activity actually provides resources they will increase in contrast to the mainly resource using activities, e.g. a resource-buying activity).

If an activity's level is increased above zero by one unit the profit made by the current solution will:

(i) Decrease by an amount equal to the sum of the drop in each basic activity's level *multiplied* by their gross margins (in some specialized cases an activity level may increase).

Let this sum = Z_j ($_j$ refers to the activity increased from zero level to one, i.e. x_j goes from 0 to 1);

(ii) Increase by an amount equal to the gross margin of the activity which is increased from zero to one. That is, by C_j.

Thus, the net effect of introducing one unit of a non-basic activity into a solution is given by:

$$Z_j - C_j$$

If this sum is

(i) *Negative*, profit will *increase* as the gain is greater than the decrease in profit,

(ii) *Positive*, profit will *decrease* as the gain is less than the decrease in profit.

Thus, given a non-basic activity with $Z_j - C_j$ negative, if this activity is introduced into a *new basic solution* (in so doing, one of the other basic activities will become zero), the total profit of the *new plan* will be greater than the total profit from the old plan (old basic solution).

Determining $Z_j - C_j$

The gross margin (or net revenue) (C_j) has already been worked out. To calculate Z_j we need to know:

(i) The amount by which each basic activity changes as the *j*th non-basic activity is increased from 0 to 1, and

(ii) Each basic activity's C_j (which is already available).

Working out how much each current basic activity will change is directly observable from the equations representing the basic solution. To see this, first define some terminology. *Let the* level of a particular basic activity be referred to as the level of the *i*th basic variable ($i = 1, 2, \ldots$ m). Thus, the left-hand side of one of the equations will indicate the level of the ith basic variable. *Let this* equation be referred to as the ith equation ($i = 1, 2, \ldots m$). Normally they are numbered sequentially from top to bottom. For example, given:

$$80 = 1x_1 + 5x_2 + 28x_3 + 0x_4$$
$$10 = 0x_1 + 2x_2 + 4x_3 + 1x_4$$

the basic solution is $x_1 = 80$, $x_4 = 10$, $x_2 = x_3 = 0$, and

(i) If x_1 *is called* the 1st basic variable, then the first equation is called equation 1.
(ii) If x_1 *is called* the 2nd basic variable, then the first equation is called equation 2.

Thus, the following statement can be made:

The amount by which the ith basic variable will decrease (or increase) if the jth non-basic activity is increased by one unit is given by the coefficient (adjusted r_{ij}) attached to the jth variable (x_j) in the ith equation. A +ve coefficient indicates a decrease while a –ve coefficient indicates an increase.

Thus, in the above example, if x_2 were increased by one unit, then:

(i) x_1 would decrease by 5 units,
(ii) x_4 would decrease by 2 units.

Given this information, Z_j can now be worked out by multiplying these figures by the C_j of each basic activity and summing the products.

Returning to the example problem originally used, consider how to estimate $Z_j - C_j$ assuming x_2 was increased from zero to one:

$$200 = 1x_1 + 1x_2 + 1x_3 + 0x_4$$

$$100 = 0x_1 + 3.5x_2 - 0.5x_3 + 1x_4$$

$$(C_1 = 100,\ C_2 = 200,\ C_3 = 0,\ C_4 = 0)$$

$$Z_2 = (1 \times 100) + (3.5 \times 0) = 100$$

$$\therefore Z_2 - C_2 = 100 - 200 = -100$$

Thus, introducing the activity coded P_2 into a new basic solution would increase profit by \$100 for each unit. Similarly, $Z_3 - C_3 = (1 \times 100) + (-0.5 \times 0) - 0 = 100$. Thus, if x_3 was increased above 0, profit would decrease.

A solving method

Given a basic solution to start with, the solving method is:

(i) Determine the $Z_j - C_j$ for each non-basic activity. Select the one with the most negative $Z_j - C_j$;
(ii) Adjust the equations so that they are in canonical form with respect to the new set of basic activities. These are the old set less one variable, plus the new activity;
(iii) Recalculate $Z_j - C_j$ and repeat the process until all $Z_j - C_j$ are zero or positive.

Marginal rates of substitution

The coefficients obtained in each adjusted set of equations are the marginal rate of substitutions between the products (activities). The coefficients in the ith equation or row that is attached to the jth variable (i.e. adjusted r_{ij}) is the marginal rate of substitution between the ith basic activity and the jth non-basic activity.

Thus, each equation is in fact the production possibility equation with respect to the ith basic activity and the non-basic activities. Thus, considering row two from the example problem:

$$100 = 0x_1 + 3.5x_2 - 0.5x_3 + 1x_4$$

$$\therefore x_4 = 100 - 3.5x_2 + 0.5x_3$$

$$\therefore \frac{\partial x_4}{\partial x_2} = 0.5 = MRS$$ (the partial derivative – the quantity by which x_4 changes as x_2 changes one unit)

Production economics states that for optimality, MRS = negative of price ratio.

In calculating $Z_j - C_j$ the procedure is seeing whether this holds. If the MRS is greater than the price ratio (so that $Z_j - C_j < 0$), the activity is introduced into the solution.

Consider the above equation from the example problem and assume $C_2 = 200$, $C_3 = 0$, $C_4 = 20$.

Then, using LP terminology:

$Z_2 - C_2 = -\ 130$ (conclude increase P_2) and
$Z_3 - C_3 = -\ 10$ (conclude increase P_3)

Using production economics methods:

(a) $$\frac{\partial x_4}{\partial x_2} = -3.5$$

$$\frac{-C_2}{C_4} = -\frac{200}{20} = 10$$

Conclusion: increase P_2 as MRS > price ratio.

(b) $$\frac{\partial x_4}{\partial x_3} = 0.5$$

$$\frac{-C_3}{C_4} = -\frac{0}{20} = -\infty$$

Conclusion: increase P_3 as MRS > price ratio.

Where the problem has many equations, the same principles apply. In effect, the $Z_j - C_j$ indicates the *marginal net return* as a result of making a marginal change to the current plan.

(Note that if the negatives are dropped from the MRS and the price ratio, a negative $Z_j - C_j$ indicates MRS < price ratio.)

11.11 The Assumptions of the LP Model

Introduction

Linear programming will determine the profit-maximizing farming system *provided* the real-world problem fits the attributes, or assumptions, of the linear model. These assumptions are listed below.

As with many models, however, it is doubtful whether many real-world problems comply perfectly with the LP model. When deciding to use an analytical technique an assessment must be made of the loss in potential profit from using an imperfect model compared to alternative analytical techniques. This assessment must consider the costs of solving the problem using the alternative planning models (e.g. for many farm consultancy jobs, budgeting may well be the optimal choice).

The assumptions

The objective function

(1) The farmer's objective
It is assumed that the farmer's objective is to maximize short-run profits (i.e. to maximize the total gross margin). If the farmer has another objective, and this can be quantified, this can be used in place of the monetary objective. However, any objective used must give rise to a function of the form:

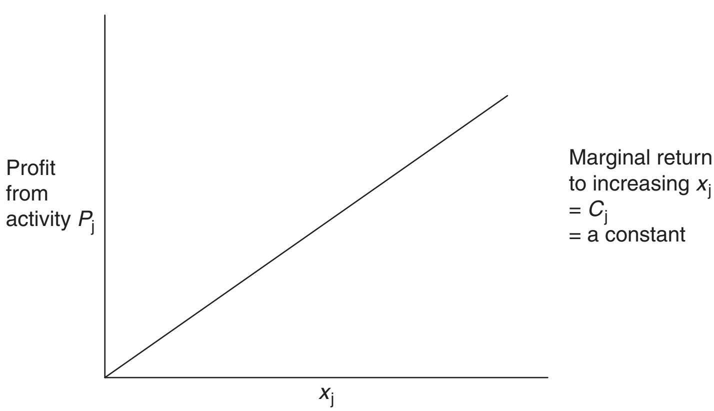
$$Z = \sum x_j C_j$$

In many problems a monetary objective is used with additional constraints being added to satisfy some of the farmer's other requirements. For example, for a farmer who enjoys producing lambs a constraint stating that a *minimum* number must be produced can be included; similarly for leisure time.

Many farmers are prepared to vary their resource structure so their decision problem has a long-term nature. To handle this situation asset buying and selling activities may need adding using advanced forms of LP to ensure only whole numbers appear in the solution (e.g. you cannot buy half a tractor). In addition, information on the resource MVPs provided by the solution indicates the changes which should be made. Also, repeated solutions could be determined for a range of problems, each with a different resource combination. Comparing the profit from each solution indicates the most profitable resource combination. This is the same technique used in gross margins analyses to indicate whether a change to resource structure would be profitable.
(2) The form of the objective function
Linear programming assumes that the objective function is linear. This implies there is:

(i) No interaction between the activities; and
(ii) The net revenues (C_j values) do not vary as x_j varies.
Consider (**i**) above. This means that the C_j of a particular activity must not vary no matter what other products are produced. In some cases this will be unrealistic. Thus, for example, it must be assumed that the C_j of a wheat crop would not change if the wheat crop follows a white clover crop compared with, say, another wheat crop. In some cases these problems can be overcome through, for example, combining activities in which these interactions occur to form an *independent activity*. Thus, an activity consisting of 1 ha of wheat and 1 ha of white clover might be defined as single entity.

Consider (**ii**) above. This assumption implies a number of things. Firstly, that the price received per unit of product does not vary as the output increases (i.e. a perfectly competitive market is assumed to exist). In general, this is realistic for primary production in the world market. Second, it implies that there will be no management efficiencies, or inefficiencies, as x_j varies (i.e. constant returns to size). Thus, for example, it assumes that the wheat yield does not vary as the number of hectares of wheat produced increases. In general, it assumes that any component of the C_j will not vary as x_j varies. Effectively, a linear profit equation is assumed.



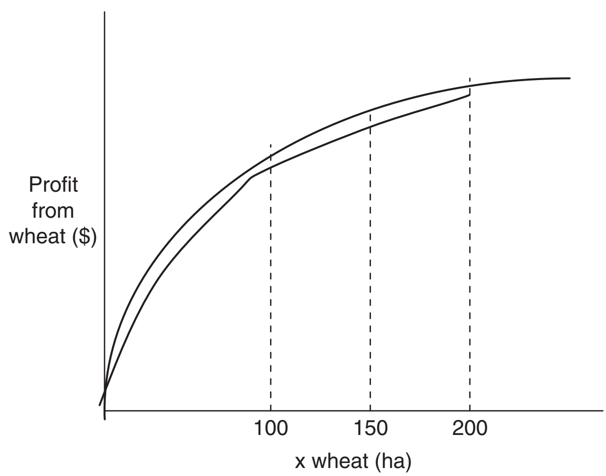
In some farming situations this assumption does not hold. Where, however, C_j decreases as x_j increases, this situation can be approximated by defining a number of different activities for the same activity. Each activity will be the same except for its C_j. These will decrease from activity to activity and each will be constrained to a maximum level commensurate with the range over which its particular C_j applies. For example, consider a wheat activity whose C_j declines as x_j increases.

This situation might be approximated using the following activities and constraints.

	$Cj \rightarrow$ \$150 Wheat 1	\$140 Wheat 2	\$130 Wheat 3	\$120 Wheat 4
Max. wheat 1 : 100 ≥	1	0	0	0
Max. wheat 2 : 150 ≥	0	1	0	0
Max. wheat 3 : 200 ≥	0	0	1	0
Land constraint 200 ≥	1	1	1	1
Labour constraint 4000 ≥	5	5	5	5
•				
•				
•				
etc. for the other constraints.				

(This simplex tableau-type table is used to represent the equation so that the values of *x* do not have to be written in.)

Thus, a curved relationship is being approximated with a series of linear segments:



Where the C_j *increases* as x_j increases the above linear segmentation technique cannot be used as the most profitable activity does not hold until all the other levels have been utilized. The LP system first selects the activity with the greatest $Z - C$, yet with increasing returns this activity should be selected last. The opposite occurs for decreasing returns and does not, therefore, violate the quantity application ranges.

The marginal rates of substitution

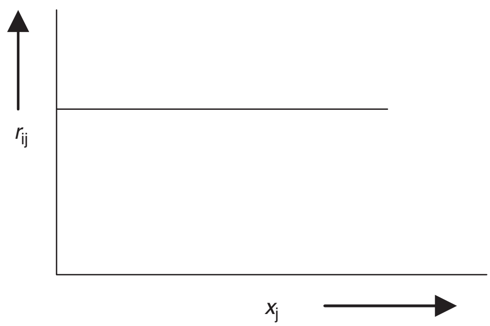
The LP model assumes that the marginal rates of substitution between products do not vary as:

(i) x_j varies; and
(ii) The combination of activities produced varies.

Recall that the basic input–output coefficients define the MRS between the products, so this assumption implies that the original r_{ij} coefficients do not vary as x_j varies and/or the combination of products produced (the farming system used) vary.

Consider (**i**) above. This assumes that, for example, the labour requirement per ewe does not vary as more ewes are produced, that the feed produced per hectare of lucerne does not vary as the hectares of lucerne is increased, and so on.

Graphically, this assumption implies:



In many cases this is probably reasonable, but not always. For example, it is likely that the per ewe labour requirement would decrease slightly as the number of ewes carried is increased (it does not take twice as long to shift a mob of sheep twice as big). Where the per unit requirement for a resource by an activity *increases* as x_j increases, this situation can be approximated using the linear segmentation approach; i.e. a number of activities would be defined, each one being identical except for the relevant r_{ij} which would increase. Each activity would be limited to a maximum with a constraint.

Consider (**ii**) above. This assumes that a particular r_{ij} will not change no matter what other activities are produced. The r_{ij} coefficients are assumed to be independent. For example, where a separate wheat activity is defined it is assumed its machinery requirement will not change no matter what crop precedes or follows it. This is clearly unrealistic in some cases. To overcome this problem those activities that are not independent must be combined.

Divisibility

Linear programming assumes that all products can be produced, all inputs purchased and all resources used *in fractional units*. For example, the optimal solution might indicate that:

(i) 6.7411 pedigree Friesian bulls should be sold;
(ii) 2.3459 tractors should be purchased and used;
(iii) 56.8431 ha should be in lucerne.

The importance of this assumption depends on the particular problem being solved. Some items can be purchased and sold in fractional units, e.g. fertilizer and seed; however, others cannot. Items that are physically small and have a low cost per unit in the case of an input, or a low return in the case of a product, can be rounded to the nearest whole number (nearest integer value) without a significant loss in *accuracy or feasibility*. For example, rounding an optimal solution to the nearest ewe will have little effect, similarly for the number of hectares of a crop.

For expensive items that do not come in fractional units, such as a tractor, a header, a labour unit, rounding to the nearest integer will most likely result in considerable inaccuracy. For LP problems where integer items can be bought and sold (i.e. the 'fixed cost' structure is being considered for possible change) the LP model will probably give an incorrect answer. It must be noted, however, that there are some special techniques available that overcome these problems.

Rounding may *give an infeasible solution* as with rounding down there may be insufficient 'facilities' available. For example, if the optimal solution indicates 3.34 labour units should be employed, rounding to 3 labour units will mean there will be insufficient labour. Further, the reason why rounding may give an *inaccurate solution* (one that is sub-optimal) is that there is no evidence to suggest that rounding to the nearest integer will give a plan with a greater profit compared with a plan having the variable rounded in the opposite direction. Thus, in the labour example above, rather than redevelop a plan having three labour units it may in fact be more profitable to use a four labour unit farming system.

As most farm problems involve products and inputs which can be rounded without a large loss in accuracy, the divisibility assumption is not extensively restrictive on the use of LP. Where it is, what is known as *integer programming* can be used. While this is computationally more demanding, modern high-speed computers can economically solve most problems. Where only a small number of variables need be integer, and the number of possible values is small (e.g. a choice between one or two tractors), it is possible to solve the problem firstly assuming one tractor, then assuming two, and finally comparing the returns of the two solutions relative to the tractor costs.

Certainty

Linear programming assumes that prices, costs, input–output coefficients, yields and resource levels are all known with certainty. Further, it assumes that the farmer's utility function is linear so that he will be indifferent to risk. Clearly, most of these assumptions are incorrect, with most variables being random variables. Conceptually, the shape of the various coefficient distributions should be taken into account when planning.

If a farmer's utility function is linear, his objective is to maximize expected profit. Thus, the normal assumption is that the farmer does have a linear utility function so expected values of the prices, costs and input–output coefficients can be used. In many cases this will give a reasonable answer, but the solution will not usually be optimal as the randomness of the variables has not been actively taken into account. (This problem was discussed for the use of expected values in budgets.) A plan that is developed using expected values may be *infeasible* in some years if a *poor combination* of the variables eventuates, and vice versa.

Some of the above problems can be overcome by adding special constraints in an attempt to recognize the fact that few farmers are indifferent to risk. For example, constraints can be added, limiting the area of risky crops, and conservative prices and yields used.

In general, this certainty assumption is the most limiting assumption of LP. It is a problem many methodological research workers have attempted to solve, with some progress. Students interested in following this through should search for articles on LP risk models. A technique known as multi-stage risk programming is relatively realistic, but very demanding on computer resources and analyst's abilities, but other less demanding methods can be used with reasonable success.

Finiteness

Linear programming assumes that there is only a finite number of different activities that need to be considered. This is an obvious assumption as there is a limit to the size of the problem that can be solved. Even with modern computers there is a limit to what can be handled in a reasonable time period (reasonable cost) and accuracy. With very large models accuracy can be a problem, as checking coefficients and assessing the logic of equations can be daunting.

The importance of this practical assumption depends on the particular problems. In normal farm system problems the limitation is more about the abilities of the analyst rather than the computer capacity. It is only where multi-stage risk models are attempted that computer power might be a problem and, as noted, the complexity of the model might be more of a problem for the analyst than the computing limits. The availability of detailed data might also be a problem.

Concluding comments

With imagination many integer, risk, multi-period (dynamic), multi-objective and non-linear problems can be adequately represented using a 'linear' programming model. To enable accuracy and completeness in these cases it is often necessary to use computer programs to generate the model as the largest desk will be too small to lay out the matrix of equations. And taking modelling even further, it is possible for computer programs to allow farmers to interact directly with LP models through having simple question and answer computer systems that obtain the farmer's data 'on screen', which is then used to automatically create an LP equation set. Once solved, the solution can then be sent to another computer program that interprets and converts the solution into farmer-based terminology, providing a report with directly useable optimal system suggestions.

In practical terms, the main task in LP analysis is the collection of the data and the creation of the equations to adequately reflect the decision problem. The next chapter contains methods to address these questions. The other problem an analyst faces is obtaining efficient and capable computer packages and learning how to use them. For simple problems most spreadsheets have an LP routine attached.

Further Reading

Students wishing for a more detailed and comprehensive explanation of the linear programming model than is provided here should refer to:

Pannell, D.J. (1997) *Introduction to Practical Linear Programming*. Wiley Interscience, New York.

12 Linear Programming – Using the Solution and Creating Realistic Farm Models

12.1 Introduction

The challenge faced by any analyst in the effective use of linear programming (LP) is interpreting the solution obtained and, prior to this stage, creating a set of equations that adequately reflect the reality of the farm-decision problem. This latter task not only involves understanding the nature of the problem, but also obtaining accurate data for all the production opportunities and management systems available. This chapter contains a discussion on the information contained in the solution tableau (matrix) and how it can be interpreted and, secondly, a series of sections covering equations representing components of typical decision problems. The chapter ends with a discussion on solving least cost problems (feed formulation and transport networks, in this case).

12.2 The Solution Tableau

Introduction

The final tableau of the simplex table (for details of the practical solving process that creates the solution tableau see Appendix 3 – this can also be obtained using the canonical form process described in the text) contains much more information than just the optimal values of the x_j and Z_j terms (the optimal farming system). This information can be invaluable in farm planning. This section contains a discussion on this information in what is called the *solution tableau*. Recall that this tableau has all the $Z_j - C_j$ equal to, or greater than, 0, and therefore represents the optimal solution.

The example solution tableau

The discussion will be based on the solution tableau of the example problem. Recall from the appendix describing the simplex method (Appendix 3) that this has the form:

	C_j	100 P_1	200 P_2	0 P_3	0 P_4	0 P_5
P_1	171.43	1	0	1.14	–0.29	0
P_2	28.57	0	1	–0.14	0.29	0
P_5	31.43	0	0	0.14	–0.29	1
$Z_j–C_j$	22,857	0	0	86	29	0

where P_1 = the wheat activity (defined in hectare units);
P_2 = the potato activity (defined in hectare units);
P_3 = the land disposal activity (defined in hectare units);
P_4 = the September labour disposal activity (defined in 1 h units);
P_5 = the maximum potatoes constraint disposal activity (defined in hectare units).

Profit maximizing plan:
x_1 = 171.43 ha of wheat;
x_2 = 28.57 ha of potatoes;
with 31.43 ha of non-used potato constraint (x_5).

12.3 Scarce Resources

As the disposal activities P_3 and P_4 are *not* in the basic solution (i.e. $x_3 = x_4 = 0$) the resources they represent are in scarce supply. That is, all the land and labour is completely used and is probably limiting production. If more land and/or labour was available profit would be increased.

Conversely, as the disposal activity P_5 is in the basic solution, the maximum potato constraint is not limiting production and, therefore, of no value. A lower constraint could be placed on the problem without affecting the optimal solution. If this constraint had been a tangible resource constraint (e.g. land or working capital) *some* of this resource could be sold off, or used elsewhere without affecting the optimal solution. Note carefully, however, that there is a limit to how much could be sold without reducing profit. Once the limit is reached the resource becomes scare and thus has a value.

12.4 The Value of Resources

If a resource is scarce it has value. The $Z_j - C_j$ in the final tableau of the non-basic resource disposal activities indicates this value. Consider, for example,

$Z_3 - C_3$. As $Z_3 - C_3 = \$86$, the *annual value* of 1 ha of land is \$86 (this is the *marginal value product* of land). The reason is the $Z_j - C_j$ for a disposal activity indicates the decrease in profit (as $Z_j - C_j$ is positive) if one unit of the disposal activity was introduced into the solution. In other words, if one unit of the resource was removed from production, profit would decrease by $Z_j - C_j$. Conversely, if an extra unit of the resource was made available, profit would increase by $Z_j - C_j$. Effectively, *the* $Z_j - C_j$ *for a non-basic resource disposal activity represents the marginal value product of that resource*.

If a farmer can purchase the resource at a price giving an *annual cost* of the resource (i.e. interest on capital, depreciation, rates, etc.) slightly less than the $Z_j - C_j$, additional units should be purchased. This will give additional profit as the MVP > marginal cost. However, there is a limit to how much should be purchased as the *MVP will decrease* as the resource level is increased and will eventually equal zero when there is enough resource available for the disposal activity to enter the basis. The availability of the other resources will determine when a resource's disposal activity will enter the basis, e.g. if *no* land was available, all labour would be in disposal. Thus, the MVP of resources depends on the ratio of resource availabilities. In the long run, the farmer should attempt to bring the resources into their correct ratio such that the MVPs are equated (and MVP with marginal cost).

In the example problem, the MVPs are:
Land = \$86/ha
September labour = \$29/h
'Maximum potatoes' = \$0/ha

This suggests the farmer should buy more land if the annual cost is less than \$86/ha and employ more labour in September if this costs less than \$29/h.

12.5 The $Z_j - C_j$ of Non-basic 'Real' Activities

Introduction

The example problem used has no 'real' activities that are non-basic; most realistic problems would (a 'real' activity is a production activity rather than a disposal activity).

The $Z_j - C_j$ of a general non-basic real activity can take on one of two general values:

(i) $Z_j - C_j = 0$
(ii) $Z_j - C_j > 0$

Case where $Z_j - C_j = 0$

This indicates there is more than one optimal solution. The particular activity with $Z_j - C_j = 0$ can be introduced into a new basic solution without a change in profit. Thus, if several of the non-basic P_j have $Z_j - C_j = 0$, there will be several optimal solutions. Each one will have the same total gross margin.

Graphically, this situation will occur if the price ratio line (iso-profit line) is parallel to a segment of the production possibility frontier. This is shown in Fig. 12.1.

Cases where $Z_j - C_j > 0$

The value of $Zj - Cj$ indicates:

1. The drop in profit per unit of production that would occur *if* a farmer insisted on producing the activity. The farmers' objective *may* mean this loss is acceptable. The solution tableau enables the extent of the profit loss to be evaluated *and thus* an appraisal of whether the gain in other objectives compensates for the profit loss; and

2. The change in the activity's C_j before it would pay to produce the activity. Given $Z_j - C_j = k$, if C_j should increase by k, then $Z_j - C_j$ would equal 0 and the activity could be introduced without profit decreasing. This has two implications:

(i) If there is some doubt about what is the true value of C_j, provided there is some surety that it lies within the range

$-\infty$ to $(Z_j + C_j) + C_j = Z_j$

there is confidence that the product should not be produced in an optimal system.

(ii) Assuming C_j is known with certainty, the optimal farming system *will remain optimal* (show stability) *even if* the C_j of the non-basic activity should vary between $-\infty$ to Z_j as a result of price changes. If it should go above Z_j, a *new* solution would be optimal.

There are, of course, many reasons why a C_j could vary and *each* must be considered when interpreting the discussion. Thus, if there is some doubt about, e.g. the yield/ha of wheat and/or expected price/kg, and/or the harvesting costs, and/or the weed control costs and so on, then the sum of these possible variations can be interpreted in the light of (**i**) above. Similarly,

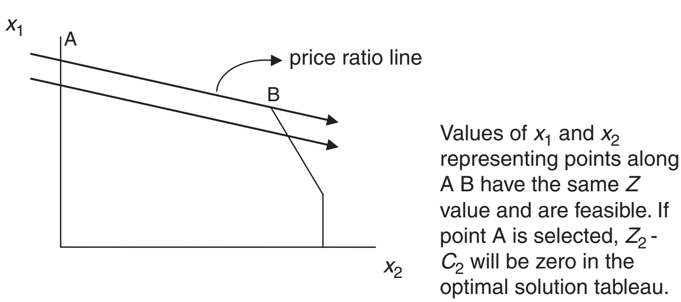


Fig. 12.1. Several optimal linear programming solutions occur where the price ratio line is parallel to the production possibility line.

if, e.g. new varieties and/or new weed control techniques meant that activity's C_j exceeded Z_j then a new plan would become optimal.

For example, assume that in the example problem there is a non-basic sheep activity. Let its solution tableau column be:

	Sheep (1 ewe) (C – \$6)
P_1	0.05
P_2	0.012
P_5	0.01
$Z_j - C_j$	1.4

Thus, if the net revenue of sheep were to increase by \$1.4, its $Z_j - C_j$ would be \$0 and the activity could be introduced into the solution without loss of profit.

If its C_j should increase above 6 + 1.4 (\$7.4), then its $Z_j - C_j$ would be negative and *it would be profitable* to farm a new optimal system with the sheep activity in the new basis.

12.6 Changes in the Basic Activity Net Revenues (C_j)

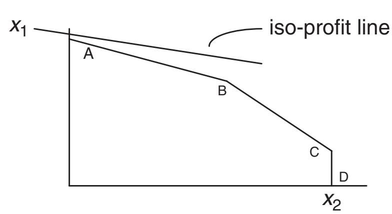
Introduction

The discussion shows non-basic activity net revenues (gross margins) can vary over a range without a solution becoming sub-optimal (a sub-optimal plan is one in which at least one $Z_j - C_j < 0$). This applies equally to the basic activities. These ranges can be used in the following way:

1. If there is uncertainty about the correct level of a C_j, the range will indicate whether this non-surety is important.
2. If a C_j changes then the range will indicate whether the optimal farming system should be changed (i.e. if the, say, price should change such that the C_j *is now* outside the stability range then the optimal plan will be different).

A graphic description of optimal plan stability

Assume a problem with a production possibility curve and iso-profit line as follows:



This problem has three constraints. The optimal solution A, consists of P_1 production with two resources in disposal. If the price of P_1 decreases, the iso-profit line will have a greater negative slope. However, this decrease can only occur up to a limit. Once the iso-profit line becomes parallel with AB then point B is also optimal. Note that in this example C_1 can increase to plus infinity without affecting the optimal solution. In the case shown in Fig. 12.2 there is an upper and lower limit to the values C_1 can take on without the solution becoming sub-optimal.

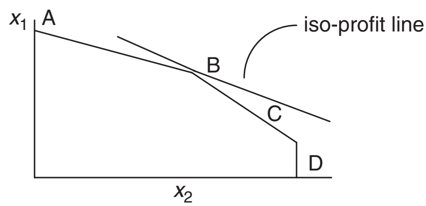


Fig. 12.2. Variation in the prices without changing the optimal point in linear programming.

Calculating the basic activity net revenue ranges

Consider a basic activity. The range over which its C_j can vary *without* making a non-basic activity's $Z_j - C_j < 0$ is calculated by the following.

1. Estimate the change that can take place without making *each* $Z_j - C_j < 0$. (The C_j of the basic activity may need to, say, fall more for one $Z_j - C_j < 0$ than for another.)
2. Select the change nearest above and nearest below the basic activity's C_j. This will ensure that *none* of the non-basic activity's will have a $Z_j - C_j < 0$ if the basic activity's current C_j should change by either of these figures.

To estimate the maximum change in a C_i (a basic activity's C_j) that can occur without making a particular $Z_j - C_j < 0$, calculate:

$$\Delta C_i = \frac{(Z_j - C_j)}{r_{ij}}$$ (*i* refers to the number given to the basic activity's row)

Thus, for example, if

$Z_j - C_j = 10$ and $r_{ij} = 2$

$\therefore\ \Delta C_i = -5$

That is, the net revenue of the basic activity can drop by 5 without giving a negative $Z_j - C_j$. Note that if the r_{ij} is negative, the basic activity's net revenue change necessary to give a $Z_j - C_j < 0$ would have to be positive.

The above calculation is repeated *for all* the non-basic activities. The smallest negative and positive ΔC_i defines the range. For example:

let $C_i = 50$ and let
$\Delta C_1 = -20$; $\Delta C_3 = 34$; $\Delta C_5 = 15$; $\Delta C_2 = 20$; $\Delta C_4 = -2$; $\Delta C_6 = 18$

$\therefore$ The range is:

50 – 2 to 50 + 15
i.e. 48 to 65

Consider the example problem, the ranges for C_1 and C_2 are:

(i) C_1

$$\text{for } P_3 \ \Delta C_1 = -\left(\frac{86}{1.14}\right) = -75.44$$

$$\text{for } P_4 \ \Delta C_1 = -\left(\frac{29}{-0.29}\right) = +100$$

Thus the C_1 range is 100 – 75 to 100 + 100 = 25 to 200.

(ii) C_2

$$\text{for } P_3 \ \Delta C_2 = -\left(\frac{86}{-0.14}\right) = +614.29$$

$$\text{for } P_4 \ \Delta C_2 = -\left(\frac{29}{0.29}\right) = -100$$

Thus the C_2 range is 200 – 100 to 200 + 614 = 100 to 814.

Normally, these calculations would need to be calculated for many more non-basic activities as real problems would have a larger number of such activities.

Virtually all LP computer packages automatically calculate the solution analysis values and ranges as part of their output in addition to the optimal values of all the basic activities. The full set of variable values gives rise to the whole-farm optimal system, though in some formulations, of course, the LP problem may represent only a sub-problem. This solution will only relate to the sub-problem and may need integrating into the whole. Such cases will only be optimal if there is no interrelationship with the rest of the farm, i.e. the sub-problem is independent.

12.7 LP Model Building – General Approach and Principles

When solving a farm decision problem using LP the biggest difficulty is the construction of a set of constraints and activities which, in total, adequately reflect the real-world problem. The other major problem is the estimation of the correct coefficients (prices, costs, yields etc.). The next sections are concerned with indicating how to set up LP models for two reasons: (i) the obvious reason is that if LP is to be used then a realistic model must be constructed (the example problem used so far has been far from realistic); and (ii) learning how to construct an LP model gives an understanding of the real structure of a farm problem and therefore makes for a more intelligent use of other planning tools such as a budgeting and gross margins analysis.

The approach is to consider a number of typical part-farm problems and develop a set of constraints or equations for each problem. That is, a whole-farm model will not be developed in each section. It is assumed that the reader recognizes that all the sub-models would have to be put together to form a whole model before solving. All aspects will not be covered. The approach will be to provide examples so that general methods will become evident.

The primary principle must, of course, be to develop a model that, as nearly as possible, reflects the real-world situation. As with budgeting and other planning techniques, experience is an important aspect of model building. Input–output coefficients and other data required are all seldom readily available in the literature and other publications so that 'considered estimates' must be used in many cases. Experience is invaluable here.

The general form a model should take will depend on the particular problem. Two basic situations can be recognized:

1. A model for a *particular* farm. The results of the study would be applied directly to the *particular farm only*.
2. A model for a *typical farm* in a defined locality. The results would be used for general extension work.

The *particular farm* model should take into account the farmer's objective, the currently available set of resources and so on.

The *general farm* model should use data typical of the conditions in the area so the solutions will provide a general idea of an optimal plan rather than a specific solution covering an action plan for the following year on a specific farm.

To adequately reflect a real-world situation the following general constraint types will usually need to be included:

- Resource supply constraints, e.g. land, labour, machinery hours etc. limits.
- Personal constraints. These are included to impose a more realistic objective than simply maximizing expected profit, e.g. a limit may be imposed on the amount of finance that can be borrowed for risk reasons; a minimum constraint may be imposed on some activity as a farmer enjoys producing it etc.
- Institutional constraints. These are necessary to take account of the limits and requirements placed on production resource use by various institutions, e.g. a local council may limit the amount of irrigation water that can be pumped from a stream, a lease may require that a certain amount of money is spent on development etc.
- Husbandry constraints. Where an input is produced by the farm, e.g. hay, for use on the farm the production and use of the input must be reconciled. Details of these constraints will be discussed later.

For a particular situation each type may not be necessary, though in most cases it is probable that there will be at least one of every type.

The example problem used suggested that only simple production activities could be used. In practice, as will become evident from the examples, many different types of activities may need to be included. For example, short-term finance borrowing can form an activity. Another example is an activity to represent the purchase of hay for stock feeding. In general, any ‘activity’ that a farmer can get involved in can be formulated as an activity in a farm LP model.

12.8 LP Model Building – Labour

Background

In general, the demand for labour by any farming system tends to fluctuate throughout a planning period. For this reason, to ensure that a farming system is developed which is feasible it must be ensured that there is sufficient labour in each sub-period of a planning period. Thus, rather than use a *single* equation to equate labour supply and demand (e.g. 3,000 man h $\geq r_1 x_1 + r_2 x_2 + \ldots + r_n x_n$), a number of equations must be used, one for each period as labour is a flow resource. i.e. it cannot be stored.

One problem is to decide on the number of different periods to use. Because labour services cannot be stored for later use if they are not used at a particular time, there must be sufficient available to carry out necessary jobs *in each period*. If it does not matter whether tasks are performed at the beginning, middle, or end of a period then the period can be considered independent. Using this principle, period lengths can be defined. The length of independent periods will probably vary with the time of year. Thus, in winter 2-month periods may be adequate whereas at harvest 2-week periods may be the largest independent units.

For example, in order to reflect the labour situation the sub-model may look like:

	Wheat activity	Turnip activity	Sheep activity	etc.
1st 2 weeks of harvest time: 160 ≥	0.25	0	0.05	
2nd 2 weeks of harvest time: 160 ≥	0.1	0.08	0.01	
Following month, etc.: 320 ≥	0.04	0.1	0.08	

All labour is in man-h units

That is, where x_1 = hectares of wheat, x_2 = hectares of turnips and x_3 = number of ewes:

$$160 \geq 0.25x_1 + 0x_2 + 0.05x_3$$
$$160 \geq 0.1x_1 + 0.08x_2 + 0.01x_3$$
$$320 \geq 0.04x_1 + 0.1x_2 + 0.08x_3$$

etc. for other periods.

Rather than write the equations out in full when giving example sub-models, the initial simplex tableau (matrix) form will be used. Thus, the heading on each column refers to a particular variable or activity and each row of the table represents a particular equation. To save space the units being used will not always be given. In general, this will be obvious.

Overhead jobs

On most farms, given a set of fixed resources, there will be a set of jobs that will have to be carried out no matter what farming system is developed, e.g. building and fence maintenance, visiting the bank manager etc. These requirements must be deducted from the man hours available for each period before the problem is solved. Thus, in the above example, if 30 h has to be set aside for such jobs in ‘the following month’ the available supply becomes 290 man h.

Hiring and selling labour

Where there are opportunities to hire casual labour (for considering permanent labour, due to the divisibility assumption, an integer routine is required) an activity representing the hiring of 1 h (or any unit) of casual labour can be included in the model. There may be one such activity for each labour period. Similarly, where there are opportunities for working off the farm, there is a need to include an activity representing the sale of labour. Thus, for one of the labour constraints the sub-model might be:

	$+C_j$	$-C_j$	$+C_j$
	Wheat ….	Hire 1 h of casual labour	Sell 1 h of casual labour
February labour 290 ≥	04+ (other activities)	….. –1	+1 etc

Notice that when an activity *supplies* a resource it has a negative r_{ij} (if it demands, the r_{ij} is positive). If the equation is rearranged all the supplying components can be made positive and put on the left-hand side of the equation, while all the demanding components can be made positive and put on the right-hand side.

Overtime

The farmer may put a limit on how much time he is prepared to put into the farm unless additional hours of work return at least a certain sum. Thus, an overtime activity can be defined that has a negative C_j.

For example:

	$+C_j$ Wheat, etc. activity	$-C_j$ Overtime labour activity	etc.
November labour 160 ≥	r_{ij}	–1	
Max. overtime labour 40 ≥		1.0	
etc.			

Thus, if the return on working longer hours than normal is greater than the $-C_j$ assigned to the activity, then the activity will be in the optimal basis. The $-C_j$ should reflect the value placed on 1 h of labour by the manager. Similarly, activities to represent overtime by employed staff could be included.

Flexibility of labour use

In some cases certain tasks can be performed equally as well at different times of the year. To obtain information on the optimal time they can be carried out, a number of activities can be defined, each with the jobs carried out at a different time. Each would be identical except for the labour coefficients. The optimal solution would then indicate when the jobs should be carried out.

Similarly, where jobs can be carried out at different times, but only at a cost in terms of a lower gross margin (e.g. lower yield), then a series of activities can be defined, each one with different labour coefficients. The LP routine will select the optimal activity. It could well be that certain jobs should be delayed after their optimal time of completion due to the demand (and therefore value) for the labour by other activities.

Data estimation

It is often difficult to obtain detailed data on labour demand. In these cases simple summary constraints can be used as an approximation. Examples include limiting the number of stock, crop hectares and the like, to a defined maximum. For example, on a mixed dairy–crop farm, assume that it was estimated that two men could

(i) Run 150 cows, or

(ii) Crop 500 ha.

Thus, define a resource 'labour', a unit of which represents the labour required by one cow. The constraint could be:

	Cow activity	Crop 1 (1 ha)	Crop 2 (1 ha)	etc.
150 ≥	1.0	0.3	0.3	

It must also be remembered that if labour is unlikely to be limiting in all but one or two periods then these are the only periods that need to be considered.

12.9 LP Model Building – Reconciliation Rows – General

As noted earlier there may be a need to reconcile the production and use of a production factor. Where the b_i (constant) of an equation is equal to 0, the equation or row is referred to as a reconciliation row. Such an equation will have both negative (supply) and positive (demand) coefficients. Thus, the sum of the negatives must be at least as great as the positives, i.e.

$$0 \geq \left(\sum -r_{ij}x_j\right) + \left(\sum +r_{ij}x_j\right)$$

$$\therefore \left(\sum r_{ij}x_j\right) \geq \left(\sum r_{ij}x_j\right)$$

For example, on a pig farm it would be necessary to ensure that the use of piglets equalled the supply. Thus, the following sub-model might be used:

	$-Cj$ sow activity	$+Cj$ light wt. Pork prodn. (1 pig)	$+Cj$ heavy wt. Pork prodn. (1 pig)	etc.
Piglet reconciliation, etc.	$0 \geq -16$	+1	+1	

The sow activity produces 16 piglets/year from two farrowings. These piglets can be used by the various fattening activities.

12.10 LP Model Building – Selling, Buying and Intermediate Activities

Reconciliation equations can be used in many different situations. A number of these will be discussed in later examples as well as in this section.

Where a number of activities produce the same product, or use the same input, and the analyser wants to vary the price of the product, or input, it is convenient to have an activity to represent the sale, or purchase, of the product or input. For example, a buy fertilizer activity and a sell wheat activity may be defined for a cropping farm model. Thus:

	$+C_j$ Sell Wheat (100 kg)	$-C_j$ Buy fert. (50 kg)	$-C_j$ Wheat 1 (1 ha)	$-C_j$ Wheat 2 (1 ha)	$-C_j$ Wheat 3 (1 ha)	etc.
Wheat reconciliation $0 \geq$	1		–18	–20	–22	
Fertilizer reconciliation $0 \geq$		–1	2	2.4	2.8	
etc.						

The C_j on the wheat activities would represent the gross margins without the gross revenue and the fertilizer costs. The sell and buy activities would account for these components of the gross margins (in the extreme case there could be a buy activity for all inputs; e.g. fuel, seed, herbicide, etc.). With the buying and selling activities isolated it is a simple matter to change the return/kg of wheat and the cost of fertilizer before resolving.

Activities that do not produce a product for sale, such as the fertilizer activity, are referred to as *intermediate activities*. Another example would be a hay-buying activity.

12.11 LP Model Building – Reconciling Feed Supply and Demand

As with most resources, feed supply must be equated with feed demand in *each* period if a farming system is to be feasible. The principle must be to construct a number of periods such that the feed production and demand is relatively constant throughout each period.

Activities that supply feed to each period and similarly those that demand feed must all be defined. For example, the following equations might be used, where the feed is defined in terms of stock equivalents:

	$+C_j$ Sheep (1 ewe)	$+C_j$ Cattle (1 cow)	$-C_j$ Pasture (1 ha)	$-C_j$ Buy hay (3 bales)	$-C_j$ G.F. maize (1 ha)	etc.
Spring feed 0 ≥	2.0	8.0	–8			
Summer feed 0 ≥	1.0	6.0	–12		–20	
Autumn feed 0 ≥	1.0	6.0	–6			
Winter feed 0 ≥	1.5	7.0	–2	–1		

In the extreme case, monthly periods could be used and metabolizable energy and digestible protein figures could be used rather than stock units.

12.12 LP Model Building – Transfer Activities

There is sometimes a need to 'transfer' a resource from one equation to another (e.g. between time periods). These transfer activities can be used to represent many different situations. Two examples are the following:

1. Haymaking

Haymaking (or silage) and the use of the hay for feeding in a period other than the one in which it was made is a transfer system. Feed is used by the haymaking process and bales of hay are produced. These are held and used in another period. Thus, a 'haymaking and using' activity is a transfer and may take the following form:

	$+C_j$ Sheep	$+C_j$ Pasture	$-C_j$ Make hay (1000 kg)	etc.
Summer feed 0 ≥	$+r_{ij}$	$-r_{ij}$	+3	
Winter feed 0 ≥	$+r_{ij}$	$-r_{ij}$	–2	
Etc.				

The C_j on the haymaking activity represents the cost of making 1000 kg and feeding it out in the winter. Note that there is a loss of feed due to the inefficiencies of haymaking (1/3 loss).

2. High and low quality feed

On many farms there will be stock requiring a production ration and others only a maintenance ration. Thus, two classes of feed need to be distinguished: low and high quality feed. However, while the producing stock cannot use the low quality feed, other stock can be maintained on the high quality feed. Thus, two constraints can be formulated in each period. One for high quality and one for low quality feed. A transfer activity can then be used to transfer any surplus high quality feed to the low quality feed so it can be used by the low quality feed-requiring animals. For example:

	0	$-C_j$	$+C_j$	$-C_j$	$+C_j$	$+C_j$
	Transfer	Lucerne (1 ha)	Wheat (1 ha)	Ryegrass Straw (1000 kg)	Milking cow	Dry cow
High Quality Feed	$0 \geq +1$	$-r_{ij}$			$+r_{ij}$	
Low Quality Feed	$0 \geq -1$		$-r_{ij}$	$-r_{ij}$		$+r_{ij}$

Thus, all feeds can be used to support the dry cow but only high quality feed can be used for milking cows.

12.13 LP Model Building – Reconciling Stock Systems

In all the examples stock activities have been represented by simple ‘sheep’ or ‘cattle’ activities. In reality there are many stock activities that might be included. The object is, of course, to develop an optimal stock and feeding system, so it is important to provide as many alternative activities as possible. As in budgeting work, the stock activities must be reconciled. In general:

- Various classes of stock can be used;
- They can be replaced in a number of different ways;
- Progeny can be sold at different ages and weights;
- Progeny can be produced at different times of the year.

An activity should be included to represent each of the possibilities.

As an example, consider part of the stock sub-model for a lamb property.

Note the variation in feed requirements for different lambing times. Only one feed equation is given for simplicity. Many other activities could be included in the overleaf sub-model. For example, wethers (castrated males) might be kept for wool production, ewe lambs could be reared and sold as ewe lambs, or perhaps as hoggets (3/4 year olds) or perhaps as two tooths etc.

P_1	P_2	P_3	P_4	P_5	P_6	P_7	P_8
Ewes Lamb July (1 ewe)	Ewes Lamb Aug (1 ewe)	Rear Replacement Stock (1 2th)	Buy Ewe lambs (1 lamb)	Buy 2ths+ (1 2th)	Sell Light wether lamb (1 lamb)	Sell Heavy wether lamb (1 lamb)	Sell Ewe lambs (1 lamb)

		P_1	P_2	P_3	P_4	P_5	P_6	P_7	P_8
		$-C_j$	$-C_j$	$-C_j$	$-C_j$	$-C_j$	$+C_j$	$+C_j$	$+C_j$
Wether lambs	0 ≥	– 0.5	– 0.51				1	1	
Ewe lambs	0 ≥	– 0.5	– 0.51	1					1
Replacement Stock	0 ≥	+ 0.2	+ 0.2	– 0.9*	– 0.95	– 1.0			
Feed	0 ≥	1.2	1.0	0.8	0.8			0.2	

+The terminology '2th' refers to two tooths (1½ year olds). Similarly 4th and 6th in later examples.

*The 'rear replacement' activity demands one ewe lamb but provides only 0.9 replacements. The difference is due to deaths between the ewe lamb and 2th stages.

12.14 LP Model Building – an Optimal Stock-Replacement System

In developing an optimal stocking system thought must be given to the age at which stock should be replaced. For example, as ewes get older their wool production declines, their death rate increases but also their fertility increases. Given the price of the products and also feed and stock replacement costs there will be an optimal replacement age. A sub-model that will help answer this problem will contain a number of activities, one for each age, so that an optimal system will be selected. Thus:

		Buy 2ths (1 ewe)	2th ewes (1 ewe)	4th ewes (1 ewe)	6th ewes etc. (1 ewe)	Sell 'old' 2th ewes (1 ewe)	Sell 'old' 4th ewes (1 ewe)	etc.
2th reconciliation	0 ≥	–1	1					
4th reconciliation	0 ≥		– 0.95	1		1		
6th reconciliation	0 ≥			– 0.93	1		1	
etc.								
Feed	0 ≥		$+r_{ij}$	$+r_{ij}$	$+r_{ij}$			

The C_j figures would be different for each activity. The increasing death rates are reflected in the decreasing numbers that are transferred from stage to stage. The optimal solution will indicate how many animals of each class to run.

12.15 LP Model Building – Working Capital

Any farming system must be organized so the working capital demand, say, in every month can be satisfied from income, working capital on hand or from borrowing. Furthermore, funds earned throughout the year help meet the demand.

To adequately represent the working capital situation a month-by-month (or some other period) cash profile must be determined for each activity. For example, a wheat activity (1 ha) might have the following *net* demand or supply in each month (monthly periods are common as this is the usual period over which businesses require payments to be reconciled).

May	+ 5	November	+ 1
June	+ 20	December	+ 1
July	+ 1	January	+ 30
August	+ 5	February	+ 5
September	+ 1	March	+ 1
October	+ 1	April	– $192

Remember that a positive sign represents a demand and a negative indicates the activity supplies. Given such a profile for each activity, to ensure feasibility the following sub-model would be necessary:

		$+C_j$	$+C_j$	$-C_j$	$+C_j$	0	0	0	
		Wheat	Barley	Pasture	Sheep	Transfer July surplus	Transfer August surplus	Transfer September surplus	etc.
July	20,000 ≥	r_{ij}	r_{ij}	r_{ij}	r_{ij}	+ 1			
August	0 ≥	r_{ij}	r_{ij}	r_{ij}	r_{ij}	– 1	+ 1		
September	0 ≥	r_{ij}	r_{ij}	r_{ij}	r_{ij}		– 1	+ 1	
October	0 ≥	r_{ij}	r_{ij}	r_{ij}	r_{ij}			– 1	
etc.									

The \$20,000 left-hand side of the July equation represents the initial working capital. This must be sufficient to meet the net demands of July (some of the r_{ij} may be negative so that at the end of the month the total supply may in fact be \$20,000 plus the month's surplus). Any surplus funds are transferred so that they become available for use in August. There is one row for each month so that the above sub-model will ensure sufficient working capital is available in every month.

Further, borrowing activities could also be included, one for each month. Each would assume that the funds are paid back at the end of the month. If this is not possible further funds could be borrowed in the following month to meet the payback of the previous month's borrowing. The C_j would reflect the monthly interest rate. There should also, probably, be constraints imposed on borrowing to limit it to a defined maximum.

For example, consider the following sub-model:

		$+C_j$	0	0	$-C_j$	$-Cj$	.
		Wheat etc.	Transfer	Transfer	Borrow (\$1) July	Borrow (\$1) August	etc.
July	20,000	r_{ij}	+ 1		– 1		
August	0 ≥	r_{ij}	– 1	+ 1	+ 1	– 1	
September	0 ≥	r_{ij}		– 1		+ 1	
October	0 ≥	r_{ij}					
etc.	0 ≥						
Max. July borrowing etc.	12,000 ≥				+ 1		

Note that to complete the model, equations must be added to ensure that sufficient living expenses are available.

12.16 LP Model Building – Soil Types

If a farm consists of several different soil types these must be treated as separate resources. For each product there must be several activities defined, one for each soil type. They will have different C_j values and possibly different r_{ij} values. For example, consider a farm with two different soil types. Part of the LP model might be:

		$+C_j$ Wheat 1 (1 ha)	$+C_j$ Wheat 2 (1 ha)	$-C_j$ Lucerne 1 (1 ha)	$+C_j$ Lucerne 2 (1 ha)	etc.
Land 1	150 ≥	1.0		1.0		
Land 2	200 ≥		1.0		1.0	
Feed	0 ≥			–12	–10	
etc.	0 ≥					

The C_j values would vary to reflect the different yields, different cultivation costs etc.

12.17 LP Model Building – Timing of Land Use

Under some situations, particularly given new winter-growing green feed varieties and cases where horticultural crops are feasible, it may be possible to occupy 1 ha of land with more than one crop per year and improve efficiency. To represent this situation in a LP model the land resource constraint must be represented by a number of equations. Each represents the land resource constraint for, e.g. a 3-monthly period. The activities are constructed so the r_{ij} values represent demand for land at each particular time.

For example, consider the following sub-models:

	$+C_j$ Wheat (Autumn)	$+C_j$ Wheat (Spring)	$+C_j$ Barley	$-C_j$ Turnips 1 (late)	$-C_j$ Turnips 2 (early)	$-C_j$ G.F. maize for silage
Land – Winter 200 ≥	1			1	1	
Land – Spring 200 ≥	1	1	1			
Land – Summer 200 ≥	1	1	1		1	1
					1	
Land – Autumn 200 ≥				1		
Feed ≥	$-r_{ij}$		$-r_{ij}$	$-r_{ij}$	$-r_{ij}$	

Note that each product might be represented by many different sub-activities. Each sub-activity may use the land for different lengths of time and have different C_j values as the yields and other coefficients will be different. The sub-activity with the greatest C_j will not necessarily be selected, as it may use land inefficiently.

12.18 LP Model Building – Independency and Rotations

LP assumes an activity's C_j and r_{ij} coefficients are constant no matter what combination of other activities are produced. If this assumption is violated, the relevant activities can be combined to form an independent activity. A typical example is in crop rotations. One case is where a wheat activity is assumed to be planted after peas (a legume) as compared with, say, after wheat. The coefficients should be different as the soil fertility will be affected by whether peas or wheat is the preceding crop.

To overcome these problems crop and pasture activities *can be combined* to form a 'rotational' activity with a certain net revenue and r_{ij} coefficients. These coefficients are combinations of the individual activity coefficients. The method of combining the activities will affect the 'combined' activity's coefficient values (other, but more complicated, approaches are also available).

For example, consider the activities 'wheat' and 'specialist white clover' (a very productive legume enhancing soil fertility). The way in which they are combined will affect their C_j and r_{ij} coefficients. Two possible methods (rotation) of using the activities are:

1. Wheat – wheat – white clover – wheat – wheat and so on with the pattern repeating.
2. Wheat – white cover – wheat – white clover and so on with the pattern repeating.

To represent these possibilities in an LP model an activity for each pattern is defined. The coefficients for each component crop, given the particular rotational system, must be estimated.

For example, assume the following coefficient values:

(i) Wheat ex white clover	C_j	= \$120/ha
	Winter feed	= 3 SU/ha
(ii) Wheat ex wheat	C_j	= \$100/ha
	Winter feed	= 2 SU/ha
(iii) White clover ex one wheat crop	C_j	= \$80/ha
	Summer feed	= 8 SU/ha
	Winter feed	= 4 SU/ha
(iv) White clover ex two wheat crops	C_j	= \$100/ha
	Summer feed	= 7 SU
	Winter feed	= 3 SU

SU = stock unit

The two rotational activities would then be defined as:

Rotation **1**: As this is a 3-year rotation before it repeats itself, define a unit of this activity as consisting of:

0.33 ha wheat ex white clover
0.33 ha wheat ex wheat
0.33 ha white clover ex two wheat crops

Thus, the coefficients of one unit of this activity are determined by taking the individual activity coefficients and multiplying by the proportion in which they occur and summing:

$C_j = (0.33 \times 120) + (0.33 \times 100) + (0.33 \times 100) = \$105.6$
Winter feed,
$r_{ij} = (0.33 \times 3) + (0.33 \times 2) + (0.33 \times 3) = 2.64$
Summer feed,
$r_{ij} = (0.33 \times 0) + (0.33 \times 0) + (0.33 \times 7) = 2.31$

Rotation **2**: Define one unit of this activity as consisting of:

0.5 ha wheat ex white clover
0.5 ha white clover ex wheat

The coefficients of the activity are:

$C_j = (0.5 \times 120) + (0.5 \times 80) = \$100$
Winter feed,
$r_{ij} = (0.5 \times 3) + (0.5 \times 4) = 3.5$
Summer feed,
$r_{ij} = (0.5 \times 0) + (0.5 \times 8) = 4.0$

Using rotational activities overcomes the need to have constraints ensuring general husbandry requirements are met. For example, rotations could be defined so they all have at least a certain proportion of pasture and lucerne to satisfy soil fertility requirements. Otherwise additional constraints would be necessary to force the correct proportions into the solution (this could be done by using a reconciliation equation with soil fertility-enhancing activities supplying 'fertility' to demanding activities such as wheat. This forces at least a minimum of enhancing activities if demanding activities are selected).

In most realistic situations many different rotational activities would be defined. Some of these might be solely feed-producing activities while others would be mixed rotations depending on the type of farm.

Having solved an LP problem, it is a simple matter to interpret a rotational activity in the optimal solution. For example, assume the solution indicates 50 ha of Rotation 2 defined above. As this activity is 0.5 ha of wheat and 0.5 ha of white clover, this infers that in any 1 year $0.5 \times 50 = 25$ ha of wheat ex white clover should be produced, and $0.5 \times 50 = 25$ ha of white clover ex wheat be produced.

12.19 LP Model Building – Complexity and the Real World

Many more examples of how to construct LP models could be given. Those presented generally indicate the possibilities. A real-world situation can produce a complex LP model containing many activities and constraints. Farm models having, say, at least 300 activities and 100 constraints are common. However, it must be emphasized that an LP model should *not be* *made more complex* than it need be. Farm resources unlikely to be limiting should be examined carefully before inclusion. Detailed labour and working capital constraints add considerable size to the model and therefore should not be included unless it is clear that they are necessary. Even where necessary, simple labour constraints, such as a limit on total stock numbers, may be sufficient.

To give some idea of what an entire model might consist of, even if rather restricted, a simplified model of a lamb farm is given in Appendix 4. In this model labour and working capital constraints have not been included under the assumption that they will not be limiting.

12.20 LP Model Building – Validation

Once a model has been developed it must be validated or tested to ensure it is a realistic model of the farm. This is always a challenging task. After an initial solution the opinion of various experts (consultants, advisors, researchers who are familiar with the area and type of farming) on whether the system is feasible can be sought. Similarly, the optimal solution can be presented to farmers, asking if they believe it is a practical solution, and if not, the reasons for its failures leading to an improved model. In the case of models of an actual farm the LP can be forced to replicate the current system (e.g. place a minimum lamb constraint on the problem to ensure currently produced numbers are replicated in the model) and check whether the same output and profit levels are produced.

12.21 The Application of LP – General

LP was developed not long after the Second World War as a result of, in part, efforts to efficiently route shipping. Since then it has been used to solve many different problems though they are all basically maximization or minimization problems. The use of LP to solve farm problems is one of the uses which first appeared around the 1960s. In order to indicate the wide applicability of the technique, and to make the discussion of LP more complete, some examples of other uses of LP are presented. Before doing this, however, some comments will be made regarding the practical use of LP in farm work.

The previous discussions have shown that LP has considerable application in farm planning (with the biggest problem being the certainty assumption). Its practical use depends on:

1. The degree of complexity; and the existence of considerable choice. Some farms may only have a limited range of possible alternative systems, and these may be quite simple. Thus, simple, less expensive, planning techniques may provide adequate results.
2. Planning costs. LP work requires access to suitable software and to analysts with LP experience who might be expensive to hire. Thus, unless LP can derive farming systems with a considerable improvement in profit compared

with techniques such as gross margins analysis, it should not be used. However, in general research work, the benefits frequently outweigh any costs. For example, developing systems which minimize greenhouse gas output may well lead to considerable worldwide benefits.

LP can be of use on individual farms where the farm is large so potential gains to good planning are high, and where there is considerable choice and the answers to the choice are not obvious. It is also, of course, useful in general advisory and research work.

Given a relatively large area of similar soils and climate, the results of an LP study on a representative farm can have considerable advisory use, particularly where the analyst uses a basic model to solve many variations of the basic resource situation, and experiments with many price, technical and cost parameter values. LP studies might be carried out to indicate the general nature of optimal plans rather than providing a blueprint of operations. They will then need interpolation for each individual situation. In carrying out LP studies of this sort it is important to estimate *as many near optimal plans as possible* rather than just the unique optimal system. This allows farmers (assisted by advisers) to select a plan that best suits their particular objective. (To determine near optimal solutions, non-basic activities with $Z_j - C_j$ figures approaching zero can be introduced into new basic solutions through adding minimum constraints and resolving, or by increasing the activity's C_j.)

Solutions obtained for a range of different prices and costs explore the effect of variations on optimal systems. Similarly, solutions for a range of resource endowments are important to enable applying the results to different farms (as noted above). Similarly, solutions for different input–output coefficients may be necessary to allow for different levels of managerial ability.

In summary, rather than obtain a single solution, considerable experimentation should always be carried out for a range of parameter values. Each solution should also be carefully analysed using the methods outlined under the solution analysis section.

One of the criticisms levelled at the use of LP is that farmers have difficulty in understanding it. This is, however, a non-criticism in the sense that the use of LP need not rely on the farmer's knowledge, for the interpretation should always be through an adviser/consultant. Some farmers, however, will undoubtedly have the skills necessary. Computerized report writers also help convert the results to a level understood by farmers who are primarily interested in a recommended farming system. This will include the land utilization, stock numbers for the different classes and details of the systems to use, e.g. the best method of supplying feed, least-cost mixes, harvesting machinery levels etc.

12.22 The Application of LP – Determining Least-Cost Feed Mixes

A firm producing, e.g., a pig (hog) fattening feed must produce a feed that has a given nutrient content for as small a cost as possible (the firm could be a farmer producing his own pig feed). To produce the feed, the firm can mix together various source materials such as wheat meal, barley meal, pea meal, soybean meal and so on. The objective of the firm is to determine the quantities of the source materials to mix so the total cost of the materials is at a minimum. Linear programming can be used to solve this problem.

Consider a simple example problem. Assume a feed firm requires a feed which has *at least* 16% crude protein and *at least* 1.4 megacals (or its equivalent in megajoules) of metabolizable energy. Assume the firm can mix together wheat meal, lucerne meal and soy meal, and that they contain:

	Crude protein %	Mcals/kg
Wheat meal	12.7	1.6
Lucerne meal	18.0	0.8
Soy meal	49.7	1.3

Letting x_1 = kg of wheat meal in the mix,
x_2 = kg of lucerne meal in the mix,
x_3 = kg of soy meal in the mix,
the problem can be stated as one of finding values for x_1, x_2 and x_3 such that the cost is minimized subject to satisfying:
crude protein constraint $16 \leq 12.7x_1 + 18.0x_2 + 49.7x_3$
digestible energy constraint $1.4 \leq 1.6x_1 + 0.8x_2 + 1.3x_3$
and, if the costs per kg of the material are:
wheat meal 0.28 cents,
lucerne meal 0.33 cents,
soy meal 0.42 cents,
the objective function is:

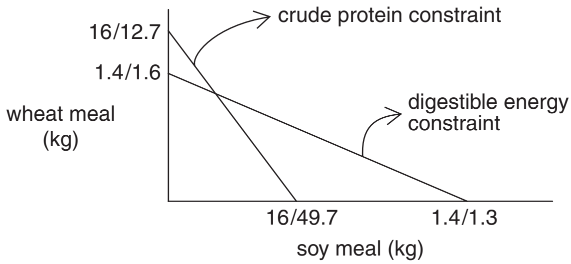
$$\text{COST} = 0.28x_1 + 0.33x_2 + 0.42x_3$$

This problem obviously has the same form as a general LP problem. That is, find variable values so a linear objective function is minimized subject to satisfying a set of simultaneous linear equations. This problem is slightly different from the farm maximization problem in that:

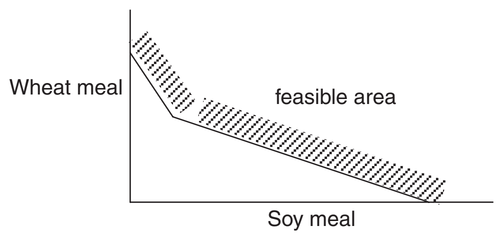
- The objective is to be minimized. Therefore select the activity to enter a new basic solution on the basis of the activity with the most *positive* $Z_j - C_j$ value. This procedure will minimize Z.
- The constraints have the inequalities the other way round compared with the maximization problem. That is, minimum rather than maximum requirements are placed on the problem. To convert such inequalities to equalities the disposal activities take a slightly different form.

Given these adjustments, the problem is solved in the same way as the maximization problem.

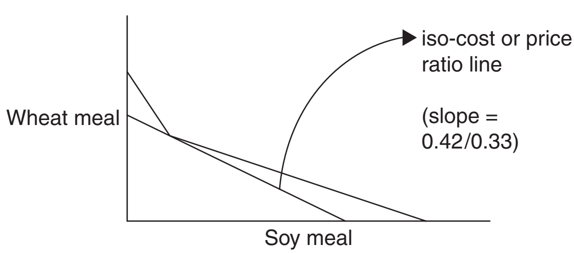
In order to further compare with the maximization problem, consider the graphical presentation of the problem. First, draw a graph of the feasible variable value combinations that satisfy the constraints. Consider only the wheat meal and soy meal activities as only two axes can be drawn. Each constraint divides the graph into two areas: the *area above* the constraint line and the area below. As the constraints are *minimum*, rather than maximum, the area above the constraints lines is feasible. Thus:



The area which satisfies both constraints (provides *at least* the required level of both nutrients) is:



To find the optimal point, an iso-cost line (price ratio line) must be superimposed on the graph. The point at which the price-ratio line has a point in common with the feasible area and *is as close to the origin* (as we wish to *minimize* the cost) as possible is the optimal point. Thus:



Note the similarity between this problem and the factor–factor problem of production economics.

To solve minimization problems the 'disposal' activities become 'surplus nutrient' activities and represent the amount of nutrient in a mix in excess of the minimum. In this case the coefficient on the surplus nutrient variable must be −1 as it records the difference between the minimum and actual content. If, for example, the crude protein content was greater than 16%, e.g. 20%, the value of the 'disposal' activity would be 4. Thus, in the equality created from the inequality

$$16 = 20 - 4.$$

As a starting solution is difficult to find using these negatives, a further set of variables are added with +1 coefficients, one for each equation, to form the identity matrix. As these are just a device to form a starting solution in the iterative process, they are given an infinitely high C_j so the first few iterations drives them all from the solution. A good computer package will achieve this requirement automatically. All the user must do is define the problem as being a minimization LP.

Note that in the formulation of this example problem no requirement was placed on the weight of the mix, just requirements on the nutrient content. Thus, an additional equation should be added:

$$1 = x_1 + x_2 + x_3$$

The solution will now have ingredients making up exactly 1 kg.

12.23 The Application of LP – Determining Least-Cost Transport Routes

Another very important use of LP is determining the least cost method of transporting a good, or product, from a number of origin points to a number of destination points.

The best way to see how LP can be used for this problem is to consider a simple example. Consider a fertilizer manufacturer that has two fertilizer-producing plants and has three selling depots. The manufacturer wants to know which plant should supply which depot in order to minimize transport costs. While this example is somewhat trivial, it will provide an understanding of the problem.

Assume that each plant produces the following quantities of fertilizer:

Plant 1 20,000 t
Plant 2 10,000 t

Assume that each depot requires the following quantities:

Depot 1 5,000 t
Depot 2 18,000 t
Depot 3 7,000 t

Assume that the costs of transporting 1 t from the plants to the depots are:

Plant 1 to Depot 1 C_1
Plant 1 to Depot 2 C_2
Plant 1 to Depot 3 C_3
Plant 2 to Depot 1 C_4
Plant 2 to Depot 2 C_5
Plant 2 to Depot 3 C_6

Thus, if x_1 = the number of tonnes transported from Plant 1 to Depot 1

x_2 = the number of tonnes transported from Plant 1 to Depot 2
x_3 = the number of tonnes transported from Plant 1 to Depot 3
x_4 = the number of tonnes transported from Plant 2 to Depot 1
x_5 = the number of tonnes transported from Plant 2 to Depot 2
x_6 = the number of tonnes transported from Plant 2 to Depot 3

then the total transport cost of a routing system is given by:

$$\text{Cost} = x_1\, C_1 + x_2\, C_2 + x_3\, C_3 + x_4\, C_4 + x_5\, C_5 + x_6\, C_6$$

Thus a linear objective function has been formulated.
Now consider the constraints that a solution to the problem must satisfy. First, the amount of fertilizer transported from Plant 1 to the depots must not exceed 20,000 t. That is:

$$20{,}000 \geq 1x_1 + 1x_2 + 1x_3 + 0x_4 + 0x_5 + 0x_6$$

Similarly, for Plant 2:

$$10{,}000 \geq 0x_1 + 0x_2 + 0x_3 + 1x_4 + 1x_5 + 1x_6$$

Second, each depot's requirements must be satisfied.
Thus, at least 5,000 t must be shipped to Depot 1:

$$5{,}000 \leq 1x_1 + 0x_2 + 0x_3 + 1x_4 + 0x_5 + 0x_6$$

Similarly, for Depots 2 and 3:

$$18{,}000 \leq 0x_1 + 1x_2 + 0x_3 + 0x_4 + 1x_5 + 0x_6$$

$$7{,}000 \leq 0x_1 + 0x_2 + 1x_3 + 0x_4 + 0x_5 + 0x_6$$

It is now clear that a LP problem has been formulated.

That is, find x_j values such that the linear objective function is minimized and the linear constraints simultaneously satisfied.

To solve this problem the simplex method would not be used. Due to the fact that the only coefficients appearing in the right-hand sides of the equations are either ones or zeros, an algorithm more efficient than the simplex method can be used. This algorithm relies on the fact that the equations can be adjusted to give the required canonical form through using a set of rules less complex than the simplex rules.

The problem could be solved using the simplex method, it is just that the computational costs would be higher. The reason is that the number of activities and constraints necessary in realistic problems is extremely large. Consider the simple example problem. With two sources and three destinations, 2 × 3 = 6 activities were required. A realistic problem could have 20 sources and 50 destinations and would therefore require 1,000 activities and 70 constraints (in some cases some of these activities could be eliminated as being obviously inefficient).

Another transporting possibility, which was not considered in the simple example, may also be relevant. Besides transporting the good (fertilizer in the example) directly from each source to each destination, it could be transported from a source to a destination via one of the other source points (trans-shipment).

This possibility provides further transporting activities, thus further increasing the size of the problem.

The example given here is *the basic transport model*. With some modifications it can be adjusted to examine such problems as the *optimal location* of production plants and depots *and* the optimal size of production plants. This latter problem is important as economies of size usually exist. Similarly, it can be used to examine, on a national or regional scale, the optimal location of different product-producing farms. For example, a problem that might be considered is whether dairy products should be produced under irrigation in one area relative to others in order to maximize total national welfare.

12.24 The Application of LP – Concluding Comments

The above examples give some idea of the problems that could be solved using LP, although many more could be given, e.g. designing oil refinery operations, aeroplane scheduling, job assignment in a factory etc. It must always be remembered, however, that when looking at some particular problem the analytical technique *that best fits the problem* must be selected. Some analysts tend to do the opposite, and adjust the problem to suit a particular analytical technique as they are only familiar with a limited number of techniques. Clearly the solutions will not comply with the real-world situation where the approximation is too far from reality.

Problems that are multi-staged, and where risk and uncertainty is a critical aspect of their analysis, may be better solved, at least in principal, by Belman's dynamic programming. This technique is the subject of the next chapter.

References and Further Reading

Dent, J.B., Harrison, J.R. and Woodford, K.B. (1986) *Farm Planning with Linear Programming: Concepts and Practice*. Butterworths, Sydney (covers model building and application).

Hazell, P.B.R. and Norton, R.D. (1986) *Mathematical Programming for Economic Analysis in Agriculture*. Macmillan Publishing Co., New York (book covering many applications both farm and macro, and also allowing for risk aspects).

Jansen, B., de Jong, J.J., Roos, C. and Terlaky, T. (1997) Sensitivity analysis in linear programming: just be careful. *European Journal of Operations Research* 101, 15–28 (on the ranges of valuation estimates (sensitivity analysis) – some advanced maths required for the detail).

McCarl, B.A. and Nuthall, P.L. (1982) Linear programming for repeated use in the analysis of agricultural systems. *Agricultural Systems* 8(1), 17–39 (use of LP in extension and other group work).

Nuthall, P.L. (1971) On the analysis of a simplex linear programming solution tableau – with reference to net revenue stability limits. *Journal of Agricultural Economics* 22(1), 49–60 (solution analysis).

Nuthall, P.L. and Moffatt, D.J. (1975) On the use of deterministic linear programming for planning in a non certain environment. *Review of Marketing and Agricultural Economics* 43(4), 184–196 (allowing for risk).

Pannell, D.J. (1997) *Introduction to Practical Linear Programming*. Wiley Interscience, New York (solution analysis, model building, least cost and other application examples).

Puterbaugh, H.L., Kehrberg, E.W. and Dunbar, J.O. (1957) Analyzing the solution tableau of a simplex linear programming problem in farm organization. *Journal of Farm Economics* 39, 478–489 (basic solution analysis).

Tozer, P.R. and Stokes, J.R. (2001) A multi objective approach to feed rationing balancing and nutrient management. *Agricultural Systems* 67(3), 201–215 (least cost rations).

13 Dynamic Programming

13.1 Introduction – Background and Computational Principles

Dynamic programming (DP) is particularly suited to risky and time-dependent problems involving part of a farm, e.g. the problem of deciding how to use limited supplies of irrigation water in an uncertain rainfall environment. Conceptually, however, the technique can be applied to whole-farm problems but the quantity of calculations required challenges the fastest of modern computers. Despite this, a study of DP is extremely beneficial for understanding the nature of the farm decision problem in the dynamic, non-certain environment in which any farm exists. It provides additional insight compared with production economics, linear programming (LP) and many other techniques. These models were largely concerned with static, certain situations. However, it should be noted that despite the computational burden, with good computer support, DP can be of use in solving whole-farm decisions for simple farm types where the choice is not great.

The name 'dynamic programming' suggests that the technique can only be used for problems involving several time periods. This *is not* the case, as will become evident later. Basically, if a problem can be divided into *stages*, it can be solved using DP.

DP is not a well defined *algorithm* like the Simplex method in LP (algorithm – a set of solving rules or procedures). It is a basic approach to problem solving, or, in other words, a way of thinking about problems. Given a specific problem, an individual solving process must be set up for the problem using the general principles of DP. To achieve this, several 'open source' computer packages are available. Alternatively a standard computer language, such as BASIC or FORTRAN, can be used by experienced programmers.

DP will be discussed within three main themes. The first (sections 13.2–13.5) will provide a simple example to show the basis of the DP approach. The next (13.6–13.12) will show how DP can be applied to a number of different farm problems and generalize the DP approach. The last theme (13.20) will discuss the assumptions used by DP and consider the lessons available on the nature of the real farm decision problem. Note that the use of examples forms the basis of the DP discussions. This method is used as DP, as noted, is not a well-defined set of rules, but a way of thinking. The examples will bring this out.

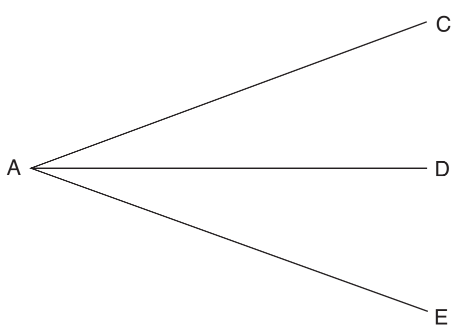
The first example is a simple example of the use of DP.

13.2 A Simple Example of the Use of DP

The problem

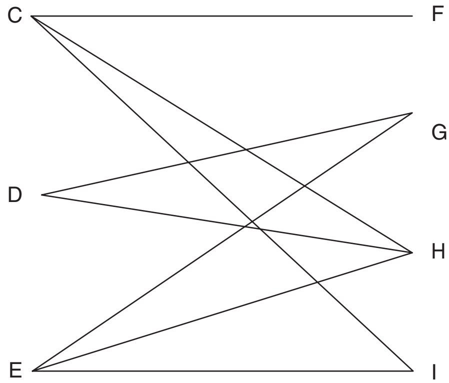
Consider a hill-country farmer who wants to put a farm road across his property so that he will have access between the back and front. Assume the objective is to minimize the total cost of the track. Also assume there is a number of alternative routes that can be followed so that choice exists. Further, assume that it does not matter which of the available routes the track follows provided it starts at point A (front of property) and finishes at point B (back of property), and construction costs are at a minimum.

In constructing the track the process can be thought of as consisting of a number of *stages*. Stage 1 might consist of getting the track to a position 500 m from the starting point and so on. Assume that point B is approximately 2000 m from point A so the problem can be seen to consist of four stages. Assume there are three alternative routes the track can follow (this will depend on the terrain, valleys available, rivers to cross, soil type etc.) from point A to the end of stage 1. That is:



From point A can move to C, D or E.

Similarly, assume that once point C, D or E is reached, the track can then be made to join up to points F, G, H or I using the following routes:



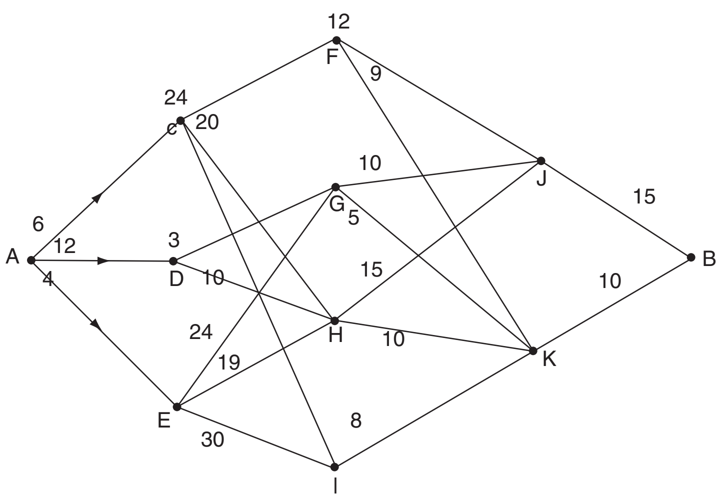
Each line represents a possible route the track can follow.

Thus, to get to point G, the track can start at D or E.

Similarly, for each of the remaining two stages, there are a number of alternative routes the track can follow.

For each possible section between two points, the track will have a fixed cost. Assume these costs are known with certainty (it will be clear in later examples how the certainty assumption can be relaxed). Thus, a summary diagram representing the farmer's decision problem can be drawn showing the alternative routes the track can follow and the costs of each segment. Assume the complete problem has the form shown in Fig. 13.1.

There are, clearly, a large number of possible routes the track can take in getting from A to B. Each of these has a given cost. For example, from A to C to F to J to B forms a possible route with total cost 6 + 24 + 12 + 15 = 57. Similarly,



Notes

(i) The figures beside each segment present the costs (perhaps $100 units)

(ii) This diagram is *not* a map of the farm. It is a *representation* of the problem

Fig. 13.1. Diagram representing the possible farm track routes and their construction costs.

another route is A to C to H to J to B with a cost 6 + 20 + 15 + 15 = 56. *The problem* is to find that route that minimizes the total construction costs.

Features of the problem

The problem can be seen to consist of a number of stages. Within each stage a decision must be made. For example, assume that it pays to construct the track so that it passes through point C. Having reached this point the problem is in stage 2 and *a decision* must be made about the direction the track should take. There are three options. The track can move from C to F; C to H; or C to I.

At the end of each stage the problem can be said to be in a *defined state*. Thus, the statement 'the track has reached point G' defines the current stage of the tracking operations.

There is obviously *a connection* between the decision made at each particular stage and the state of the system at the end of the stage. Thus, given the process is at point C, and assuming the alternatives are labelled in the following way:

C to F – decision 1
C to H – decision 2
C to I – decision 3

then, for example, taking decision 3 means the decision maker has decided the system should end up in state I at the end of this particular stage.

The decision taken at each stage results in a particular cost (in some problems this will be a return rather than a cost). Thus, in each particular stage, *a decision has two effects*: (i) it gives rise to a particular cost; and (ii) it defines the *state* the system will be in at the end of the stage.

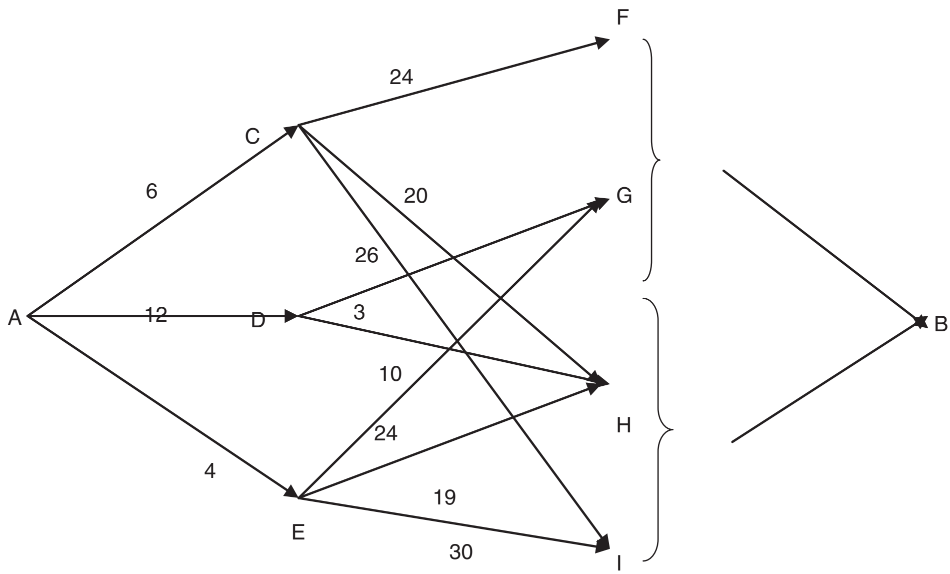
Note that for a particular stage, the *state* of the system dictates what the available alternatives (decisions) are. Thus, for example, if the system is at state H there are two alternative decisions available. That is, move from H to J *or* H to K.

13.3 Solving the Problem

One method would be to determine all possible routes and evaluate the cost of each and select the route with minimum cost. This is obviously inefficient. Another approach is to split the solving process up into a number of stages. (In general, a dynamic programming problem consists of n stages.) A solution to the n stage tracking problem is then obtained by determining the optimal solution to each stage *and combining these* n *optimal solutions* to form one, overall, optimal solution.

Consider the example problem. The first two stages are repeated below.

Consider the first stage. The optimal decision for the first stage cannot be determined until it is known what the minimum cost is from moving from C to B using an optimal route, similarly from D to B and E to B. However, *the*



least-cost way of getting to each *of the possible states at the end of stage 1* can be determined. In this case the answer is trivial as there is only one way of getting from A to each of the possible states C, D or E. If there was in fact a number of alternative ways to each stage (as there is in the second stage), the selection above would give the least-cost way of getting to each stage. Each of the least-cost ways would have a defined cost (in the example, A to C = 6, A to D = 12, A to E = 4).

Now, given the optimal route (optimal decision) for getting to each stage at the end of the first stage, the decision about the minimum cost route to get to *each* possible state at the end of the *second* stage can be determined. Consider each of the second stage states:

(a) F – to get to F there is no choice.
Total cost for stages 1 and 2 is = 6 (A to C) + 24 (C to F) = 30
(b) G – alternatives
  (i) A to D to G, total cost = 12 + 3 = 15
  (ii) A to E to G, total cost = 4 + 24 = 28
  ∴ optimal route to G is ADG with cost = 15.
(c) H – alternatives
  (i) H to C to A, total cost = 20 + 6 = 26
  (ii) H to D to A, total cost = 10 + 12 = 22
  (iii) H to E to A, total cost = 19 + 4 = 23
  ∴ optimal route to H is = ADH with cost = 22.
(d) I – alternatives
  (i) I to C to A, total cost = 26 + 6 = 32
  (ii) I to E to A, total cost = 30 + 4 = 34
  ∴ optimal route to I is ACI with cost = 32.

Thus, the *optimal* route from A to *each* of the possible states at the end of stage 2 has been calculated. Summarizing, these are:

State	Route	Cost
F	ACF	30
G	ADG	15
H	ADH	22
I	ACI	32

Which of these stages should be 'aimed' for depends on the cost of getting from these states to B. What has been achieved so far is to determine the least-cost route of getting to each of the possible states by doing two sets of calculations:

(i) determining optimal routes to get to each possible state at the end of stage 1 (in the example this calculation was trivial); and
(ii) using *the answers to the first calculation*, the optimal route to get to each possible state at the end of stage 2 was determined.

Consider the third stage. Using the information calculated, the least-cost way of getting to *each* of the possible states at the end of stage 3 can be determined. Consider state J. There are three routes to get to this stage:

(a) F to J,
(b) G to J,
(c) H to J.

Note that the cost of each of these routes is:

(a) the cost of getting to F using an optimal route up to this stage (i.e. least-cost route to F) plus the cost of going from F to J. This is 30 + 12 = 42;
(b) the *minimum cost* of getting to G *plus* cost of G to J. This is 15 + 10 = 25;
(c) the *minimum cost* of getting to H *plus* cost of H to J. This is 22 + 15 = 37.

Thus the optimal route to get to J is ADGJ with a cost of 25.

Using the same procedure the alternative least-cost routes to K, the other possible state at the end of stage 3, can be determined. That is, to determine these costs, evaluate the *minimum* cost of getting to each particular state at the end of stage 2 (this has already been determined) *plus* the cost of getting from that state to the desired state at the end of stage 3.

This gives the following alternative routes and costs:

(a) ACF and FK = 30 + 9 = 39
(b) ADG and GK = 15 + 5 = 20
(c) ADH and HK = 22 + 10 = 32
(d) ACI and IK = 32 + 8 = 40

Thus, the least-cost route to get to state K is ADGK with a cost of 20.

There is only one stage left to evaluate. As there is only one state at the end of this stage, this being the destination, the solution to the last stage is obvious. There are two routes to get to B from the start of the last stage. These are J to B and K to B. The optimal path is given by the lowest of:

(a) the cost of an optimal path to J plus J to B cost;
(b) the cost of an optimal path to K *plus* K to B cost

i.e.

(a) 25 + 15 = 40
(b) 20 + 10 = 30

Thus, the optimal path or route for all *n* stages has been determined. It is:

ADGKB with a cost of 30.

13.4 Features of the Solving Process

The important features of the solving process have largely been noted. The *n* stage process has been solved using the answers to a series of one stage problems. The procedure at each stage was:

1. Using the previously calculated information for an optimal set of decisions or policies, and therefore the minimum cost (or maximum return depending on the problem), to get to each state at the start of the stage in order to,
2. Determine the optimal policy for the stage in order to get to *each* possible state occurring at the end of the stage at least cost (or maximum return in some problems).

This process is continued, stage by stage, until the final stage is reached.

13.5 A Backwards Solution

The solving process started at the first stage and proceeded to the last stage. The optimal solution could have been obtained by following the opposite procedure. In practice, DP usually uses this backwards solution process and most texts and research projects use the backwards solution, though it is immaterial which is used. The following discussion uses the backwards approach.

Consider the simple example:

Stage *n*: (Stage 4)

(a) An optimal policy between stages B and J is BJ with a cost of 15.
(b) An optimal policy between states B and K is BK with a cost of 10.

Stage *n*-1: (Stage 3)

(a) An optimal policy from F to B is given by the alternative with the least cost:
  (i) 12 + 15 = 27 (BJF)
  (ii) 9 + 10 = 19 (BKF)
  i.e. policy **(ii)**.

(b) An optimal policy from G to B is given by the alternative with the least cost:

(i) 10 + 15 = 25 (BJG)
(ii) 5 + 10 = 15 (BKG)
i.e. policy **(ii)**.

(c) An optimal policy from H to B:

(i) 15 + 15 = 30 (HJB)
(ii) 10 + 10 = 20 (HKB)
i.e. policy **(ii)**.

(d) An optimal policy from I to B – only one alternative:

(i) 8 + 10 = 18 (JKB).

Stage *n*-2: (Stage 2)

(a) An optimal policy for C to B:

(i) Cost of C to F *plus* cost of an optimal policy from F to B (from previous calculation) = 24 + 19 = 43
(ii) Cost of C to H *plus* cost of an optimal policy from H to B (from previous calculation) = 20 + 20 = 40*
(iii) C to I *plus* I to B = 26 + 18 = 44.

Optimal choice is **(ii)**, i.e. CHKB.

(b) An optimal policy for D to B:

(i) DG + GB = 3 + 15 = 18*
(ii) DH + HB = 10 + 20 = 30.

Optimal choice is **(i)**, i.e. DGKB.

(c) An optimal policy from E to B:

(i) EG + GB = 24 + 15 = 39*
(ii) EH + HB = 19 + 20 = 39
(iii) EI + IB = 30 + 18 = 48.

Optimal choice is either **(i)** or **(ii)**.
Say **(i)**, i.e. EGKB.

Stage *n*-3: (Stage 1)

(a) Optimal policy from A to B through C is AC + CB = 6 + 40 = 46.
(b) Optimal policy from A to B through D is AD + DB = 12 + 18 = 30.
(c) Optimal policy from A to B through E is AE + EB = 4 + 39 = 43.

Thus, optimal choice is **(b)** above, giving an optimal path of AD + DB.

To find the full optimal path, go back to the calculations for stage 2 and determine the optimal route for DB. This is DG + GB. Similarly, the details of the whole route are ascertained giving ADGKB.

This is the same path or route determined using the forwards solution.

13.6 Farm Example (with some general terminology introduced)

Introduction

The previous discussion covered the DP approach for problems that are conceptually multi-stage. The example used, however, was not a particularly important or difficult farm problem, and the DP approach used was not generalized. This section will give a more realistic problem, though it will still assume a certainty environment, and will start to derive a general statement for the DP approach. It is important to make this statement as DP can be used to solve problems that appear to be, and are, quite different.

Using DP to solve a typical part farm decision problem

The problem – possible states

Consider a farm on which store (unfinished) cattle are purchased and fed on purchased concentrate feed. The problem is to decide on the feed to give per head in order to maximize profit. This is a realistic problem that involves considerable complexity. In order to simplify the description, the amount of choice (the number of decisions) will be limited.

Assume the farmer keeps the cattle for exactly three 40-day periods before selling, giving a total of 120 days. Thus, the problem is being divided into *three stages*. Assume that when the cattle are purchased they will be either 300 or 320 kg LW (i.e. they can be put into groups so their mean weights are 300 or 320 kg). The farmer must decide the quantity of feed to give each group during each stage. Assume that the feeding rate is not varied throughout a particular stage.

Assume that the farmer has a choice of three feeding rates:

(a) a maintenance ration (growth rate = 0 kg/day);
(b) a growth rate of 0.5 kg/day;
(c) a growth rate of 1.0 kg/day.

As a result of choosing a feeding rate, the animal(s) will be in a defined state at the end of the 40-day period.

Consider a 300 kg animal. After 40 days it will be in one of the following states, depending on the feeding rate chosen:

(a) 300 kg – using feeding rate (a) above;
(b) 320 kg – using feeding rate (b) above;
(c) 340 kg – using feeding rate (c) above.

Similarly, depending on the feeding rate used in the next 40 days, the animal(s) may end up at a number of possible states. Summarizing, considering both starting weights and all three stages, the possible states and the 'paths' connecting them are presented in Fig. 13.2.

Notice in the figure that a fourth stage has been added to take account of selling. After three feeding stages the animals are sold at the beginning of the fourth stage.

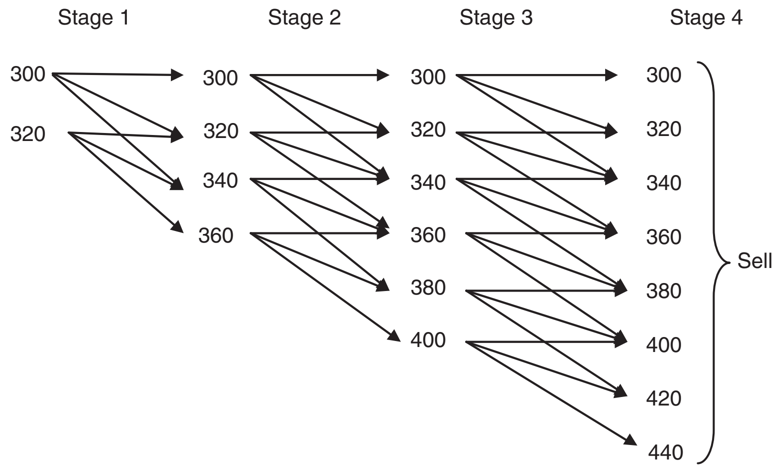


Fig. 13.2. Stages and paths in a beef production system – live weight options (kg).

13.7 The Problem – Terminology for Defining Possible States

Rather than use kg of live weight to describe the state of the animals, the problem can be generalized through the following terminology:

S_j^i = ith possible state at the *beginning* of stage j (*or* at the end of stage j–1).

Let i = 1 be 300 kg LW
i = 2 be 320 kg LW
i = 3 be 340 kg LW
. .
. .
. .
i = 8 be 440 kg LW

Thus, for example:

S_3^2 = a state of 340 kg at the beginning of stage 3 (or end of stage 2)
S_4^8 = a state of 440 kg at the beginning of stage 4 (or end of stage 3).

13.8 The Problem – Terminology Representing Decisions and Action (feeding alternatives)

Three alternative feeding programmes (decisions) within each stage are possible (i.e. plan for 0 kg/day growth rate, *or* 0.5 kg/day *or* 1.0 kg/day). This forms the decision variable. In general, let x_j refer to the decision taken during stage j where:

x_j = 0 represents feeding to give zero growth rate;
x_j = 1 represents feeding to give 0.5 kg/day growth rate;
x_j = 2 represents feeding to give 1.0 kg/day growth rate.

The value given to x_j affects what state the animal will be in at the end of the stage (as previously noted). It does not, however, completely determine the ending state. The starting state *together* with x_j determine the ending state. That is, the starting state (S_j^i) and the decisions made (x_j) determine the ending state for a particular stage.

In general:

$$S_{j+1} = f(S_j x_j)$$
$$S_j = f(S_{j-1} x_{j-1})$$

etc. where S_j is a general variable describing the state of the system at the beginning of stage *j*. In our example, S_j can take on the values $S_j^1, S_j^2, S_j^3, \ldots, S_j^8$, i.e. S_j^i.

13.9 The Problem – Costs and Returns

In order to solve the problem, feeding costs and the value of the animal at different weights must be known.

Assume that the price per kg received varies with the LW. Therefore, assume the following prices per head:

300 kg LW at 30 cents = \$90.0 (i.e. return for beast in state S^1)
320 kg LW at 30 cents = \$96.0 (i.e. return for beast in state S^2)
340 kg LW at 30 cents = \$102.0 (i.e. return for beast in state S^3)
360 kg LW at 29 cents = \$104.4 (i.e. return for beast in state S^4)
380 kg LW at 27 cents = \$102.6 (i.e. return for beast in state S^5)
400 kg LW at 25 cents = \$100.0 (i.e. return for beast in state S^6)
420 kg LW at 25 cents = \$105.0 (i.e. return for beast in state S^7)
440 kg LW at 22 cents = \$96.8 (i.e. return for beast in state S^8)

In order to estimate the feed costs the kg of feed necessary to give the specified growth rates must be known. Assume the following feed requirements:

	Feed per day (kg) to give a growth rate of		
LW	0	0.5 kg/day	1.0 kg/day
300	5	6	9
320	5.2	6.2	9.4
340	5.4	6.4	9.8
360	5.6	6.6	10.2
380	5.8	6.8	10.6
400	6	7	11.0

Given that the heavier the cattle are, the greater the feed requirement to gain 1 kg (the maintenance requirement is higher), then taking cattle to heavy weights is not necessarily profitable. Similarly, the faster the growth the greater the food requirement per kg of gain so that faster growth rates are not necessarily profitable. To make such decisions the marginal costs and returns must be considered. To get very high rates of gain very expensive feed may have to be used in order to get sufficient energy into the animal within the animal's appetite limit.

Assume that as the season progresses the cost/kg of feed increases. Assume a cost of 2.5 cents/kg, 3.0 cents/kg and 3.5 cents/kg in stages 1, 2 and 3, respectively. Thus, the cost to get from each state to another state can be determined. These are given below:

$ Cost of moving from S_j^i to S_{j+1}^i

Stage 1

		End state			
		S_2^1	S_2^2	S_2^3	S_2^4
Starting	S_1^1	*5.00*	*6.00*	*9.00*	
state	S_1^2		*5.20*	*6.20*	*9.40*

Stage 2

		End state					
		S_3^1	S_3^2	S_3^3	S_3^4	S_3^5	S_3^6
	S_2^1	*6.00*	*7.20*	*10.80*			
Starting	S_2^2		*6.24*	*7.44*	*11.28*		
state	S_2^3			*6.48*	*7.68*	*11.76*	
	S_2^4				*8.40*	*9.80*	*15.40*

Stage 3

		End state							
		S_4^1	S_4^2	S_4^3	S_4^4	S_4^5	S_4^6	S_4^7	S_4^8
	S_3^1	*7.00*	*8.40*	*12.6*					
	S_3^2		*7.28*	*8.68*	*13.16*				
Starting	S_3^3			*7.56*	*8.96*	*13.72*			
state	S_3^4				*7.84*	*9.24*	*14.28*		
	S_3^5					*8.12*	*9.52*	*14.84*	
	S_3^6						*8.40*	*9.80*	*15.40*

13.10 The Problem – the General Return Function

The return (where it is assumed that a cost is written as a negative return) in any stage is a function of:

(i) the decision made in that stage, i.e. x_j value; and
(ii) the starting state in that stage, i.e. S_j^i.

Thus, let R_j = the return in stage j:

$R_j = f(S_j, x_j)$

Consider our problem. If, for example:

(i) $S_j = S_2^2$ and $x_2 = 0$, then $R_2 = -6.24$
(ii) $S_j = S_2^2$ and $x_2 = 2$, then $R_2 = -11.28$
(iii) $S_j = S_3^2$ and $x_3 = 1$, then $R_3 = -8.68$

etc. (These figures are read off the preceding tables. Also recall that $S_{j+1} = f(S_j, x_j)$.)

In the final stage (stage 4), x_4 cannot take on any values as the cattle are sold. Thus,

R_4 = \$90 if $S_j = S_4^1$ (i.e. 300 kg animal is sold)

R_4 = \$96 if $S_j = S_4^2$

. .

. .

. .

R_4 = \$96.8 if $S_j = S_4^8$

13.11 The Backwards Solution to the Problem

General method

To solve the problem, use the backwards solution. Thus, when the calculations for a particular stage are being carried out, the return from using *optimal* decisions for the rest of the stages (to the end of the process) is known from previous calculations. This is known for each possible state. Therefore, to determine the optimal decision (in this stage) for each particular starting state, the return from moving from this state to the various possible ending states in the stage is combined with the above information.

In order to define this statement using symbols, let,

1. $f_j\left(S_j^i\right)$ = the \$ return if an optimal policy (optimal decisions are made) is followed from stage j to the end of the process assuming the process is in state i at the beginning of stage j.
2. $r_j^i\left(S_{j+1}^i\right)$ = the return associated with moving from state *i* (the superscript on *r*) at the beginning of stage *j* to state *i* (the superscript on *S*) at the beginning

of stage j+1 (or *end* of stage j); the return depends on the values of the decision variables necessary to give S^i_{j+1}.

Thus, $f_j\left(S^i_j\right) = \max_{i \text{ on } S^i_{j+1}} r^i_j\left(S^i_{j+1}\right) + f_{j+1}\left(S^i_{j+1}\right)$

(This is referred to as a *recurrence relationship*.)

In words:

the return if an *optimal* policy is followed from stage j to the end of the process *assuming* the process is in stage S^i_j	=	The maximum value of the following calculation: (1) the return from the jth stage if a decision is made to go to state S^1 at the end of the stage *plus* (2) the return from following an *optimal* policy from S^1 in j+1 *to the end* of the decision problem (process) where this is calculated for each S^i_{j+1} that can be reached from S^i_j.

The third stage

Using the above formulae an optimal policy for stage 3 can be calculated for each possible starting state. For example, consider the starting state S^1_3 (i.e. 300 kg LW at the beginning of the last 40-day period). Depending on the value of x_3, the animal will be fed to end up in one of the states S^1_4, S^2_4, S^3_4 (i.e. 300, 320 or 340 kg LW). The decision about which of the alternatives is optimal relies on the evaluation of the recurrence relationship:

$$r^1_3\left(S^i_4\right) + f_4\left(S^i_4\right)$$

$$S^1_3 \begin{cases} \nearrow S^1_4 (\text{i.e. } i = 1) \; - \; 7 + 90 = \$83.00 \\ \rightarrow S^2_4 (\text{i.e. } i = 2) \; - \; 8.4 + 96 = \$87.60 \\ \searrow S^3_4 (\text{i.e. } i = 3) \; - \; 12.6 + 102 = \$89.4\,^* \end{cases}$$

where these $ figures are obtained from the cost and return tables developed earlier.

Note that in this final stage the $f_j\left(S^i_j\right)$ figures are simply the sale prices. The optimal i value is clearly 3.

Thus, *an optimal policy* for stage 3, given that the process *is in stage* S^1_3, is to feed so the animal(s) *end up in state* S^3_4. This then indicates that $f_3\left(S^1_3\right)$ = $89.4. That is, the return to following an optimal policy from S^1_3 to the end of the process is $89.4. This information is used in calculating an optimal decision in stage 2.

Similarly, the optimal policies and associated optimal returns ($f_j\left(S^i_j\right)$) can be calculated for S^i_3 (i = 2, 3, 4, 5 & 6).

The results are presented in Table 13.1 (the calculations for S^1_3 are presented for the sake of completeness).

Table 13.1. Costs, returns and optimal polices for each stage of the beef production problem.

S_j	S_{j+1}	$r_j^i(S_{j+1}^i)$	$+ f_{j+1}(S_{j+1}^i)$	= $ Total
	S_4^1	–7.0	+ 90.0	= 83.0
S_3^1	S_4^2	–8.4	+ 96.0	= 87.6
	S_4^3	–12.6	+ 102.0	= 89.4*
	S_4^2	– 7.28	+ 96.0	= 88.72
S_3^2	S_4^3	– 8.68	+ 102.0	= 93.32*
	S_4^4	–13.16	+ 104.4	= 91.24
	S_4^3	– 7.56	+ 102.0	= 94.44
S_3^3	S_4^4	– 8.96	+ 104.4	= 95.44*
	S_4^5	– 13.72	+ 102.6	= 88.88
	S_4^4	– 7.84	+ 104.4	= 96.56*
S_3^4	S_4^5	– 9.24	+ 102.6	= 93.36
	S_4^6	– 14.28	+ 100.0	= 85.72
	S_4^5	– 8.12	+ 102.6	= 94.48*
S_3^5	S_4^6	– 9.52	+ 100.0	= 90.48
	S_4^7	– 14.84	+ 105.0	= 90.16
	S_4^6	– 8.4	+ 100.0	= 91.6
S_3^6	S_4^7	– 9.8	+ 105.0	= 95.2*
	S_4^8	– 15.4	+ 96.8	= 81.4

The * locates the optimal policy and return for each S_j.

To summarize the results for stage 3 the following set of figures can be constructed.

	Optimal policies and returns for Stage 3	
Starting state S_3^i	Optimal policy Move to state:	Optimal returns $f_3\left(S_3^i\right)$ $
S_3^1	S_4^3 (implies $x_3 = 2$)	89.4
S_3^2	S_4^3 (implies $x_3 = 1$)	93.32
S_3^3	S_4^4 (implies $x_3 = 1$)	95.44
S_3^4	S_4^4 (implies $x_3 = 0$)	96.56
S_3^5	S_4^5 (implies $x_3 = 0$)	94.48
S_3^6	S_4^7 (implies $x_3 = 1$)	95.2

The second and first stages

Similarly, using the recurrence relationship, the optimal policies for the alternative possible starting states in each stage can be estimated, bearing in mind that the results for stage 2 must be estimated before stage 1 results can be estimated. Once the first stage results have been calculated an optimal policy for the whole three stages becomes evident.

Second stage			First stage		
S_2^i	S_3^i	$r_2^i(S_3^i)\ f_2\ (S_3^i)^+$	S_1^i	S_2^i	$r^i(S_2^i)\ f_2\ (S_2^i)^+$
	S_3^1	−6.0 + 89.4 = 83.4		S_2^1	−5.0 + 86.12 = 81.12
S_2^1	S_3^2	−7.2 + 93.32 = 86.12*	S_1^1	S_2^2	−6.0 + 88.00 = 82.00*
	S_3^3	−10.8 + 95.44 = 84.64		S_2^3	−9.0 + 88.96 = 79.96
	S_3^2	−6.24 + 93.32 = 87.08		S_2^2	−5.2 + 88.00 = 82.80*
S_2^2	S_3^3	−7.44 + 95.44 = 88.00*	S_1^2	S_2^3	−6.2 + 88.96 = 82.76
	S_3^4	−11.28 + 96.56 = 85.28		S_2^4	−9.4 + 89.84 = 80.44
	S_3^4	−6.72 + 96.56 = 89.84*			
S_2^3	S_3^5	−7.92 + 94.48 = 86.56			
	S_3^6	−12.24 + 95.20 = 82.96			
	S_3^4	−6.72 + 96.56 = 89.84*			
S_2^4	S_3^5	−7.92 + 94.48 = 86.56			
	S_3^6	−12.24 + 95.20 = 82.96			

+ The $f_j(S_j^i)$ are obtained from the table for the previous stage.
* denotes an optimal policy.

Optimal Policies

Second stage			First stage		
Starting state	Ending state	$f_2(S_2^i)$ ($)	Starting state	Ending State	$f_1(S_1^i)$ ($)
S_2^1	S_3^2	86.12	S_1^1	S_2^2	82.00
S_2^2	S_3^3	88.00	S_1^2	S_2^2	82.80
S_2^3	S_3^4	89.84			
S_2^4	S_3^4	89.84			

13.12 The Optimal Solution

Description

Having reached the first stage in the backwards solution, the optimal solution can be determined by using the information on the optimal policies in each stage given the system is at a particular state. For convenience, the table giving the optimal policies is repeated below:

Third stage		Second stage		First stage		
Starting state	Optimal ending state	Starting state	Optimal ending state	Starting state	Optimal ending state	Optimal return
S_3^1	S_4^3	S_2^1	S_3^2	S_1^1	S_2^2	$82.00
S_3^2	S_4^3	S_2^2	S_3^3	S_1^2	S_2^2	$82.80
S_3^3	S_4^4	S_2^3	S_3^4			
S_3^4	S_4^4	S_2^4	S_3^4			
S_3^5	S_4^5					
S_3^6	S_4^7					

An optimal policy for the whole system is now evident. An optimal feeding system depends on the weight of the animal when it is brought into the feed lot. Given:

1. 300 kg animal (i.e. state S_1^1):
   (i) The first stage table shows that the animal should be fed to reach S_2^2 (i.e. 320 kg at the end of the stage (i.e. $x_1 = 1$)).
   (ii) Thus, for the second stage the animal starts in state S_2^2. The second stage table shows that if the animal starts in this stage that an optimal policy is to end in state S_3^3 (i.e. 340 kg, this $x_2 = 1$).
   (iii) Thus, for the third stage given the animal is starting in S_3^3, an optimal policy is to end in state S_4^4 (i.e. 360 kg, thus $x_3 = 1$). The gross return is $82.00

2. 320 kg animal (S_1^2): the tables show that the optimal feeding policy for this animal is the same as for the 300 kg animal except that in the first stage $x = 0$ rather than 1 (i.e. the animal is only fed a maintenance ration).

Summarizing, the optimal policies are:

	Stage 1	Stage 2	Stage 3
300 kg animal	$x = 1$	$x = 1$	$x = 1$
320 kg animal	$x = 0$	$x = 1$	$x = 1$

Recall that $x = 0$ implies feeding for zero growth rate; $x = 1$ implies feeding for 0.5 kg/day growth rate; $x = 2$ implies feeding for 1.0 kg/day growth rate.

Note that in this example it does not pay to feed for the maximum growth rate. This occurs as Marginal Return < Marginal Cost for the high growth rate.

13.13 A Feature of the Solution Tables

The solution tables indicate more than just the optimal solution. They indicate what an optimal policy should be, given the process is started *at any stage* and *in any* of the possible states. For example, say that due to a truck load of good feed the mean weights of a batch of animals at the end of Stage 1 was 340 kg (S_2^3) rather than the planned 320 kg (S_2^3). Given that the end state is different, the optimal policy should change. The tables show that the new optimal policy should be:

Stage 2: End in state S_3^4 ($x_2 = 1$)
Stage 3: End in state S_4^4 ($x_3 = 1$)
Final weight = 360 kg

13.14 A Diagram of the Optimal Policies

In order to reinforce an understanding of the nature of the optimal policies, a diagram showing the optimal paths for each possible starting state is given in Fig. 13.3.

Optimal policies, no matter what the starting state at any particular stage, are now quite evident.

It is re-emphasized that the problem used as an example has been simplified to facilitate the explanation. For example, there is no reason why each animal should be kept for exactly 3 × 40 = 120 days. It may pay to sell after, say, 80 days, or a large range of other weights.

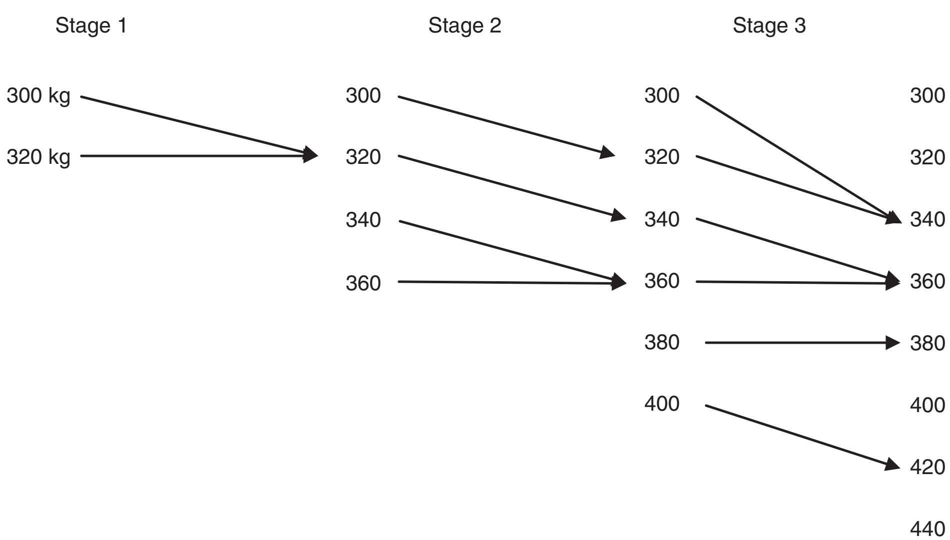


Fig. 13.3. The diagram of the optimal policy sequences for the beef fattening example.

13.15 General Statement and Further Examples

The general form necessary

- For a problem to be solvable using DP, it must be capable of being broken down into stages. These stages *do not* have to be separated in time (an example at the end of the chapter will show this).
- The state of the process at the end of each stage must be capable of being described, *and* there must be a range of alternative possible states at the end of each stage (if only one state is *possible*, the solution to each stage can be determined independently of the other stages).

The state variables

Let the state of the process (in general this will be a production process) be defined by the general variable S_j^i where:

j refers to the stage, $j = 1, 2, \ldots n$

i refers to the ith possible state, $i = 1, 2, \ldots m$

In most processes a number of variables are required to describe the state of the system. These are referred to as the *state variables*. For example, consider the store beef fattening example. The variable 'live weight' was used to describe the state of the system. However, another important variable in some feedlot problems is 'feed on hand'.

Thus, two state variables (q_1 = LW, q_2 = t of feed) might be necessary to describe the state of the system.

Then, for example, S_j^i *might be defined* as the overall state where $q_1 = 300$, $q_2 = 180$.

Further definitions might be:

$S^2 = (q_1 = 300, q_2 = 200)$

$S^3 = (q_1 = 300, q_2 = 220)$

$S^4 = (q_1 = 320, q_2 = 180)$

etc.

Similarly, variables such as q_1 = working capital balance, q_2 = ewes on hand, q_3 = rams on hand, q_4 = area of good turnips, q_5 = area of poor turnips etc. would be required to describe the state of a *whole* farm. To completely describe a large system at any particular time would require many state variables. Thus, the number of possible value combinations for these variables would be extremely large so that i on S^i would have to range from 1 to approaching infinity. (Computational limitations mean such problems become impossible to solve using DP.)

The decision variables

In any particular stage, a number of decisions must be made. Let each decision be described by a variable x_j^i where j refers to the stage, and i refers to the particular decision variable ($i = 1, 2, \ldots m$).

For example:

x_j^2 might refer to the feeding decision. Thus, if $x_j^2 = 0$, this refers to a maintenance diet in the feed lot example.

x_j^3 might refer to the animal health expenditure. Thus, if $x_j^3 = \$2.40$ this refers to a decision to spend $2.40 on animal health within stage *j*.

x_j^4 might refer to the stock sale decision. Thus, $x_j^4 = 1$ might represent the decision to sell, while $x_j^4 = 0$ might represent the decision *not* to sell.

In the store beef fattening example the superscript was not required as there was only one decision variable. In realistic problems there would be many.

13.16 Relationships Between States and Decision Variables, Associated Returns and the Optimal Policy

Given the system is in a particular state, the values of the decision variables (the decision made) affect the ending state. It is assumed that there is a definable function relating the decision made and the resulting end state. If the decisions taken do not affect the ending state, then the solution to each stage can be determined independently.

Thus,

$$S_j = f\left(S_{j-1}, x_{j-1}^1, x_{j-1}^2, \ldots x_{j-1}^n\right)$$

The return received in a *particular* stage depends on the decisions made and the state of the system at the start of the stage. (The return in a particular stage may be negative.)

Thus, letting R_j = the return in the *j*th stage:

$$R_j = f\left(S_j, x_j^1, x_j^2, x_j^3, \ldots, x_j^n\right)$$

Note that, because $S_j = f(S_{j-1}, x_{j-1})$,

$$R_j = f(f(S_{j-1}, x_{j-1}), x_j)$$

This could be further split up as

$$S_{j-1} = f(S_{j-2}, x_{j-2}) \text{ etc. so that}$$

$$R_j = f(S_1, x_1, x_2, x_3, \ldots, x_j)$$

Now as the ending state of a particular stage is a function of the starting state and the decisions made ($S_{j+1} = f(S_j, x_j)$), the return in a stage is a function of the ending state. Thus,

$$R_j = f(S_j, S_{j+1})$$

To determine an optimal policy:

let $F_N(S_i)$ = the return from an *optimal* policy for the *n* stage process assuming the process starts in state S^i (i.e. for the whole problem);

let $f_j\left(S_j^i\right)$ = the return from an optimal policy for the stages *j* to *n* assuming the process is at state S_j^i (i.e. from stage *j* to the end of the process);

let $f_j\left(S_j^i, S_{j+1}^i\right) = R_j$ where the process moves from state S_j^i to state S_{j+1}^i. Then,

$$F_N(S_i) = \max\left(f\left(S_1^i, S_2^i\right) + f_2\left(S_2^i\right)\right) \text{ across all } i = 1, 2, \ldots m \text{ on } S_{j+1}^i$$

That is, to determine the optimal first stage return, given a particular starting state, determine the *n* stage return from moving to each possible ending state for the second stage from the given starting state and select the maximum value.

However, $f_2\left(S_2^i\right)$ is not known at this stage. Thus, the solving process is started at the last stage. *At each stage* the general recurrence (or recursive) relationship given below is used to determine an optimal policy:

$$f_j\left(S_j^1\right) = \max\left(f\left(S_j^i, S_{j+1}^i\right) + f_{j+1}\left(S_{j+1}^i\right)\right) \text{across all } i = 1, 2, \ldots m \text{ on } S_{j+1}^i$$

13.17 The Dynamic Programming Principle

DP was largely developed by Bellman (1957). The solving process used rests upon his *Principle of Optimality*. Bellman states that 'An optimal policy has the property that whatever the state and initial decision are, the remaining decisions must constitute an optimal policy with regard to the state resulting from the first decision.'

In setting up solving systems that meet this requirement, the terminology used to describe DP tends to vary from text to text, and from computer package to computer package. Further, as DP can be used for many different problem types, the symbols used will tend to vary. Thus, when doing general reading make sure that you carefully read all symbol definitions.

13.18 A Stochastic Example

Introduction

One of the major advantages of the DP approach is that stochastic problems, i.e. problems where the variables are random so that planning is taking place under a non-certainty environment, can be solved with the technique. An example is given as an understanding of the method leads to a better comprehension of the nature of the general farm decision problem. It will be assumed that the decision maker's objective is to maximize expected profit. Almost as simply, a model to maximize expected utility could be developed.

The problem

For the sake of simplicity, the store beef fattening problem will be used. Assume, however, that:

- The beef prices are a random variable;
- The feed costs are a random variable; and
- The growth rate is a random variable, i.e. weather conditions, incidence of disease and disorders, the quality of the particular batch of feed etc. are all random variables and affect the growth rate.

The fact that beef prices and feed costs are random variables does not affect the optimal solution or solving process provided the expected values of the random variables are used. If it is assumed that the prices and costs used in the example were expected values, then the solution is optimal. Note, however, that if there was a constraint on the maximum amount of funds that could be spent on feed in each stage, then for the years in which feed costs turned out to be very high, the suggested 'optimal system' may become infeasible. DP can, in fact, handle such situations, but this is beyond the scope of this text.

The stochastic growth rate variable cannot be handled so easily. Its existence (and this is the realistic situation) means that in any particular stage the ending state 'aimed' for may not be reached. Thus, a new solution to the – what is now a stochastic – problem must be found.

Assume that when an animal is fed at a particular rate, the range of possible ending states and their associated probabilities are as follows:

1. Fed to give a growth rate of 0 kg/day (x = 0)
 (i) $S_j^i = 0$ kg $p = 0.8$
 (ii) $S_j^i = 20$ kg $p = 0.2$
 (iii) $S_j^i = 40$ kg $p = 0$

2. Fed to give a growth rate of 0.5 kg/day (x = 1)
 (i) $S_j^i = 0$ kg $p = 0.3$
 (ii) $S_j^i = 20$ kg $p = 0.6$
 (iii) $S_j^i = 40$ kg $p = 0.1$

3. Fed to give a growth rate of 1 kg/day (x = 2)
 (i) $S_j^i = 0$ kg $p = 0.1$
 (ii) $S_j^i = 20$ kg $p = 0.4$
 (iii) $S_j^i = 40$ kg $p = 0.5$

(Note that these probability figures might well vary according to the state of the animal. This is not assumed in this case.)

The effect of a stochastic growth rate on the calculation

Given the process is at a particular state, to estimate the optimal policy for this state the calculations must recognize that the system may not end up in the state expected given a decision regarding x_j. Thus, in calculating the recurrence relationship, the *expected value* of an optimal policy for the remainder of the process must be used instead of the certain $f_j\left(S_j^i\right)$.

Given that the process is at state $i = 2$ in some stage, the *expected* return for this and the rest of the stages, assuming the cattle are fed so that they will end up in state, say, 3, will be:

$$\text{E(return from states 2 to 3)} = f\left(S_j^2, S_{j+1}^3\right) + \sum_{i=1}^{m}\left(f_{j+1}\left(S_{j+1}^i\right)\right)p_i$$

where p_i = the probability of the system ending up in state S^i.

For example, consider the third stage of the example problem and a starting state of S_3^2. Assume the animals are fed to give a growth rate of 0.5 kg/day. Thus, from the tables provided above:

1.

$f\left(S_3^2, S_4^3\right) = -\$8.68$

$f\left(S_4^2\right) = \$96.0$

$f_4\left(S_4^3\right) = \$102.0$

$f_4\left(S_4^4\right) = \$104.4$

2.

$p_1 = 0$
$p_2 = 0.3$
$p_3 = 0.6$
$p_4 = 0.1$

from the probabilities given above

∴ E(return from stage 3 to the end) = −8.68 + 0.3(86) + 0.6(102) + 0.1(104.4) = \$91.76

Similarly, given a starting state of S_3^2 but where the animals are fed a growth rate of 0.0 and 1.0 kg/day the return figures are:

1. E(return from S_3^2 to S_4^2) = −7.28 + 0.8(96) + 0.2(102) = \$89.82
2. E(return from S_3^2 to S_4^4) = −13.16 + 0.1(96) + 0.4(102) + 0.5(104.4) = \$89.44

The maximum E(return) occurs when state S_4^3 is 'aimed' for.

Thus, the optimal policy, *if* the starting state in stage 3 is S_3^2, is to feed for 0.5 kg/day giving an expected return of \$91.76. (This is in fact the same policy as in the certainty case. This is due to the choice of probabilities in the sample problem.)

The optimal policies in the stochastic example

The preceding examples were taken from the third stage. The results of the other calculations required for the third stage are given in Table 13.2.

To save space, the results of each calculation for the other two stages will not be presented. (These calculations use, of course, the optimal E(return) figures from above.) However, the optimal policies for these stages together with those for the third stage are given in Fig. 13.4.

Table 13.2. Optimal policies for the stochastic example.

Starting state	Ending state	E(Return)	Starting state	Ending state	E(Return)
	S_4^1	84.2		S_4^4	96.2*+
S_3^1	S_4^2	86.4*	S_3^4	S_4^5	93.64
	S_4^3	85.8+		S_4^6	87.20
	S_4^2	89.92		S_4^5	93.96*+
S_3^2	S_4^3	91.76*+	S_3^5	S_4^6	91.76
	S_4^4	89.44		S_4^7	87.92
	S_4^3	94.92*		S_4^6	92.6
S_3^3	S_4^4	94.54+	S_3^6	S_4^7	93.16*+
	S_4^5	89.54		S_4^8	85.56

*designates the optimal policy; + designates the policy that was optimal in the non-stochastic case. In two cases the optimal 3rd stage policies are different.

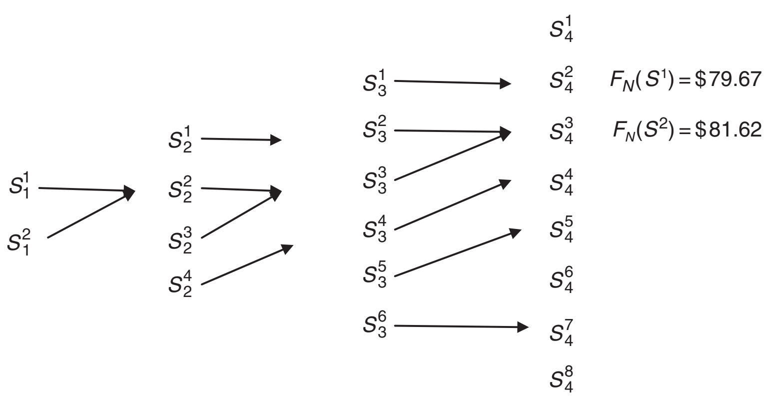


Fig. 13.4. Optimal policies for all stages in the stochastic example problem.

Thus, the optimal policies for each possible starting state are:

$$\left.\begin{array}{l} S_1^1 \xrightarrow{x=1} S_2^2 \\ S_1^2 \xrightarrow{x=0} S_2^2 \end{array}\right\} \xrightarrow{x=1} S_3^3 \xrightarrow{x=0} S_4^3$$

These policies are slightly different to the certainty case in that x in the third stage represents maintenance feeding rather than feeding for 0.5 kg growth/day. The fact that the certainty and non-certainty results are different in this case does not necessarily mean they will always be different. In some cases the ending state can be replaced with the expected ending state and calculation carried out as for the certainty case. This depends on the shape of the return functions and the effect of decisions on subsequent states (remember the utility situation – if the utility function is linear (U(E(Profit) = E(U(Profit)). This situation is somewhat similar.)

Note that in the stochastic, or non-certainty case, the table giving the optimal policies for *all* starting states in *all* stages becomes important. The reason is that it is not sure what the ending state will be at any stage. Given the system actually ends up in other than the expected state, the table indicates the *new* optimal policy without further calculations.

13.19 An Example Not Involving Time

In an earlier chapter it was shown how linear programming (LP) could be used to solve constrained, one-period problems. LP can in fact be used to solve multi-period problems as well. Similarly, DP can be used to solve one-period constrained problems (as well as the multi-period problems as already shown). This section will briefly indicate how this is done.

Consider a very simple problem. A small fruit producer has three alternative products and the only constraint on production is land. Assume he has 4 ha.

To solve this trivial problem (it is trivial as, given a linear return function, the answer is to grow 4 ha of the most profitable fruit) using DP, *synthetic* stages are developed. Assume that the producer thinks in stages. In the first stage he decides how much of fruit 1 to produce; in the second and third stages he does the same for fruits 2 and 3, respectively. Thus, where the land area *not* allocated to crops forms the state variable, the problem in diagram form is as shown in Fig. 13.5.

Consider stages 1 and 2. If the farmer decides to grow 0 ha of fruit 1 he will end up having 4 ha for use in stage 2 (i.e. he is at S_2^1). Thus, his choices in the second stage are to move to S_3^1 (i.e. produce 0 ha of fruit 2 and thus have 4 ha available for the last stage), or move to S_3^2 (i.e. produce 1 ha of fruit 2 and thus have 3 ha available for the last stage), or move to S_3^3 etc. (These decisions are not, of course, separated by more time than it takes the farmer to work out his plan.) On the other hand, if the farmer decides to grow, say, 3 ha of fruit 1, then there will be 1 ha available for allocating to fruit 2. Alternatively, he can decide not to allocate any land to fruit 2. Thus, the feasible ending states are S_3^4 or S_3^5.

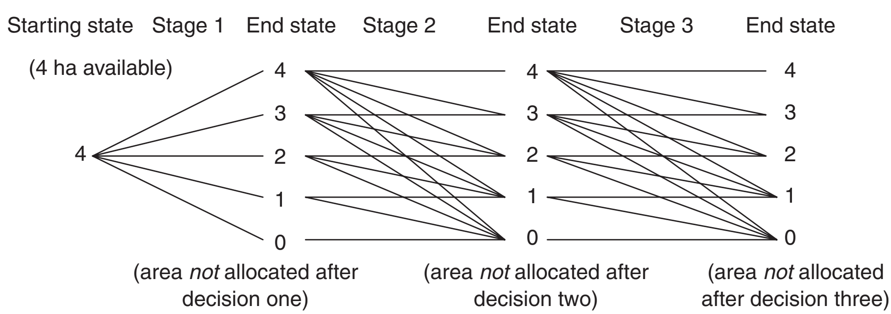


Fig. 13.5. Synthetic stages in determining an optimal crop allocation problem (hectare units).

Given the return figures (gross margin/ha) for each crop the optimal policy can be calculated in the usual manner. For example, let the gross margins per hectare be \$3000, \$3600 and \$2500 for fruits 1, 2 and 3, respectively. Thus, consider an optimal policy for state S_3^1. Alternatives and returns are:

	S_4^1	0 × gross margin of fruit 3 = 0
	S_4^2	1 × 2,500 = \$2,500
S_3^1	S_4^3	2 × 2,500 = \$5,000
	S_4^4	3 × 2,500 = \$7,500
	S_4^5	4 × 2,500 = \$10,000

Therefore, the optimal policy is to allocate 4 ha to fruit 3, *assuming* this stage is started at state S_3^1.

If the calculations were continued it would be found that the optimal policy is:

$S_1^1 \qquad S_2^1 \qquad S_3^5 \qquad S_4^5$

i.e. grow 4 ha of fruit 2. If the return function had been non-linear (as it might be for some specialized horticultural crops), an optimal policy may involve more than one crop as the price would decrease as output increased allowing one of the other crops to be profitably introduced into the solution.

This simple example clearly points out *the effect constraints have* on a DP problem (*and a farm problem*). This is to:

- Limit the number of possible states the system (farm) can be in (in the example, the state $S_j^6 =$ 5 ha not allocated is not permissible); and
- Prevent the movement from a particular state to *all* other possible states at the end of the stage (in the example moving from S_2^4 to S_3^2 is impossible due to the land constraint).

13.20 Assumptions, Problems and Use of DP

Introduction

Throughout the previous discussions the assumptions implicit in the DP approach have emerged. For the sake of clarity these will be brought together in a complete list. Similarly, the important features of the DP approach that emphasize *the nature of real-farm decision problems* have been introduced but will now be brought together in one list. Thus, part of this discussion reviews previous comments, but two new topics will also be discussed and some further comments made regarding the practical applications of DP.

The assumptions of the DP approach

The number of assumptions is small reflecting the realism of the model. They are:

1. It is assumed the problem can be divided into stages;
2. That the state of the process can be described through the use of state variables;
3. That the decisions made in one stage, together with the state of the system, determine the state of the system at the end of the stage;
4. That the returns in each stage are a function of the decision taken in *that* stage and the state of the system;
5. That all the relationships can be quantified;
6. That at any stage the system can only be in a *finite* number of discrete states.

Thus, for example, it is assumed that a beef animal can only be in one of a finite number of weight states. In reality, of course, an animal can be *any* weight between, e.g. 30 and 1000 kg. In general this assumption is not very restrictive, except if very small divisions exist between each state then the number of calculations required becomes very large.

It will be noted that there are *no assumptions* relating to the shape of the return functions (objective function), cost functions and the functions relating decisions to states. (Compare this with the LP situation.) Similarly, constraint equations can take any form necessary to describe the real-world problem. Furthermore, there is no assumption about the existence of the unrealistic certainty situation.

It should be clear why this lack of restrictive assumptions exists. Neither the calculations, nor the optimization technique, are based on a well-defined fixed solving technique. At each stage the return from a particular decision is simply determined by evaluating the relevant return function. Similarly, to determine the ending state as a result of some decision the relevant function is evaluated. These 'functions' may of course be represented by tables worked out through various budgets. For example, in the beef fattening example, the feed cost 'function' was reduced to a table containing the relevant figures.

It will also be noted that there is no divisibility assumption, as was the case in LP. This is because it is a simple matter to evaluate the various functions for *integer values only* if this is required. Similarly, if the (or only some) state variables are only permitted to take on integer values, then only states that satisfy this requirement are defined when the problem is set up.

Calculation problem

The major problem with DP is that the number of calculations required can be extremely large. The calculational costs are often significant, as are the costs of developing the computer code for each specific problem.

The need to calculate an optimal policy for *every alternative state at each stage* is the real problem. Thus, DP *is not an efficient* approach, although it is more efficient than enumerating the return from *all* alternative plans (farming systems). In contrast, for example, the LP solving technique is very efficient as it only has to examine a small number of the alternative feasible plans (these are the possibility curve corner points which are no more than m, where m = the number of constraints).

The examples used did not contain many states, unlike most realistic problems. For example, in the beef production example, it would be realistic to use at least three state variables. Assume:

V_1 = LW of an animal (in kg);
V_2 = t of feed in store;
V_3 = age of the animal (in months).

Further, assume that,

V_1 can vary between 200 and 500 kg in 10 kg units. Thus, V_1 can take on 31 values.

V_2 can vary between 10 and 200 t in 5 t units. Thus, V_2 can take on 41 values.

V_3 can vary between 6 months and 30 months in 1 month periods. Thus, V_3 can take on 25 values.

As a state must be defined (provided any constraints do not prevent some of the states being attained) for each possible value combination of V_1, V_2 and V_3 taken together, there will be $31 \times 41 \times 25 = 31{,}775$ alternative states at each stage. Each would require calculations at each stage to estimate the costs and returns as well as the resulting state from each of the possible decisions.

A whole farm problem with, say, 20 state variables becomes a difficult processing load. In general, only problems involving a small number of state variables can be solved using DP. For example, a problem with, e.g. five state variables, each of which can take on, say, 15 values, there will be 759,375 possible states at each stage. Of course, some may be impractical at some stages and can be ignored. Nevertheless, the calculations will be challenging.

13.21 DP and the Nature of Real-World Farm Decision Problems

A study of DP leads to an extremely clear understanding of the overall farm decision problem in an uncertainty environment. Some of the features emerge from a study of production economics, LP and the other techniques discussed, but DP tends to emphasize some of these particularly well. The important points that emerge about *farm decision problems* are listed below.

1. Decision problems consist of a multi-stage process with decisions required at each stage. When budgeting is carried out, for example, there is a tendency to think in terms of making a set of decisions at the start of the year only. This is clearly not the real situation.

2. While the plans are designed to have the farm in a particular state for each period in the future, the desired state may not be attained. Thus, a new optimal policy may need to be implemented, thus the need to have a recording system to show when the system is diverging from the desired path. Budgetary control attempts to do this.
3. The comments imply that the *initial* starting state, or the initial condition of the farm (the state when planning was carried out), are irrelevant to the optimal farming system to use in stages other than the first. The optimal policy depends only on the *current state* of the system (e.g. it does not matter what the initial bank overdraft was when plans are being revised at, say, Christmas time. What is important is the current bank overdraft level). Further, past decisions should not directly affect the future policy or system. They are history. Nothing can be done to alter them. They are reflected in the current state, which is the new all important starting state.
4. The value of each resource and input depends on the contribution made to future returns. This conclusion (somewhat obvious) is derived from the fact that the $f(S_j^i)^*$ figures are valuations of the alternative states, which depend on future returns. Consider the store beef fattening example. The $f_j(S_j^i)$ figures indicated the value of a beast at particular weights as they represent the eventual sale value less feeding costs from an optimal policy.
5. As the expected future prices, costs and input–output coefficients change compared with the original estimates, the farming system or policy may need to be changed. In DP terms, this will occur as the $f_j(S_j^i)$ for future stages will change.
6. When planning a farm, only a certain number of future states needs to be considered. That is, there exists a planning horizon beyond which future stages do not need to be considered when deciding on the plans that should be implemented at the current time. Decisions for periods beyond the *planning horizon* can in no way affect current decisions through the value ($f_j(S_j^i)$) of the state variables.

This should become clear after studying the example shown in Fig. 13.6.

The figures on the lines leading to each state (denoted with alphabetic letters) are the returns for the stage for the particular decisions. Assuming the forward solution method is used rather than the conventional backward solution, the figures beside each state (letter) are the values (returns) of an optimal policy from the starting state to the particular state. The black-headed arrow lines represent optimal sub-policies while the large double-lined hollow arrows represent the optimal policy for all stages. *Now*, consider stages 1 and 2 only. If these were the only stages considered when deciding on an optimal decision for the first period, the decision would be to move to state C. However, this decision is in fact *the optimal decision even if all the stages are considered*. Thus, the planning horizon need only consist of two stages. Note that if only one stage was considered when making the first period decision the *incorrect* decision would be made.

The example clearly indicates the idea of, and the relevance of, selecting the correct planning horizon. It does not, however, indicate how to determine the minimum planning horizon. In general, there is no rule indicating the

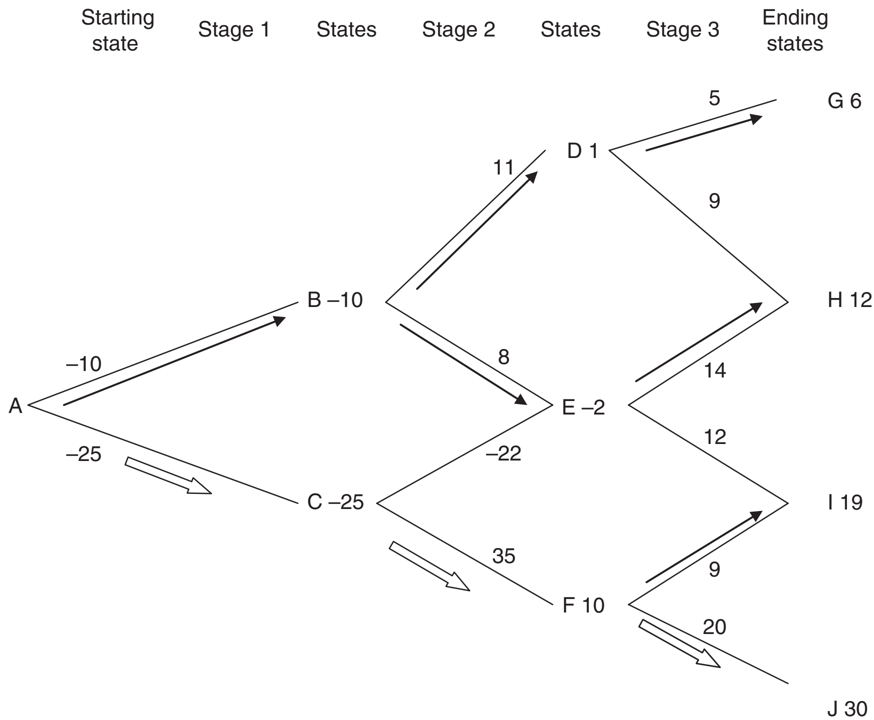


Fig. 13.6. An example demonstrating an optimal planning horizon.

length of the planning horizon so estimates must rely on trial and error calculations through including additional stages until the first-stage decisions tend not to change as further stages are added. (Note: in a stochastic environment it is pointless constructing plans for more stages than is necessary as they are unlikely to be carried out unless all the conditions expected eventuate.) The only decisions absolutely necessary are those for the 'here and now', the rest can wait until the next stage becomes the 'here and now' requiring a re-run of the model given the new state is now known.

13.22 The Practical Use of DP

DP has considerable use at a research level for many *part farm* problems because of its ability to handle uncertainty and the realities of most situations. Problems such as the beef feeding problem considered, an optimal weaning policy, an optimal irrigation policy (to name a few) can all be solved using DP. The restriction lies, of course, in the number of state variables necessary to describe the problem. An analyst operating in a large, environmentally similar area might well use DP on some representative farms. The results would then be interpolated for each individual. The analyst will need access to a fast computer and a programmer who would compile the necessary programs.

One of the practical advantages of DP is the form the results are in. The optimal policies for all possible states are provided, so if the expected outcomes do not occur (as is usually the case), the results still provide the new optimal policy for the current state. For example, consider a lamb weaning problem. The results of a DP study may indicate the lambs should be weaned at 9.5 weeks of age. This would assume the expected feed conditions, expected lambing percentage and so on occurred. If a particularly good season eventuated so the feed state was good, the DP results would indicate the new optimal policy. The new decision might, for example, involve weaning at 11.8 weeks.

As noted, a major advantage of studying DP is the insights provided into the nature of the farm problem. This understanding should assist in the successful use of the less-sophisticated planning techniques allowing subjective adjustments to better represent the true decision problem.

DP has extensive use throughout all industries and other areas too. It is used extensively, for example, by wholesalers and retailers to calculate when the products they handle should be re-ordered from the manufacturers and how many units should be purchased (an inventory problem). A totally different example is determining an optimal trajectory for a space capsule being sent to the moon. The state variable in this case consists of alternative points in space. If something goes wrong, provided the current location of the capsule is known, the DP answers will indicate a new optimal trajectory. The objective in such a case could be to minimize oxygen and fuel consumption. Clearly, any analytical system that mimics the real world will have numerous applications as is the case for DP. It is also interesting to note an LP model can be set up to mimic a DP situation – can you think how this might be achieved? (Hint: each period's sub-matrix is repeated with resource rows linking each period.)

References

Bellman, R. (1957) *Dynamic Programming*. Princeton University Press, Princeton, New Jersey (one of the original texts on the idea of DP).

Huirne, R.B.M., Dijkhuizen, P., van Beek, P. and Hendriks, Th.H.B. (1993) Stochastic dynamic programming to support sow replacement decisions. *European Journal of Operational Research* 62(2), 161–171 (example of incorporating risk into DP).

Johnston, J. (1964) Dynamic programming and the theory of the firm. *Journal of Agricultural Economics* 16(4), 532–547 (interesting early review of DP related to production economics).

Kennedy, J.O.S. (1986) *Dynamic Programming Applications to Agriculture and Natural Resources*. Elsevier Science Publishing Co, New York (extensive review of the use of DP in agriculture with examples).

Wagner, H.M. (1975) *Principles of Operations Research with Applications to Managerial Decisions*. Prentice Hall, Englewood Cliffs, pp. 253–261 (general review of DP).

14 Systems Simulation

14.1 Introduction

Some farm management problems cannot be solved using the well-structured analytical techniques such as linear programming, gross margins analysis and so on. Under these circumstances it is necessary to develop a model *designed specifically* to solve the particular problem. For example, to estimate the effect of using forward contracts on the shape of the profit distribution, conventional models are unlikely to be suitable.

The development of 'specific problem' models and their analysis is referred to as *systems simulation*. Strictly speaking this is incorrect, as all analytical techniques which do not physically experiment with the problem can be described as systems simulation. However, in line with general usage, the term *simulation* is used to indicate the development and use of symbolic models for solving specific problems that are often not 'well behaved', i.e. not easily solved with simple maths.

A study of simulation is dissimilar from a study of e.g. linear programming, which is a well-defined analytical model. As a systems simulation model is developed for each particular problem, the approach is to provide *a general* set of guidelines. These list the stages, or steps, that must be followed. Furthermore, it is useful to study an example problem to demonstrate how the stages can be followed through. While the example will emphasize many points, it cannot cover all aspects. Many examples would be necessary for this. Indeed, in most dedicated systems simulation texts the main approach is studying case examples to cover the range of methods.

Due to each model's uniqueness and, therefore, their high cost, their main use lies in research on representative farm situations where the results have wide application. However, some analyses common to many farms have been set up for individual farm use via the World Wide Web (e.g. feed planning). Of course, systems simulation should only be used where none of the more computationally efficient techniques, such as mathematical programming, can be used (this will be where their assumptions are a gross simplification of the problem).

A major reason for potential consultants and advisers to study simulation is to understand simulation generated research results, and to be able to interpret them for their locality. Furthermore, students wanting to follow through their studies to a higher level and specialize in simulation and computer programming will find this introduction a starting point. Whatever the case, the problem-solving guidelines provided for simulation not only help both these groups, but also have application to a wide range of problem-solving techniques.

It should be noted that budgeting is a simple form of simulation as it is a model of a farming system. A specific budget is set up for each situation, but the assumptions on the shape of the various relationships, such as the return functions for alternative products, are not formalized. Once a budget is set up it can be used to experiment with improved farming systems by changing various parameters and comparing the results. In other words, a comparative whole- and part-farm budgeting exercise is carried out. All the input–output relationships are intuitively created by the budgeter rather than using formal equations that are required in systems simulation.

There are four topics, or discussions, in this chapter. The first (section 14.2) and third (14.7–14.8) cover the uses of simulation and general simulation methodology while the second (14.3–14.6) contains a discussion on the simulation of stochastic systems. The remaining discussion (14.9) covers an example problem. Some readers may find glancing through this example before reading the earlier sections beneficial to their understanding. Students who like to go from the particular to the general will fall into this category. The finer details of the example can then be discovered at the second reading.

14.2 The Uses of Systems Simulation

General

The systems simulation emphasis on ‘experimentation’ will be noted as the discussion proceeds. Models are seldom used directly for optimization due to the complexities of real-life models and their mathematical expression. The maths is simply too complicated for calculus and differentiation in particular.

This implies that simulation can be used to explore problems that are realistic and complex as often the linking relationships between system components are complicated, particularly where uncertainty is important. In order to see why simulation can handle such problems, a brief introduction to the method is first provided. This will be expanded later.

The first step is to develop a set of relationships, which link the output of the product/s of interest (this may be a simple monetary output) to the factors affecting the production of the product. In its simplest form this may be a simple production function. Given this symbolic model, the second step is to experiment with the system by determining output as a result of using various input combinations. Thus, specific values for all the variables are put into the relationships and the output evaluated. Finally, the results of these 'paper' experiments are used to draw conclusions about the system.

As the relationships and functions are not mathematically manipulated, their complexity is not important. All that is necessary is to *evaluate* (rather than manipulate) the relationships for given variable (input) values. Contrast this with the use of calculus in determining optimal solutions particularly to uncertainty situations where there is a need to manipulate the equations of stochastic variable distributions (e.g. try manipulating the equations of normal distributions).

Finally, it must be noted that simulation should only be carried out provided it is difficult and/or too expensive to use actual systems to experiment with (e.g. replicated experimental small farms). This is often the case due to the expense of employing experienced technicians and staff, though small farm trials may be profitable in their own right given the product produced in the experiments can be sold. However, once extensive knowledge is available on all the technical relationships, systems simulation can take over to provide an ability to explore input combinations for which field trials are not available, and similarly be used to explore profit relationships under a wide range of price and cost values.

Input–output data

Small plot trials, small farm and even whole farms provide data for use in farm planning. There are, however, many areas where such experiments have not been carried out or are too expensive in relation to potential returns, so that other approaches are necessary. One is making 'considered estimates' using experienced people. Alternatively, a simulation model can be developed to provide the data.

The usual approach is to work from basic soil, plant and animal relationships in order to synthesize the required data. For example, consider the problem of estimating wool production per ewe given the ewe will be grazing a pasture in a newly developed area for which no past experience exists. There are two parts to this system: (i) the pasture production; and (ii) the animal production, given a certain level of pasture production.

Plant physiologists have developed relationships that predict growth as a function of soil moisture, nutrient levels, temperature, radiation and so on. Consequently, given observations for the particular area on rainfall, temperature and the other variables, it is possible to estimate what the expected pasture production will be at different times of the year.

Similarly, animal scientists have determined the feed requirements of ewes for maintenance and production. Thus, given the pasture production figures obtained from the pasture simulation section of the model, the expected animal production for different stocking rate levels can be estimated, and thus the per ewe feed intake.

The input–output data provided can then be used in other planning models. The above description might suggest the determination of input–output data in this way is relatively simple. In general this is not the case, as the description is a simplification of the problem. For example, pasture production figures must indicate the production of the various proteins, carbohydrates, vitamins and so on, as these will affect animal production. Some of the difficulties will emerge when the example problem is detailed later.

In evaluating pasture and animal production, the model would commonly be evaluated for varying levels of the decision variables. Thus, for example, provided the basic functional relationships are known, the effect of varying the pasture fertilization policy and the stocking rate could be explored.

Evaluating management systems

Rather than use the pasture–animal model to predict only wool production it could be extended to predict the profit from a whole (or part) farm. Given a symbolic model (probably a series of equations) of the whole farm, the profit from setting inputs at different levels can be evaluated. Similarly, the carrying capacity can be altered and the profit determined for each different level. In effect, the farm is being experimented with on paper.

While it is easy to read and write about this approach, it is both difficult to estimate all the necessary relationships and time consuming to evaluate a large number of alternative management systems. For example, assume a model of a fat lamb is available. Consider three decision variables, e.g. stocking rate, lambing time and weaning time. Given that each can take on, say, ten different values (e.g. consider the lambing time variable; this might take on the following alternative 'values' – aim to lamb all the flock on 25 July, all the flock on 8 August, all the flock on 22 August, ½ the flock on 25 July and ½ on 8 August and so on for a wide range of possible combinations), then at least a thousand possible combinations is quite common, each of which would require evaluation using the complex whole-farm model.

Parametric budgeting can be a simplified form of the 'evaluation of management systems'-type simulation. In parametric budgeting, however, the basic physical relationships are not quantified. It is assumed that these relationships have been summarized in the normal budget underlying the parametric budget and their results appear as the constants in the equation.

While using simulation to explore different management systems will become clearer when the example problem is studied, a simple example may help at this stage. Consider a simple beef fattening enterprise using a feed lot system. The farmer must decide on how to feed the animals, and when to buy and sell. Assume the growth rate is given by the following generalized function (this would have to be quantified):

$$\begin{aligned} \text{kg/day} &= g\ (\text{age, weight, kg of protein etc.}) \\ &= g(X_i),\ i = 1,\ \ldots .\ m \end{aligned}$$

where there are m possible inputs (Note: g represents 'function of').

Thus, the weight of an animal at a particular time will be given by:

$$\begin{aligned} \text{Weight (kg)} = ((&\text{starting weight}) + (\text{days held at feeding system one}) \times g\left(X_i^1\right) \\ &+ (\text{days held at feeding system two}) \times g\left(X_i^2\right) \text{ etc.}) \end{aligned}$$

where X_i^1 and X_i^2 represent the values of the input values for each of the possible feeding systems.

Thus, where W = weight in kg, the profit from one beast is given by:

$$\begin{aligned}\text{Profit / beast} = {} & (W \times \text{dressing } \% \times \text{price / kg}) \\ & -\left(\sum X_i \times \text{cost / unit of input}\right) - \text{animal purchase price}\end{aligned}$$

Given that the farmer has a feed lot that will hold a given number of beasts at any one time, the objective must be to maximize the return *per unit of feed-lot space per time period*. Using the above functions the profit/beast could be calculated from varying purchase ages, purchase weights, keeping times and feed policies. The combination of policies (a management system) giving the maximum return/unit of time would then be selected (this is determined by dividing the profit per beast by the number of days it is kept). It may also be necessary to consider the fact that beasts of varying ages take up different quantities of feed lot space.

The above example *assumes certainty*, i.e. assumes the variables are non-stochastic. A simple estimate of profit/beast is made for each set of input variable levels. *It is quite feasible however, to assume a non-certainty situation*. In fact one of the major reasons why simulation techniques were developed was to take into account non-certainty. For example, rather than assume the price received per kg was non-stochastic, a large number of profit/beast estimates could be made using, say, a large number of past year's prices after adjustments for trends. This would give some idea of the profit/beast distribution. The methods that should be used to take account of non-certainty will be discussed in some detail in a subsequent section.

Determining the profit distribution for a defined farming system

Given a farmer with a non-linear monetary utility function it may be important to determine the shape of the profit distribution for alternative farming systems. One method is to use the probability tree approach. In complex situations this approach becomes cumbersome. Alternatively, given the equations of the constituent random variable distributions, the equation of the whole-farm profit can occasionally be analytically determined. Usually, however, this approach is impossible due to the complex maths involved.

Another approach is to use simulation, which, in many cases, is the only practical approach. The procedure is first to develop a profit equation or series of equations for the farming system/s under study. (This/these may be somewhat similar to a parametric budget.) Given the equation, one approach is to use a past series of yields, prices and costs (suitably adjusted to remove trend and inflationary effects) to obtain a large number of profit estimates for the farming system, one for each set of variable values. Using these estimates (e.g. 50 years of simulated profit estimates) the probability of profit occurring within a number of defined ranges can be determined on a frequency basis. In other words, the profit distribution is derived. There is also an alternative approach to estimating the profit distribution. This will be discussed in the next section.

Concluding comments on uses

Systems simulation is being increasingly used in all fields. Agronomists, for example, are using it to experiment with pasture management systems before actual physical experiments are carried out. The ‘paper’ experiments suggest the more promising areas for actual physical research.

Similarly, in farm management, simulation can be used to explore and develop radically different farming systems before actual farm trials are carried out.

In-depth simulation studies are said to have many side advantages. For example, they tend to highlight the areas where there is a lack of basic physical data. Similarly, the act of developing detailed models tends to suggest new, alternative, management systems for later detailed analysis using the model. Further, the act of analysing systems using the models tends to provide a good understanding of the system and an idea of where the likely management problems will be.

14.3 Monte Carlo Simulation

Introduction

Where analytical methods cannot be used to determine the profit distribution for a farming system the alternative is to use experimental methods. As noted in the previous section, given a symbolic model, which enables the profit from the system to be determined for given values for all the inputs, ‘paper experiments’ can be carried out for many different values of the random variables. Given the results, the shape of the profit distribution can be estimated.

There are two different approaches to this procedure: (i) use a series of historical values of the random variables and so obtain a series of profit estimates for the farming system; or (ii) rather than use historical values, a series of values for the random variables can be synthetically generated on the basis of their distributions, and these used to estimate a series of profit values. These profit estimates are then used to estimate the profit distribution. This second approach is referred to as Monte Carlo simulation and will form the major discussion in this section.

An example problem

For simplicity, consider a part-farm problem, although a whole-farm problem could have been chosen as the principles are the same for all cases. Assume it is necessary to estimate the profit/beast distribution for a particular feeding system and feeding period for the feed lot problem introduced earlier.

Assume that the equation giving profit/beast is:

$$\text{Profit/beast} = 100G \times P \times D - 450F - 3$$

Where G = growth rate/day (kg) (assume this is the average over all animals in one batch);
100 = the number of days the animal is kept;
P = price/kg of dressed meat;
D = dressing percentage expressed as a fraction;
F = feed cost/kg;
450 = kg of feed consumed;
3 = other variable costs (health, etc.).

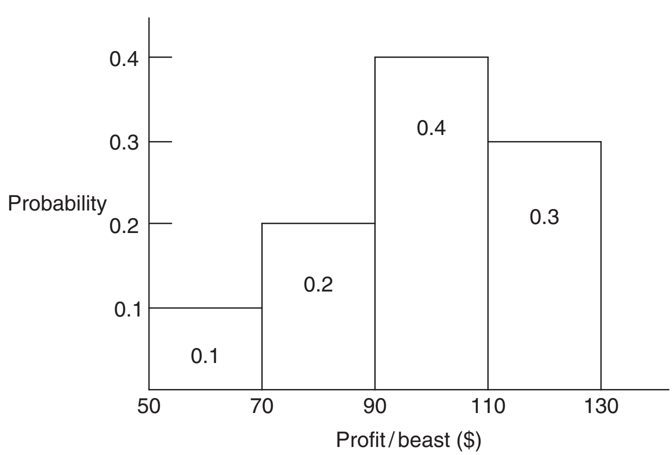
It is assumed that the only random variables affecting profit are G, P, D and F. Thus, it is assumed that all other variables are known with certainty. Note that to obtain this equation a number of feed and cash budgets must be calculated, e.g. to estimate the health costs.

Historical data method

Given a number of years' records indicating the value each random variable (after adjusting them for inflation, trends, etc.), the profit is estimated for each set of values. Thus, given, say, 50 years' records, 50 estimates of profit would be obtained, which are then used to estimate the profit distribution. For example, the number of estimates falling within a number of ranges might be:

Profit/beast ($)	No. of times estimate falls within range
50–69.9	5
70–89.9	10
90–109.9	20
110–129.9	15

This gives a distribution of:



The implicit assumption in this historical data approach is that the available records indicate the shape of each random variable's distribution that is *expected to occur in the future*. However, this assumption may well be violated in many cases. Thus, the second approach (Monte Carlo approach) tends to have greater use. The historical data method also requires information on all the variables. In many cases such data will not exist. Consider the example problem. Data on the growth rate achieved for the given feeding system probably do not exist as there are no farmers who have used exactly this system for a large number of years, let alone actually recorded the growth rates.

Monte Carlo method

Introduction

The profit estimates are made using *synthetically generated* random variable values rather than a historical series. The data are generated on the basis of the random variable distributions, assuming the distribution of each of the random variables is known. Some comments will be made about this assumption.

Generating data

Consider the random variable G (average growth rate/day/head in kg). Assume for a particular feeding system the variable exhibits the following discrete distribution:

Event (kg/day)	Probability
0.8	0.2
1.0	0.4
1.2	0.3
1.4	0.1

For 50 batches of animals the growth rates expected would be 0.8 kg/day in 0.2×50 of the trials (10), 1.0 kg/day in 0.4×50 of the trials (20) and so on. Thus, to synthetically generate a series of growth rate observations from the distribution, select a number of observations *according to the proportion we would expect to occur* in real life.

To achieve this a random selection process is used. Assume 100 identical marbles are numbered 1 to 100, each of which has an equal chance of selection. Now give the variable 'growth rate/day' a specific value if the number on the marble drawn should fall within a specific range. These ranges are based on the chance of each value of the random variable occurring. Thus, as a growth rate of 0.8 kg/day has a probability of 0.2, assign a range of 0.2×100 (20) numbers to this value. Let the numbers 1–20 represent a growth rate of 0.8 kg/day. Similarly, as a growth rate of 1.0 kg/day has a probability of 0.4 assign a range of $0.4 \times 100 = 40$ to this value, so numbers 21–60 represent a growth rate of 1.0 kg/day. Assignments of numbers are determined for the other values of the variable using the same principle. Thus:

Variable value (kg/day)	Assigned no. range
0.8	1–20
1.0	21–60
1.2	61–90
1.4	91–100

A series of growth rate values can now be generated. Randomly select a marble, which, for example, might be 67. As this lies between 61 and 90, the first growth rate observation is set at 1.2 kg/day. This procedure is then repeated for as many values as are required.

For example:

Value on marble selected	Growth rate
1	0.8
38	1.0
24	1.0
68	1.2
95	1.4
42	1.0
etc.	etc.

This procedure produces a random series of 'observations' coming from a defined probability distribution. Each is selected on the basis of the probabilities.

In the example problem there are four random variables, so for each variable the procedure described is carried out. Assuming the random variables are independent, i.e. covariance between each pair is 0, to determine one set of values for all the variables four numbers would be randomly selected and, depending on the numbers selected, a value given to each random variable. This process is then repeated for as many times as sets of observations are required.

This process can be compared with the historical data approach in which 'nature' does the selecting of the variable values for each trial. In the synthetic approach, the researcher does the selecting but on the same basis that 'nature' would use as the selections are based on the probability distribution.

Random numbers

The numbers selected using the marble process described are referred to as *random numbers*. This is a number selected from a defined range such that each number has an equal chance of selection compared with all other numbers. A marble-drawing process is one way of generating a series of random numbers. Various other, less tedious, mathematical methods have been devised. As a result, tables of random numbers are readily available and most statistical books have them as an appendix. They also provide tests to check whether a series of numbers is truly random. This testing is not a trivial matter.

The following is an extract from such a table of random numbers:

Table of Random Numbers								
136	124	987	652	312	486	134	951	
831	261	658	164	288	396	789	546	
321	690	830	348	911	201	016	353	
168	754	382	086	169	350	687	456	etc.

Most simulation models will be computerized and will utilize the commonly available random number functions built into most computer languages.

Generating data for the example problem

Assuming random numbers from within the range 0–999, rather than the 1–100 assumed earlier, each random variable must be considered and number ranges allotted. Thus, for each value a random variable can take on, a random number range is assigned. This range is based on its probability of occurrence.

Assume that in the example problem the ranges assigned are:

Growth rate			Price/kg of dressed meat		
Probability	Value (kg/day)	Random no. range	Probability	Value ($/kg)	Random no. range
0.2	0.8	0–199	0.3	2.40	0–299
0.4	1.0	200–599	0.6	2.80	300–899
0.3	1.2	600–899	0.1	3.00	900–999
0.1	1.4	900–999			

Dressing %			Feed costs/kg		
Probability	Value (%)	Random no. range	Probability	Value (cents/kg)	Random no. range
0.3	45	0–299	0.2	18.0	0–199
0.4	50	300–699	0.5	20.0	200–699
0.3	55	700–999	0.2	22.0	700–899
			0.1	24.0	900–999

To generate one set of data for one trial, four random numbers are selected. Their values determine the random variable values. Take the first four numbers from the random number table:

136, 124, 987 and 652

Thus, the values for the first trial are:

Growth rate: 0.8 as 136 falls within the range 0–199
Price/kg: 2.40 as 124 falls within the range 0–299
Dressing %: 55 as 987 falls within the range 700–999
Feed cost/kg: 27.0 as 652 falls within the range 200–699

The process is then repeated for as many sets of data as are required. Using the next four random numbers from the table of random numbers, the generated data are:

Random nos.	Growth rate (kg/day)	Price/kg	Dressing %	Feed cost c/kg
312, 486, 134, 951	1.0	\$2.80	45%	24.0
831, 261, 658, 164	1.2	\$2.40	50%	18.0
288, 396, 789, 546	1.0	\$2.80	55%	20.0
321, 690, 830, 348	1.0	\$2.80	55%	20.0
etc.		etc.		

14.4 The Profit Distribution

Using the sets of generated data, the symbolic model of the system is evaluated for each set of data. This produces a series of profit estimates, which enable the shape of the profit distribution to be estimated in the same way as that demonstrated for the historical data approach. This distribution will obviously depend on the shape of the profit-determining random variables' distributions.

Consider the example problem. Given the first set of generated data of 0.8 kg/day, \$2.40/kg, 55% and 20.0 cents/kg (\$0.20), the profit is:

$$\begin{aligned}\text{Profit/beast (\$)} &= (100 \times 0.8)\,(2.40)\,(0.55) - (450 \times 0.20) - 3\\ &= 105.60 - 93\\ &= \$12.60\end{aligned}$$

This is obviously one of the poor outcomes and it would not be expected to occur frequently. For the other four sets of data in the above table, the profit estimates are –\$1.50, \$12.60, \$9.50, \$9.50.

Given that 100 sets of data were generated, 100 profit estimates might exhibit the following frequencies:

Profit/beast (\$)	No. of simulated observations
–24.60 (lowest value)	1
.	2
.	.
.	.
. etc.	.
.	.
47.0	12
.	.
. etc.	.
.	.
.	.
97.5 (highest value)	1

Given these data, the probability of each value can be estimated on the basis of its frequency, which leads to the probability distribution.

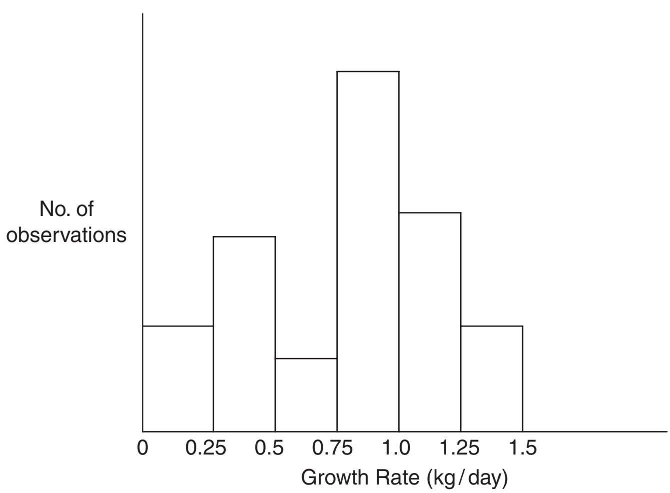
14.5 Use of the Monte Carlo Approach

Where the historical data approach cannot be used, the Monte Carlo method is the only alternative. This will occur under the following conditions:

1. Where it is believed subjective probabilities should be used. This might be because:
(i) Objective probabilities are not available; or
(ii) Objective probabilities are available, but they differ from the farmer's beliefs (subjective probabilities).
2. Where historical data are available, but it is considered that the historical data represent a poor sample of possible values, or a sample of insufficient size.

Under these conditions the historic data must be capable of suggesting the shape of the profit distribution to enable the Monte Carlo approach to be used.

For example, assume that historical records of beef growth rate gave the following frequency graph:



It is likely that the low observed frequency of the 0.75 kg/day rate is due to chance and is not a true indication of the distribution shape. Thus, the distribution could be 'smoothed' and then used to generate data.

14.6 Probability Tree Approach

It will be recalled that the profit distribution for a farming system can be determined using the probability tree method. This involved determining *all* *possible* outcomes and their associated probabilities. This technique, however, is clearly only suitable where the number of random variables is small, and the values each can take are limited. If this is not the case the method becomes cumbersome due to the number of alternative possible outcomes. For example, consider a problem in which there are ten random variables and each can take on ten values. The number of possible outcomes is 10^{10}. It is impossible to solve such a problem using a probability tree.

The alternative is to use the Monte Carlo simulation technique. This is basically *a sampling technique*. Rather than determine all alternative outcomes, sets of data are generated and from these the shape of the profit distribution is *inferred*. The larger the sample (number of data series generated), the more accurate the estimate of the distribution.

14.7 General Procedures

Introduction

In carrying out any analysis, a number of well-defined steps must be followed. These tend to be the same for all problems (scientific method), no matter whether the problem is a 'simple' farm advisory problem or a complex pure research problem. It follows that in using simulation the same general steps must be followed. In this section, an outline of the steps necessary for general simulation are provided.

The steps are:

1. Formulation of the problem.
2. Creation of the mathematical (symbolic) model.
3. Data collection.
4. Creating the computer program.
5. Validation.
6. Experimental design and experimentation.
7. Data analysis.

Formulation of the problem

This should be as detailed as possible in order to give as much direction to the following steps as possible. Study of the example provided later will give ideas on a typical approach.

The first step in model building should be to construct a *flow diagram* of the system. As the model is developed, this diagram would probably be changed and improved. The diagram enables the whole system to be visualized in its entirety and as a result tends to suggest areas that should be included in the overall model, which might otherwise be initially overlooked.

A flow diagram consists of a number of decision-making and event-occurring 'boxes', linked together by arrows indicating the relationship that exists. For example, consider the beef fattening feedlot as depicted in Fig. 14.1.

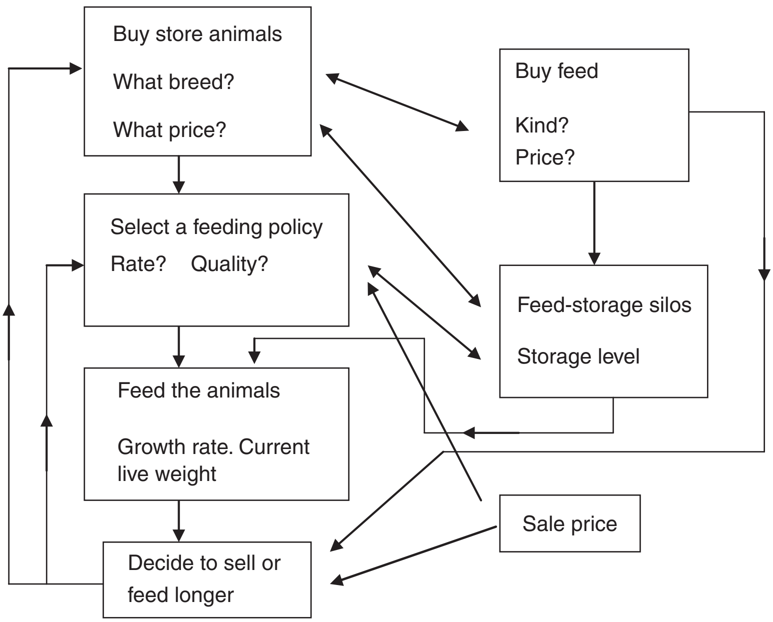


Fig. 14.1. A flow diagram for a systems simulation model – an example for beef production.

The second step is to quantify all the relationships in a mathematical model as described in the next section. This may involve going back and changing the flow diagram as more is learnt about the system. Eventually, the first model is completed. In general, this would not occur until after some trial runs with the model.

Creation of the mathematical model

This is usually the most difficult task and is where all the basic thought must go. Once the model has been developed, the nature of the results are largely determined.

Modelling is in part an experimental process. Given a problem, the first suggested models tend to be incorrect. Given more research and reading the model becomes closer to reality. Construction of the model is linked to data collection, as it is obviously of no use constructing a model that cannot be used due to lack of data. Model construction also tends to be a learning process. Faced with a problem, the analyst has initial ideas on the structure of the model, but as data relationships and problems become evident (e.g. the effect of feed quality on animal intake), the analyst obtains a better idea of the structure of the problem and how to solve it.

Some of the decisions that have to be made include:

- Which variables are going to be determined outside the model (e.g. the decision variables, the prices received, the costs paid) and which will be determined by the model (e.g. the quantity of dressed meat produced, the wool production/head etc.). This latter variable could well be supplied as given data rather than determined by the model on the basis of such variables as the feed input.

- Which variables are going to be assumed to be constants and which random variables?
- How detailed and complex should the model be? That is, which simplifications are included? This must depend on the expected increase in accuracy resulting from a more detailed analysis. Complexity for its own sake is of no use.

A major area of model construction is determining the relationships tying together all the variables. These will usually be obtained from specialists in the various physical sciences (animals, plants, etc.), though in some cases they may have to be derived from the results of plot trials. For example, consider the relationship between pasture growth and irrigation water applied. Physical scientists tend to report the results of such experiments in terms of total water used and total increase in dry matter production. For detailed management systems it may be necessary to have the relationship between water applied in each 2-weekly period and pasture growth. (These are probably non-linear relationships.) In order to do this, the basic experimental data may have to be obtained.

Data collection

Data collection methods and problems depend on whether the simulation study is stochastic or non-stochastic.

In the non-stochastic models, estimates must be made of all the variables determined outside the model. Thus, estimates must be made of the prices, costs and all the technical relationships not determined by the model itself such as, perhaps, the growth rates expected given a range of different feeding methods. (This same procedure must be gone through by all planners whether it is for budgeting studies or any other kind of study.)

In the stochastic case the data collection becomes more difficult and complex. Estimates must be made of the distributions of all random variables that are determined outside the model. The simplest situation is where subjective probabilities are being used, though in determining these some thought should be given to probabilities determined using historical data.

The steps which must be followed in estimating distributions from historical data are:

1. Collect the information.
2. Remove any trends affecting the data. For example, in attempting to determine the wool price distribution, many of the changes from period to period are due to supply and demand effects rather than simple random disturbances. These effects must be removed so that the observations only indicate the effect of the simple random disturbances. This could be done by determining

the average change in price from one period to the next and deducting this change from the observations. Similarly, the observations would have to be deflated to a common point *before* any trends are removed.

3. *Test to see whether any autocorrelation exists*, i.e. test to see whether there is a correlation between successive observations. For example, it is likely that the pasture growth during a particular week will be, in part, dependent on the pasture growth during the preceding week. In other words the variable pasture growth/week is autocorrelated. The reason for testing for autocorrelation is that when data are generated in a Monte Carlo simulation a particular value selection cannot be made independently of the previous selection if autocorrelation exists. For example, in the pasture growth example, when selecting this week's growth the figure generated for last week would form the basis of the estimate. Thus, 200 kg of dry matter (DM) might have been last week's growth, so this week's figure would be determined by randomly selecting a correction term and adding or subtracting this to/from the 200 kg. This assures that the current week's growth is determined by:

kg of DM = last week's production ± random part

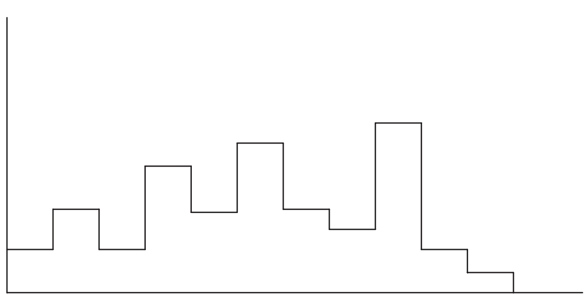
In reality, there would also be other factors affecting growth.

4. *Test to see whether there is any correlation between the random variables*. For example, two random variables in a simulation model of a mixed cropping farming system will probably be wheat and barley yield/ha. These two variables tend to be highly correlated so the values generated for a particular trial of the system, to be realistic, must allow for this. Thus, given that a wheat yield has been generated, a barley yield would be generated using the wheat yield as a base:

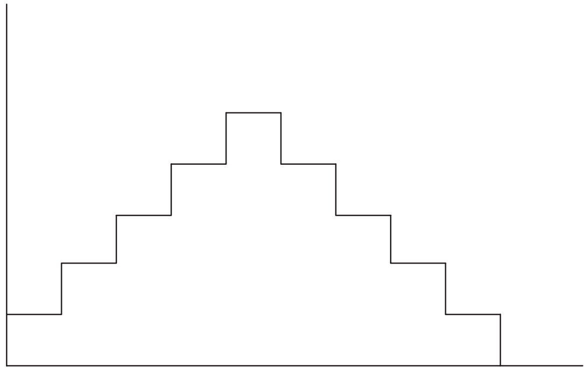
barley yield = wheat yield ± randomly selected value

The size of the randomly selected value would depend in part on the degree of correlation between the variables.

5. *Distribution smoothing*. Where the historical data give rise to an oddly shaped and/or irregular distribution, there may be reasonable grounds for smoothing the distribution (using techniques devised for doing this) before data generation. For example, assume that a spring rainfall distribution is being estimated for some area (this might be necessary to determine spring pasture growth). Meteorological records for the specific area might be limited and give a frequency distribution of the following form:



However, there may be a large number of observations for surrounding areas and these might give a regular distribution:



This would suggest that the distribution for the specific area of interest should be smoothed before use.

Developing a computer program

As simulation model experimentation usually involves a large number of calculations it is necessary to use a computer, particularly for Monte Carlo simulations. To control and automate the calculations, which will follow the flow diagram of the problem, a computer program will need to be written using a standard computer language, or possibly one of the packages specifically developed for simulation. A search of the World Wide Web will quickly provide details of the latest packages available. For inexperienced analysts it pays to seek advice on an appropriate package for the particular problem to hand.

If the analyst cannot efficiently write computer programs, which will often be the case unless he/she has had special programming courses, it will be necessary to employ a programmer to prepare the necessary computer code from the developed flow diagram and mathematical model. This part of the operation is somewhat mechanical with the main work being in creating the model concept and working out all the relationships between the variables as well as searching out the data.

Validation

Before using a model it must be tested to ensure a realistic representation of the real world. This process is known as *validation*. (This applies to all models, not just simulation models, ranging from simple budget models to complicated stochastic dynamic programming models.) The problem is that validation is seldom simple, as models are usually developed for systems for which little empirical data are available (if the data were available there would be no need to build the simulation model).

Some of the *suggested procedures* are listed below (which ones can be used depend on the particular situation):

- *Compare the model output with real-world farm results*, where the model output is determined using the input values actually used on farms. For example, consider the beef fattening feedlot example. Given records of profit/beast for a real farm, the simulation model can be used to estimate the profit/beast where the same input values, feeding policies and prices as experienced by the farm are used in the model and then comparisons made. These comparisons tend to generally validate the model but not necessarily validate it for conditions under which it will be used (the model is likely to be used to explore systems that have not been extensively used in practice. Otherwise, there is little advantage in developing the model).
- *Compare parts of the model with actual farm results, plot trial results and experimental results*. This is the same as above but rather than use the final output from the model the comparison is broken down to constituent parts. For example, in the beef fattening example, physical growth rates for a defined feeding system might be compared. In a grazing system model, the pasture production might be compared rather than the final profit figures. This approach will be necessary where there are no actual farm records available for the whole system. This is likely, as the model will be used to explore new systems.
- Conceptually, provided only proven fundamental physical relationships are used in the model, the model should not require further validation. The validation in this situation *occurs through selecting only proven relationships* (e.g. relationships between animal live weight and maintenance requirements are generally considered to be proven). In practice, this approach is seldom possible.
- *Ask the relevant experts* whether the model output complies with what their general experience would suggest the results should be. For example, given a model of irrigated lucerne and beef fattening, ask an agronomist whether the lucerne production figures compare with what would be expected; similarly, for the animal production part of the model. As the models will be used to explore new systems, this subjective-type validation is usually important as actual comparisons will not be available.

Model validation is part of the overall model development process. As a result of the validation exercises the model may need to be adjusted and the whole process repeated.

Experimental design and experimentation

As systems simulation is a special form of experimentation, experimental design is just as important as it is in physical trials. By *experimental design* is meant the choice of: (i) the decision variable values for which the system should be evaluated; and (ii) in stochastic models, the number of trials that should be conducted for each set of decision variable values in order to obtain meaningful results.

Consider a non-stochastic model. The object is to develop an improved management system. Thus, the profit is estimated for different combinations and values of the decision variables (alternative systems). However, it is usually impossible to evaluate all combinations, so a decision must be made over the combinations to evaluate. For example, which feeding policies should be evaluated in the beef fattening example? Where an 'optimal' policy is required in contrast to simply exploring a particular farming system, the usual approach is initially to obtain profit estimates for markedly different decision variable combinations and values. The results suggest the area of the most efficient system, so a more detailed analysis using small decision variable value changes can then be carried out around what appears to be a good system.

Consider a stochastic model. As with the non-stochastic case a decision must be made regarding the decision variable value combinations to evaluate. Further, however, the number of trials to carry out *with each* set of decision variable values must be determined. If insufficient trials are carried out: (i) the profit distribution obtained will be an incorrect estimate of the true distribution; and/or (ii) it will be impossible to decide whether the different systems evaluated are significantly different (difference in the average profit is not sufficient justification for stating the systems are different in a profit sense).

For example, in the beef fattening case, how many trials should be carried out for one of the possible feeding, buying and selling policies to reflect a true indication of the profit distribution?

To answer the questions discussed above, a knowledge of statistics and experimental design is necessary. Many analysts may need to seek expert advice in this area. Generally, however, the greater the variance in the constituent random variables, the greater number of replicates will be required to obtain a true picture of the output distributions. If only a few experiments are carried out, the differences between policies may well be due to chance in contrast to the different farm systems.

Data analysis

From the model evaluations, the usual procedure is to rank the alternative systems (sets of decision variable values). In the stochastic case this will involve using statistical methods to test for significant differences (analysis of variance). In addition, in some cases regression analysis may be used to summarize the systems model data output. Consider, for example, a model designed to determine pasture production as a result of fertilizer and irrigation levels (the decision variables). The output from such a model will be yield figures for each input combination. Using these data a relationship of the form:

$$\text{Yield} = f(\text{N, P, K, H}_2\text{O})$$

can be estimated. This function can then be used to assist decision making (the data from the simulation model is directly analogous to the data obtained from field plot trials designed to achieve much the same results) once combined with price and cost information. In this example, the systems simulation model has enabled a pasture output production function to be estimated.

As with experimental design, expert advice in this area is often necessary to ensure significant and useful equations are devised. Of course, this potential output should be in the mind of the analyst when first planning the model.

14.8 Concluding Comments on the Methods of Simulation

The discussion on simulation methodology has tended to be quite general and has not provided a set of definite rules that should be followed in simulation studies. The reason for this is threefold: (i) no such set of rules exists, as the method is a general approach. Each problem must have a model specifically designed for the particular case; (ii) system simulation draws upon many fields, so that a discussion of simulation must rely on a good knowledge of other areas besides management science, operations research and economics; and (iii) the discussion is intended to provide a general introduction to the ideas and problems rather than equip an analyst with all the skills necessary for a systems simulation study.

14.9 An Example of a Systems Simulation Study

The case of irrigation decision rules

Introduction

As noted, the best way to understand systems simulation is to consider examples. Most texts on systems simulation largely consist of case studies. Given that this chapter is intended to provide an introduction to systems analysis a single case study will suffice, though it does cover many of the problems encountered in simulation.

The study covers the development of irrigation decision rules for irrigating a particular cash crop. While the crop in the example is cotton, the principles discussed apply in general to all cash crops (details of this example study were taken from Nuthall, 1972).

The problem

When an irrigated crop is grown, two important irrigation decisions are:

1. At what times should the crop be irrigated?
2. How much water should be applied each time the crop is irrigated?

Note that in some cases the irrigation system, particularly communal systems, *may dictate* how much water must be applied at each irrigation and when irrigation can occur. It will be noted that the irrigation decision problem is not simply a matter of deciding how much water in total to apply to a crop.

To decide when to irrigate, the logical variable to use is the soil moisture level as plant growth is directly related to soil moisture. Once soil moisture falls to a certain level the crop should be irrigated if profit is to be maximized.

The problem, however, is somewhat more complicated than this.

1. Irrigation decisions for a particular crop cannot be made independently of other crops due to the opportunity costs of the water.
2. Irrigation cannot be decided without considering limitations on water quantities. Many irrigators have two constraints on water use: (i) a limit on total water use in one season (e.g. a community Water Board Restriction); and (ii) a limit on the amount of water that can be applied in any one irrigation (e.g. due to pumping facility limitations, flow rate limitations etc.).
3. As crops respond differently to water at different stages of growth, the soil moisture level at which irrigation should occur will probably vary throughout the season. Thus, for example, it is more important to apply water frequently during grain filling in most crops.

The objective, therefore, is to develop a systems simulation model that will indicate the effect of *varying the soil moisture level* at which irrigation water is applied on crop yield and variable costs and at the same time account for the opportunity cost of the resources concerned (including water) and any constraints on water use.

The method

The method used estimated the shape of the crop yield/irrigation water production function. Using this production function the optimal irrigation policy was determined using calculus, given the relevant cost and price data.

The first step was therefore to develop a model that enabled crop yield to be determined given a particular irrigation policy. This was constructed to allow different values of the following variables: (i) the total water available; and (ii) the maximum amount of water that could be applied at each irrigation.

The second step was to use the developed model to predict the crop yield for many different combinations of the irrigation decision variable (soil moisture level at which to irrigate). This was repeated for a range of total water and water per irrigation constraint values, as on any particular farm a decision is required on how much water to allocate to each crop.

A stochastic approach was taken as decision-making estimates of the gross margin variance per hectare were required, i.e. irrigators may well have a non-linear utility function so that information on more than just the expected returns of alternative policies were required. To obtain estimates of the mean and variance, repeated runs of the simulation process were carried out *for each* alternative irrigation policy using a series of randomly generated data.

As the gross margin per hectare depends on the yield, the amount of water used (and thus irrigation water costs) and the number of irrigations carried out (and thus the labour costs), the relationship between these variables and the irrigation decision variable had to be estimated. This was achieved through the results of the simulation model. Each run of the model estimated the total water use and the number of irrigations performed. Similarly, in order to estimate the gross margin variance, the relationship between the irrigation decision variable and the following variables had to be determined:

- the standard deviation of yield;
- the standard deviation of the total water used;
- the standard deviation of the number of irrigations;
- the covariances between all these variables.

These parameters were estimated from the results of repeated simulation trials for each irrigation policy. This enabled quantifying how each variable changed as the decision variable was varied in subsequent trials.

It was assumed that fertilizer costs, insecticide, herbicide and other crop costs were held constant. Thus, these costs were ignored in the study. You could argue, however, that fertilizer should also be included as there is likely to be an interaction between water and fertilizer, particularly nitrogen (one of the models does include N application). Effectively, the model presented assumes water is the most limiting resource.

The simulation model

Introduction

As the model was constructed in a number of stages it is similarly discussed in these stages: (i) consider the amount of water used by the crop; (ii) develop an equation recording the soil moisture level throughout the season (water budget); (iii) develop the relationship between soil moisture and crop yield; and (iv) construct a model incorporating all these relationships enabling the prediction of yield.

Crop water use

A plant requires water to transport nutrients, as an ingredient to some of the biochemical processes and as a cell component. Consequently water availability affects the growth and production of plants. For maximum growth a plant must have access to sufficient water to allow it to freely transpire. This potential rate of water transpiration is determined by the atmospheric conditions existing on any one day. If the soil moisture level is below a certain level, the plant will not be able to transpire at this potential rate so growth is below the maximum possible. Where the potential is not attained the plant is said to be *stressed*. If stress occurs, the yield may fall.

Moisture loss from a soil-crop system occurs not only through leaf transpiration but also as evaporation from the soil surface. Thus, rather than estimate simply transpiration, estimates must be made of both evaporation and transpiration losses. These are collectively referred to as *evapotranspiration*. Thus, a crop exhibits a *potential evapotranspiration*, but whether this can be attained depends on the soil moisture level. The actual water loss is referred to as *actual evapotranspiration*. The difference between potential and actual evapotranspiration gives rise to a measure of water stress. The amount of water stress affects eventual yield.

To estimate water use, the relationship giving potential evapotranspiration must be determined. By searching through various agronomic journals research relating water use to atmospheric conditions was found. Using these data the following relationships for the crop were estimated:

$$\text{Potential Evapotranspiration } (E_t) = (0.387 + 0.103T - 0.005T^2)E$$

where (1 inch = 25 mm):

E_t is measured in inches of water per period;

T = age of the crop in weeks;

E = the evaporation from a free water surface in inches of water per period.

As would be expected, E_t depends on the age (stage of growth) of the crop.

$$\text{Actual Evapotranspiration } (E_a) = (0.053 + 0.295Q - 0.023Q^2)E_t$$

where:

E_a is in inches of water per period;

Q = the soil moisture level in inches of *available* water (i.e. the quantity of water which, if removed, would leave the soil at wilting point).

Summarizing, with these two relationships (and information on evaporation from a free water surface over a defined period) and the soil moisture level and the crop age, an estimate can be made of the moisture lost from the soil (i.e. E_a). An estimate can also be made of the amount of stress ($E_t - E_a$).

The water budget

In order to estimate stress over a period, information must be available on the level of soil moisture.

It will be noted that all relationships defined have estimates of water use for a non-defined period. For complete accuracy, the simulation model should be based on daily periods (or even hourly). Given daily estimates of evaporation for a whole growing season the *total* potential evapotranspiration can be estimated. However, if daily periods are used the number of calculations required would be extremely large. Thus, as an approximation, the simulation model was based on weekly periods.

Returning to the soil moisture-level problem, where weekly periods are used, the soil moisture level at the end of a week will be given by:

$$SM_i = SM_{i-1} + I_i + R_i - O_i - P_i - E_i$$

where,

SM_i = soil moisture level at the end of week i (in inches of available water).

I_i = irrigation water applied during week i (inches).

R_i = effective rainfall during week i (inches).
0_i = runoff during week i (inches).
P_i = deep percolation during week i (inches).
E_i = actual crop evapotranspiration during week i (inches), based on a soil moisture level of: $SM_{i-1} + I_i + R_i - O_i - P_i$

Given this equation and values for the variables throughout a growing season, the soil moisture level for each week can be traced and thus the *actual* evapotranspiration determined. This leads to an estimate of the stress which occurs during each week.

The method of determining some of the above variables requires some explanation. The rainfall figures were obtained from meteorological data (discussed in detail later) and the irrigation water applied was set at a defined figure depending on the value of the decision variable (discussed later). The deep percolation was estimated using the relationship:

$$P_i = SM_{i-1} + I_i + R_i - O_i - 7$$

where 7 = the total available water holding capacity of the soil in inches.

When P_i was negative, it was set equal to zero, and when P_i was positive, it was set equal to the value given by the equation.

To estimate the runoff (O_i) during a week it was necessary to have the soil water infiltration rate, as if the rain fell fast enough infiltration would not cope and runoff take place. Through searching a number of agronomic and soils journals, information was obtained which gave the following relationships for the soils in the area studied:

$$Y = 3.269 - 0.979Q + 0.076Q^2$$

where,

Y = infiltration rate in inches per hour;
Q = soil moisture level in inches of available water (max. Q = 7 inches).

Given R_i and *the rate* at which the rainfall occurred, an estimate could be made of the water lost through runoff during a particular week. Thus:

$$O_i = R_i - \left(\frac{R_i}{D_i}\right)Y$$

where,

D_i = the rate at which the rainfall occurred in inches per hour.

$\frac{R_i}{D_i}$ = the number of hours over which rainfall occurred.

If this equation gave a negative value, O_i was set equal to zero.

Summarizing, given last week's soil moisture level and values for the following variables:

- rainfall during the week,
- the rate at which the rain fell,
- the irrigation water applied during the week,
- the evaporation from a free water surface during the week and

- the age of the crop,

then the following variables can be estimated:

- potential evapotranspiration for the week (inches),
- actual evapotranspiration for the week (inches),
- plant water stress for the week (inches) and
- the soil moisture level for the start of the next week.

Crop yields and soil moisture levels

Background

Yield is a function of water stress, and water stress a function of soil moisture. Thus, to relate soil moisture and yield a relationship between stress and yield must be estimated so the model developed in the preceding section allows predicting stress throughout a growing season.

As a relationship between stress and yield was not available it had to be developed through information obtained from many small paddock (field) crop trials over many years. These trials were not specifically for this purpose but sufficient information was available for the necessary deductions. For each trial, irrigation records had been kept and daily observations on rainfall, rate of rainfall and free water surface evaporation were available. Yields were also recorded.

Using the water budget approach, the daily stress experienced by the plants on each trial was estimated giving many hundreds of observations on stress levels at different crop stages and the resultant yields. These observations gave the following yield/stress relationship (using regression analysis):

$$Y = 2651.54 - 810.1X + 778X_{22} - 427.7X_{31} - 1779.9X_{32} + 317.9X_{11}X_{21} + 241.1X_{11}X_{31} - 924.3X_{11}X_{41} - 167.9X_{21}X_{31} + 537.7X_{21}X_{41} + 8.8\,N$$

where Y = crop yield per acre (lbs);

X_{11} = the sum of weekly stress for weeks in which stress was > 0.15 inches up to the stage where the first flower buds appeared;

X_{22} = the sum of weekly stress for weeks in which stress was ≤ 0.25 inches for the period from the appearance of the first open flowers;

X_{31} = the sum of weekly stress for weeks in which stress was > 0.15 inches for the period from the appearance of open flowers to the appearance of the first seed heads;

X_{32} = the same as X_{31} except for weeks in which stress was ≤ 0.15 inches;

X_{41} = the sum of weekly stress for weeks in which stress was > 0.15 inches for the period from first seed heads to harvest;

N = lbs of elemental nitrogen applied.

The prediction of yield

All relationships necessary for predicting yield have now been established. Given weekly observations on rainfall, rate of rainfall, evaporation from a free surface and irrigation applied, the weekly stress in inches of water can be estimated. Given these stress estimates they can be used to predict yield using the yield/stress equation.

The next step was to bring all the relationships together into a detailed, workable set of calculational steps amenable to computer operation. (Before such a model could be used, however, information on weekly rainfall and the other variables had to be obtained and decisions made about which alternative irrigation policies were to be explored using the model. These problems will be discussed later.) One of the problems still to be resolved is that of how the model can itself determine when to apply irrigation water.

The determination of irrigation water applied

The crop growing season can be divided into four periods: (i) from planting to first flower buds (6 weeks); (ii) from first flower buds to first open flowers (4 weeks); (iii) from the first open flowers to first seed heads (8 weeks); and (iv) from first seed heads to harvest (4 weeks), giving a total season of 22 weeks.

To ensure crop establishment it is necessary to irrigate before planting so that the soil is at field capacity. Given this pre-planting irrigation it is usually not necessary to irrigate in the first 6 weeks as the plants, being small, have low evapotranspiration. Thus, it was assumed irrigation water is never applied during the first 6 weeks. Similarly, it was assumed water was never applied during the last 4 weeks prior to harvest as encouraging growth adversely affects the quality of the crop and any increase in yield is more than offset by a decrease in the price received due to the low quality.

The irrigation period is consequently weeks 7 to 18, inclusive. Given these 12 weeks it could well be profitable to have a different policy in each (strictly, rather than talk about crop age the stage of growth should be used to describe development). However, there is an extremely large number of irrigation policy combinations if 12 periods are used. For example, if four different policies are recognized and can be used in any one week, the number of combinations for the whole season is 4^{12}. For this reason it was decided to divide the 12 weeks into two periods. These periods coincided with the crop growth stages, so one policy might be used over weeks 7 to 10 inclusive and possibly a different policy in weeks 11 to 18 inclusive. This reduced the number of simulation experiments required.

An irrigation policy is, of course, the soil moisture level at which water is applied. Thus, for example, the policy might be to irrigate once soil moisture drops to 50% of the soil water holding capacity. Given the two periods, an irrigation policy for the whole year is a decision to irrigate at defined soil moisture levels in each of the periods. As the holding capacity of the soil in question was 7 inches, a policy might be to, say, irrigate once available water dropped to 3 inches over weeks 7 to 10, and to 4 inches in weeks 11 to 18. The weeks in which irrigation will occur is then determined by monitoring soil moisture levels week by week.

The amount of water to apply at each irrigation is still to be determined. In most flood-type irrigation schemes (this was the type of scheme in the area of interest) there is a limit to how much water can be applied in one irrigation depending on the design of the system. Thus, it was assumed that when irrigation occurred sufficient was applied to bring the soil to field capacity. This could have been varied by applying sufficient to bring the soil to varying proportions of field capacity. To explore this option many more simulations would have been required according to the number of levels experimented with.

There is usually water wastage in any scheme. The quantity of water pumped from a bore or entering the farm gate in a race seldom equals the quantity entering the soil, due to evaporation losses, race seepage losses and the like. While this loss is a random variable, for simplicity it was assumed it occurred at a constant 25% of total water received. Thus, to get 3 inches of water entering the soil

$$3 \times \frac{1}{0.75} = 4 \text{ inches}$$

must be received and paid for.

It can now be seen how the value of I_i (irrigation water applied per week) can be determined. Given that a decision has been made regarding how much water to apply per irrigation (this depends on the irrigation scheme available and can be varied in the analysis), I_i equals this amount multiplied by $\frac{1}{0.75}$ (loss factor) whenever soil moisture at the end of the preceding week (SM_{i-1}) drops to a pre-defined level. There is, however, one more factor to be considered when estimating I_i. This is the question of whether there is sufficient water available to be able to use the amount I_i.

Recall that in this problem, there is a limit to how much water can be used in any one season, and also applied in any one irrigation (this will depend in part on the area to be watered at much the same time and the rate at which water is received). Thus, if all the water has already been used, even though the soil moisture level is such that I_i should be set at a positive level, I_i must be set equal to zero. Similarly, if it is not possible to apply all of I_i due to rate of availability constraints, I_i must be set as near to the originally set value of I_i as the flow rate permits.

The flow diagram and its equivalent statement form

One of the first steps in developing any model should be to develop an overall diagram of the system. In this description of the problem and its solution this step has been left until this stage as some preliminary discussion was necessary. The flow diagram should now be self-explanatory and is presented in Fig. 14.2.

The simulation model can now be written out as a series of steps to be sequentially carried out. This will be simplified compared with the system actually used in order to give a general idea. Assume that information is available on the weekly rainfall, rate at which this falls and the free water surface evaporation:

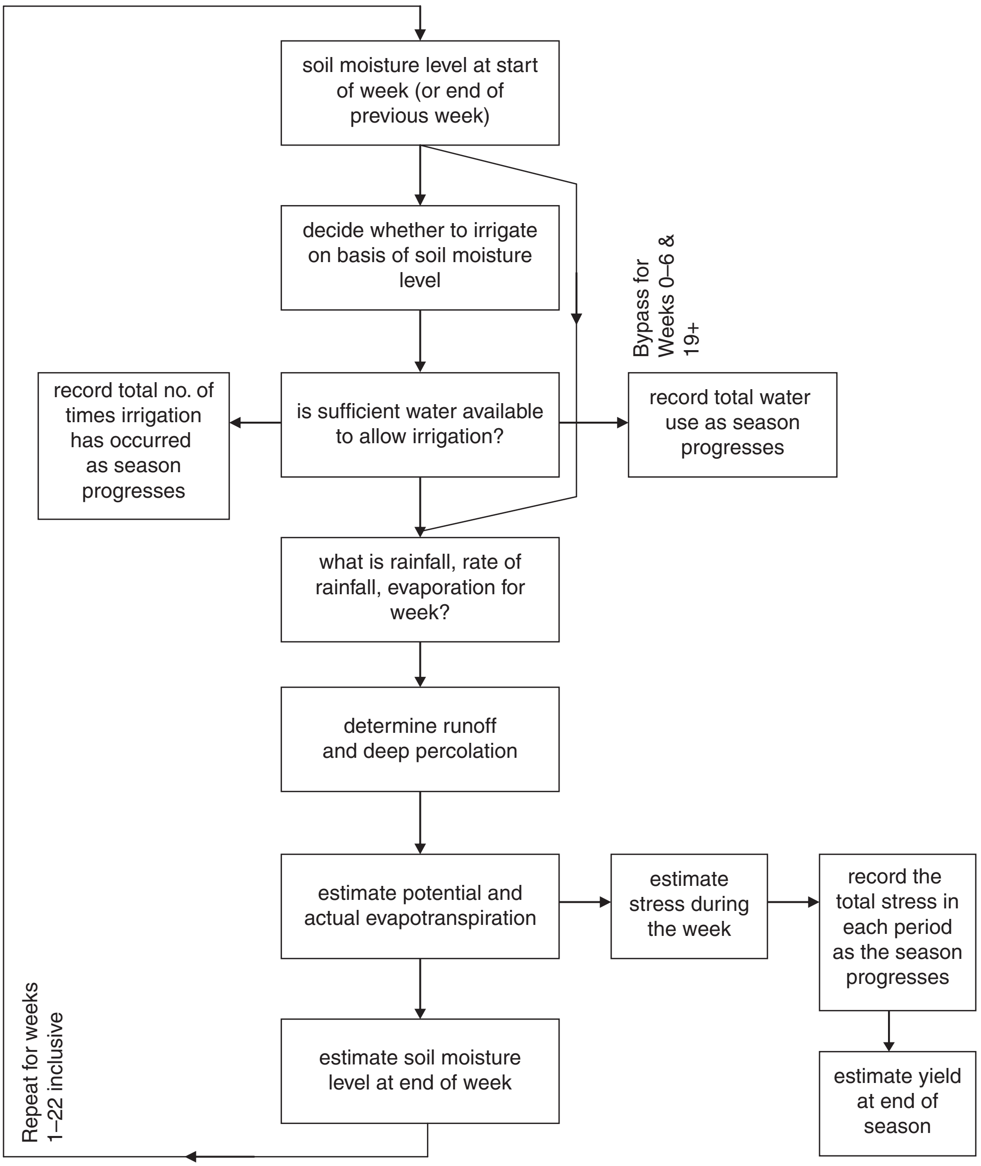


Fig. 14.2. Diagram of a crop yield simulation model.

1. If the week no. is one of 1–6 inclusive, or 19–22 inclusive, proceed to step 5, thus missing out the calculation of irrigation water to be applied.

If the week number is 7–18 inclusive, proceed to next step.

2. If SM_{i-1} (soil moisture level at end of last week) is less than the soil moisture level set as being the level at which to irrigate, provisionally set I_i equal to the maximum quantity of water than can be applied per irrigation given the particular irrigation scheme.

3. Adjust I_i *if* there is not sufficient water left to permit the quantity I_i to be applied. Set it equal to the maximum quantity that can be applied given the quantity of water still available for application. *Similarly*, adjust I_i if the application of I_i will put soil moisture above field capacity (7 inches). In this case, set I_i equal to the quantity necessary to bring the soil to field capacity.

4. (a) Record how much water has been used during the season up to and including this particular week. (This is necessary to calculate step 3 above for the following week.)
(b) Record how many irrigations have occurred up to and including this week.

5. Note rainfall, rate of rainfall and evaporation for the particular week.

6. Calculate runoff (O_i) for the week.

7. Calculate deep percolation (P_i) for the week.

8. Calculate potential evapotranspiration (E_{ti}) for the week.

9. Calculate actual evapotranspiration (E_i) for the week.

10. Calculate stress occurring during the week.
Let stress in week $i = S_i$
Therefore $S_i = E_{ti} - E_i$

11. Record the total stress occurring within *each* period.
Let TS_i = total stress to date. Thus,
$TS_i = TS_{t-1} + S_i$
(In practice, different levels of stress have to be recorded as separate sums rather than joined to form one TS as suggested above.)

12. Calculate SM_i
$SM_i = SM_{i-1} + I_i + R_i - O_i - P_i - E_i$
If the week is 1, set $SM_o = 7.00$, it being assumed that the soil is at field capacity at the start of the season.

13. Return to step 1 if week 22 has not been reached (thus, the whole process is repeated 22 times.) If week 22 has been reached, proceed to step 14 below.

14. Using the total stress figures (TS), calculate the crop yield for the season.

15. Stop calculating if no more seasons are to be simulated. Otherwise, repeat the whole process for another season.

As this process follows a logical structure it can easily be performed using a computer.

Determining the input data

Introduction

Estimates of the variances and average values of the yield, the amount of water used and the number of irrigation events are all required for each alternative irrigation policy. Clearly many simulations were necessary allowing for replicates and a range of seasons based on the probabilities.

As noted in the methodology section, two approaches to obtaining data are possible. Either use a series of historic data to obtain the repeated trials for each irrigation policy, or generate data using the Monte Carlo method. The latter approach was used, as it was decided the historic data did not provide a sufficient number of sequences of rainfall and the other variables. However, the historic data were used to indicate the nature of the distributions and these were then used to generate new sequences of data. (Personal subjective probabilities were not used as the results were intended to be of use to many farmers in the area.)

The rainfall data

Hourly observations from meteorological stations in the area were collected for as many years as records had been kept. These were used to estimate the weekly rainfall and the average rate of rainfall (inches/h) for each week for each year.

Consider the weekly rainfall figures. These were tested to see if there was any correlation between rainfall in one week and rainfall in any of the succeeding weeks. This was not the case, so when the data were generated using the Monte Carlo method rainfall in a particular week could be simulated independently of the rainfall in the preceding week. The weekly figures were then used to develop a rainfall probability distribution for each week. These required smoothing.

Similarly, the weekly rate of rainfall data proved to have no autocorrelation, i.e. succeeding weeks are independent. However, it was found that the correlation coefficient between the quantity of rain in one week and the rate at which this rain fell was quite high. This meant the weekly rate of rainfall figure could not be generated independently of the quantity of rain. Thus, once a week's rainfall figure was generated, the rate of rainfall level was calculated from the weekly rainfall plus or minus a randomly selected factor. The randomly selected factor was estimated from smoothed weekly distributions based on the weekly rate of rainfall observations after making adjustments for the correlations.

The evaporation data

There were no meteorological stations in the area that had been recording free water surface evaporation for a large number of years. Those in the area had a number of monthly observations rather than the required weekly observations. Thus, extrapolation was required. Records were obtained from the nearest stations with a large number of years' observations on weekly evaporation distribution. It was found (using chi-square tests) that the distributions were normal. It was therefore assumed that the area of interest's evaporation distributions would also be normal. The monthly observations that were available were used to estimate the mean and variances for these distributions (these being a normal distribution's only parameters). By observing the trend from month to month in the means, and assuming the same basic trend would occur from week to week, the weekly distributions were derived.

The data were tested for correlation with the rainfall, and rate of rainfall, and found to be independent. However, autocorrelation (the evaporation in one week was correlated with evaporation in the following week) was found to be present. Thus, when generating the evaporation data for a particular week this was based on the preceding week's level plus or minus a randomly selected value in much the same way as the rate of rainfall data were generated from the rainfall data.

The number of simulations and the experimental process

It was decided (largely because of the computing funds available rather than on a good experimental design basis) to estimate the crop yield 60 times for each irrigation policy. This meant that the Monte Carlo process had to be used to generate weekly data for 60 seasons for each of the input variables. This was done using the distributions discussed above.

Before proceeding, a decision had to be made regarding which irrigation policies to evaluate. There is obviously no point in evaluating all alternative policies as the results from a few enable conclusions to be made about others. Thus, a range of quite different policies was selected. As early results were obtained, the policies that were evaluated were changed slightly as the results suggested other likely policies.

Each set of irrigation policy experiments was repeated many times with a different assumption about the total water available per season and the rate at which this would be applied (max. quantity/irrigation). This was necessary to ensure that the results would apply to many different farm situations.

The results for a typical situation

General

For each irrigation policy, total water availability and maximum quantity/irrigation situation, 60 yield estimates were available. These were divided into four sets of 15 simulated observations (replicates). For each set of 15 the mean and variance of all the variables were estimated. Using these estimates regression analysis was used to determine the relationship between the soil moisture level at which irrigation occurred and the average yield, average water used and so on.

To provide the general form of the relationships, rather than all the actual results, some are presented below (details are in Nuthall, 1972). The terms used are:

W_1 = total water available for use in weeks 7–10 inclusive;

W_2 = total water available for use in weeks 11–18 inclusive;

A_1 = maximum quantity of water than can be applied per week in weeks 7–10 inclusive;

A_2 = maximum quantity of water that can be applied per week in weeks 11–18 inclusive;

D_1 = available soil moisture level at which irrigation is set to occur in weeks 7–10 inclusive;

D_2 = available soil moisture level at which irrigation is set to occur in weeks 11–18 inclusive;

$$E(\text{Yield})\ (\text{lbs/acre}) = 1497 + 97W_2 + 115D_2 - 7.7W_2^2 + 6.2W_1W_2 + 2.3A_1D_1 + 2.8D_1W_1 - 7.5D_2W_1 + 8.6D_2W_2$$

$$E(\text{Water used})(\text{in}) = 0.65W_1 + 0.27W_2 - 0.4W_1W_2 + 0.1D_1\,D_2 + 0.1D_1\,W_1 - 0.05D_1\,W_2 + 0.02D_2\,A_2 - 0.08D_2\,W_1$$

E(Number of irrigations)

$$= 0.47W_1 - 0.11A_2 + 0.366D_1 + 0.318D_2t + 0.004A_1^2 - 0.02W_1 - 0.043D_2W_1 + 0.042D_2W_2 - 0.002A_2W_1$$

Variance (Yield) = 374^2 (was independent of policy);

Variance (Water used)

$$= (0.05A_2 + 0.17W_1 + 0.29W_2 - 0.01W_2^2 - 0.05D_2^2 + 0.03D_1D_2 - 0.01D_1A_2 + 0.02D_2W_2^2)^2$$

Variance (No. of irrigations)

$$= (0.2W_1 + 0.25D_1 - 0.02W_1^2 + 0.01W_2^2 - 0.04D_1^2 + 0.02D_1W_1 - 0.01D_1W_2 - 0.001A_1W_1)^2$$

Covariance (yield, water used) $= 22.1D_2^2 - 157D_2 - 3.5D_2 + 0.5A_1W_2$ (and similarly for all the other covariance terms).

Using the results

AVERAGE RETURNS Given a price for the crop, and the opportunity cost of 1 inch of irrigation water and the labour required to carry out one irrigation, the equations giving the expected yield, water use and number of irrigations (i.e. labour use) can be used to develop a gross profit equation:

E(Gross Profit) = (Price/lb) (E(Yield)function) – (Water cost/inch) × (E(water used) function) – (Labour cost) × (E(no of irrigations) function) × (no. of hours required per irrigation)

Given values for W_1, W_2, A_1 and A_2 for a particular farm, calculus can be used to estimate the values of D_1 and D_2 (the decision variables – soil moisture levels at which to irrigate), which maximize the gross profit. Using typical prices and water availability the optimal policy was to set D_1 to 0, and D_2 to

7 inches (weekly irrigations), which gave profit per acre of $279 with an expected yield of 3135 lbs. The marginal cost of increasing D_1 above zero was $0.24 whereas D_2 had a marginal return of $17.42 clearly showing the value of water in the second period. The relationships can be used to explore a whole range of critical values such as, for example, the price of water which would make applying water in the first period profitable. Other values for the W and A parameters, and the prices, would of course give a different optimal policy.

The variance of gross profit

Using the estimated variance and covariance relationships, an equation giving the variance of gross profit can be developed in a similar way to that used to estimate the expected gross profit. This will be, however, somewhat complicated so it is not presented here. Just how complex this is will depend on whether the prices and costs are also random variables. The results showed, for example, that under typical prices the standard deviation of profit was $40.20, which is relatively low and reflects the value of irrigation in reducing variance.

Simplifications

In describing and discussing the model some simplifications have been introduced. Some of these are no doubt evident. There are also many ways in which the model could be improved. The example, however, does give a good idea of a typical simulation study.

Concluding comments

The major comment must be that it should be clear just how complicated and time consuming simulation studies can be. It has reinforced the main use of simulation studies must be in developing management systems on representative farms that are typical of very large areas. For small areas of similarity, the expense of a simulation study is just not warranted in general. However, where a model can be set up to allow individual farm parameter values to be entered easily, simulators can be made available for individual farm use via the World Wide Web. These systems need to be very robust and self explanatory otherwise expert help is necessary to obtain benefit from their use.

Despite these cautionary words, it is clear that systems simulation can be extremely useful for situations where other analytical techniques cannot be used due to their assumptions being violated. This statement only holds, of course, provided the results will have extensive use. Otherwise it may be better to do a few comparative budgets. It must also be noted the literature contains the details of many simulation models – these should always be utilized as starting points in developing a model (see the references below).

References and Further Reading

Dent, J. and Anderson, J. (1971) *Systems Analysis in Agricultural Management*. Wiley, New York (early text on systems simulation).

Keating, B.A. *et al*. (2003) An overview of APSIM, a model designed for farming systems simulation. *European Journal of Agronomy* 18(3–4), 267–288 (a review of a collection of sub-models suitable for plant simulation).

Meinke, H., Baethgen, W.E., Carberry, P.S., Donatelli, M., Hammer, G.L., Selvaraju, R. and Stockle, C.O. (2001) Increasing profits and reducing risks in crop production using participatory systems simulation approaches. *Agricultural Systems* 70(2–3), 493–513 (general review of crop system simulations).

Nuthall, P.L. (1972) Irrigation decision rules for cotton growing in the Brookstead area of the Darling Downs. *Review of Marketing and Agricultural Economics* 40(3), 123–133 (full details of the example system described in the text).

15 The Structure and Analysis of Specific Part-Farm Problems

15.1 Introduction

The discussion in earlier chapters has covered analytical methods for whole-farm decision situations, though some can also apply to part-farm analyses. In contrast, this chapter will concentrate on specific part-farm problems and how they can be analysed. The emphasis lies on understanding the structure of the problems rather than developing detailed computational methods. Some of the analytical techniques already discussed can in fact be used to solve some of the problems as will become evident. Readers interested in developing more detailed models should refer to a modern operations research text.

As the number of decision problems on a farm are innumerable the approach is to cover a small range of the more important typical problems. An adviser/consultant/analyst will be able to use the ideas and methods introduced to set up models useful in answering more than just the problems covered.

The decision problems discussed are the inventory and replacement analysis problems. However, introductory comments will also be made about critical path and queuing analysis. These latter two are less important in farm settings compared to inventory and replacement analysis. As most primary producing properties hold inventories of various inputs and outputs it is important to work out an optimal inventory level. Similarly, a farm holds many assets that need replacing from time to time, so getting the timing right is important to ensuring least-cost farming. Critical path analysis is about organizing projects to provide cost and time efficiency, and queuing analysis is about providing facilities to ensure queued jobs are dealt with economically. While these latter problems are not a major source of decision difficulty on a farm, it is useful to be aware of their structure.

15.2 Inventory Analysis – Introduction

An *inventory* is a stock of goods usually held for the purposes of making a profit. Most firms hold inventories of one form or another, e.g. the corner grocer holds an inventory of soap powder, an oil refinery holds an inventory of crude oil, a farmer holds an inventory of fuel for the tractors, and of animal feed and so on. All these firms must make decisions on a range of problems directly related to this inventory. The major question is deciding the level of inventory, as it is costly to hold large quantities of resources sitting idle, and similarly costly if production or sales must be halted when stock is not on hand.

Inventory problems take many forms. This discussion is primarily concerned with the feed reserve problem faced by most stock farmers. Farmers have other inventories but on stock farms the feed problem tends to be the most significant profit wise. However, depending on the farm type, many other inventory problems can be important, e.g. grain farmers might be faced with significant grain-holding questions. In all cases, the same basic set of principles apply.

To round out the discussion, comments will be made about incorporating the feed inventory model into a whole-farm model. Feed inventory is about minimizing costs, yet this is not the overall farm objective. The problem should not be solved without recourse to the rest of the farm and the inter-relationships involved.

Before discussing the specifics of the feed problem, comments are made about inventory problems in general.

15.3 The General Inventory Problem

Inventories enable the smooth and efficient operation of a firm. An inventory may be necessary for a number of reasons as listed below.

1. To enable a firm to satisfy a fluctuating demand for a product. In this case stocks of the finished product may be held, or alternatively, stocks held of semi-finished products, which can be rapidly processed to meet the demand. For example, a turkey producer may hold an inventory of frozen turkeys to meet a fluctuating demand.
2. To enable a firm to overcome the problems of a fluctuating supply of an input. These goods might be completely, or partially, processed inputs. For example, an intensive crop producer may hold an inventory of nitrogen fertilizer as it is impossible to obtain large quantities when required due to production bottlenecks, ordering delays and the like.
3. To obtain inputs at least cost it may be necessary to purchase at certain times of the year and hold for later use. For example, it may pay a farmer to purchase feed barley at harvest time and store it for later use rather than purchase it at the time of requirement due to the cost differences.
4. Similarly, to obtain the best price for a product it may be worth holding production for sale in a period of high prices. For example, a farmer holds wheat, barley, hay, potatoes and similar.

5. For input costs, if large quantities are purchased at a particular time there may be savings due to discounts offered. This gives rise to an inventory.
6. For some products (e.g. washing machines) there are considerable set-up costs when a production line is organized: these may involve re-training, adjusting the machines etc. If a large quantity of the product is produced in one run the average cost per unit declines. Thus it may pay to hold an inventory of the product rather than have many small production runs. For example, it may pay a flower seed producer to produce 2 years' supply in one season (the effect of time on germination would be an important variable in this analysis).

15.4 Analysing the Inventory Problem

As the decision variables in each problem vary, a model must be developed for each. Further, in some cases the problem may be stochastic (i.e. it involves non-certainty), others non-stochastic (e.g. where orders have already been received for flower seed so that the demand is known). The problem is determining the decisions that minimize the cost of producing a given level of output. Given the answers for several output levels a decision can be made on the profit maximizing level. The output levels considered should be only those attainable with the available level of resources.

In some cases profit-maximizing solutions will be the major aim of the analysis. For example, the question of deciding how long (and how much) to store barley before selling falls into this category. Others will be cost minimizing, though in the end this leads to profit maximization.

In general, the two basic questions which must be answered are: (i) the level of inventory to hold; and (ii) the level to which inventory should be allowed to fall before new supplies are obtained, as these may not be available immediately.

The reason for maintaining a minimum level of stock may be due to an expected delay in acquiring new stocks (e.g. ordering delays, a poor season leads to low hay production, etc.) and/or a fluctuating demand given immediate replenishment is not possible. An optimal inventory policy may consist of a simple rule stating that quantity y should be obtained once stocks have fallen to quantity x. This might vary throughout the year.

As cost analysis forms the major part of any inventory analysis a good understanding of the types of costs which will be encountered is necessary. Two of the major costs are the *cost of providing and maintaining the storage facilities* (repairs and maintenance, depreciation, interest etc.) and the *opportunity cost of having funds tied up* in the good being stored (the money could alternatively be invested in shares, the overdraft reduced etc.). In some firms additional personnel may need to be employed to operate the storage facilities. Other costs are ordering costs, transport costs, the varying cost of an input (good) purchased at different times, selling costs, set-up costs on production lines etc. and all the accounting and clerical costs of maintaining inventories, though these tend to be negligible in a farm business. A further opportunity cost is the loss of potential sales through not being able to meet a demand as a result of an insufficient inventory, e.g. a farmer producing turkeys for a chain of restaurants may well loose market share if he fails to supply in a critical period. Similarly, if an input is not available when required there will be a loss in production, e.g. not having nitrogen fertilizer available at the critical time. A few days' delay can reduce yield markedly. This gives rise to an opportunity cost.

15.5 The Feed Inventory Problem on Grazing Farms

The nature of the problem

As noted, a simple feed inventory problem provides an important example of a farm inventory problem. A farmer operating a stock system needs to supply appropriate feed quantities to maintain production. However, pasture production fluctuates throughout the year and from year to year. Thus, feed must be made and stored and/or purchased to maintain production. The problem is to determine the quantity of feed to conserve and store, and purchase, to meet deficits in the variable pasture supply. This is a stochastic problem as the pasture supply will fluctuate from year to year, the amount of hay/silage that can be made fluctuates from year to year, the cost of purchasing feed fluctuates and so on. On feedlot-type farms without grazing (or cutting and carting), similar problems apply as the demand and prices vary throughout the year and between years.

In planning a farming system the feed inventory problem is only part of the overall problem. Consideration must be given to such factors as, for example, an optimal stocking rate as this affects the feed demand and the potential for feed conservation in surplus growth periods. Thus, given a *particular* stocking rate there will be a particular optimal feed inventory. Another factor is whether to provide a sub-maintenance diet rather than maintain large inventories. Many other possibilities exist and are discussed later.

The nature of inventory management

Inventory management consists of continuous re-evaluations of the state of the system and a continuously changing optimal policy as conditions change. A simple optimal policy for average conditions exists only in theory, as average conditions seldom occur.

The feed problem can be divided into two phases, which may occur more than once in each year: (i) the phase during pasture surpluses in which feed is usually accumulated; and (ii) the phase during which feed stocks must be used due to pasture deficits. The state of the farm (the state variable values) at the start of a potentially cumulative period will affect the policy that should be followed. For example, stocks of feed on hand and animal numbers will dictate in part what policy should be followed. Similarly, the level of cash reserves (liquid assets) should affect the decision made. Given, for example, full hay barns and high cash reserves it may pay to assign land to a risky cash crop rather than make hay/silage.

As each phase progresses, the optimal policy may change within the period depending on whether each season progresses as expected. The winter 'no growth' period may have started earlier than expected so it may pay to alter the rate at which it was planned to feed the stock. Barley/maize prices may be lower than expected at harvest time so that it may be profitable to purchase supplies that were not originally planned, and so on.

Summarizing, while an average optimal feed inventory level can be envisaged, feed inventory plans must be continuously reviewed and changed if necessary as: (i) the state of the farm differs from that expected; and (ii) expectation of the future conditions change (e.g. barley/maize prices are expected to be lower than usual).

Further, as a farm is developed, different stock numbers are carried, so the optimal feed inventory policy may also change.

Some alternative strategies

Introduction

In the real world there is more to the feed inventory problem than simply deciding on a feed level to maintain. Rather than store feed, various other methods of bridging shortages are available. Similarly, as conditions change and the state of the farm varies from the expected there are alternative strategies besides simply altering the feed inventory policy. Some of the possible strategies are listed in this section. Some require action during feed surplus periods (or, at least, non-deficit periods), some during feed shortage periods and others a combination. The alternatives listed are intended to cover the full range of possibilities. Some will not be viable under some conditions and, on the other hand, many are not mutually exclusive. This list is provided to ensure an analyst keeps inventory problems in perspective and stresses the need to consider any business 'as a whole'.

Alternative strategies

CEASE BREEDING This reduces the feeding demand through the smaller number of stock to be fed and through the breeding animals not requiring special preparturition feeding. Thus, the demand for winter feed is lessened as parturition usually takes place at the end of winter and it may be possible to conserve more feed for future use due to the smaller demand in the spring/summer period.

The actual number of animals not mated can be varied.

Under this policy income will be reduced as there will be no offspring to sell. In extreme cases, however, there may in fact be an increase in income due to the saving in feed costs that would otherwise be necessary. If income is expected to decrease this must be offset against any expected increase in fodder reserves made possible by not mating and therefore having no offspring to feed in spring/summer. Another factor is the effect on the cash profile.

SELL STOCK Feed demand can be reduced by selling stock. This can occur either during a feed shortage thus reducing the requirement for stored feed, or during a surplus period thus increasing the quantity of feed that can be stored for future use.

The type and quantity sold off will affect: (i) saleable production; (ii) amount of feed demand reduction; and (iii) future development of the property (e.g. where breeding stock are sold). In most cases, finishing stock would be sold off first.

STOCKING RATE Over the long term, the average stocking rate has a marked effect on feed shortages, ability to conserve feed and profit variability. To reduce the need for large reserves, for ease of management and to reduce income variability a lower stocking rate may need to be considered. There may well be a stocking rate at which the cost of maintaining feed reserves and associated policies does not warrant the additional returns.

ADJUSTING THE FLOCK OR HERD STRUCTURE This policy involves selling and buying to alter the age and/or sex structure of the flock or herd. This may achieve feed savings but more likely will provide greater flexibility and 'hardiness'. For example, if it appears there will be feed stress it may pay to sell off young, growing and old animals and replace them with animals better able to withstand stress conditions. The success of such a policy will depend in part on the buying and selling prices.

REDUCING THE FEED INTAKE OF SPECIFIC STOCK CLASSES To reduce feed consumption, some animals can be fed less than normal. As a regular policy, and as a policy to be used in unexpected shortage periods, animals can be fed at any level between maximum production down to maintenance for long periods of time, and below maintenance for short periods. Such policies clearly reduce physical production but also reduce feeding costs as feed may not have to be purchased, or allow more feed to be conserved. These policies will affect different classes of stock differently. Thus, for example, reducing the feed intake of pregnant ewes prior to lambing may result in considerable mortality whereas wethers (castrated males) will only be affected through a marginal drop in wool production. In feed shortage periods, therefore, this approach of reducing the feed intake of certain classes of animals may be useful.

COMPLETE DE-STOCKING An extreme policy under severe feed stress is to sell off all stock and to re-stock once feed conditions improve. The economics depends not only on the selling and buying prices but also on the quality (productivity) of stock purchased to replace the sold-off stock. This policy is more likely to be acceptable for non-breeding programmes. The cost of buying feed will also be an important variable. The geographical size of the area experiencing extreme feed shortages will influence stock prices and transport costs, as will general farmer confidence and the availability of finance.

MAINTAIN A NORMAL POLICY (TAKE NO ACTION) Given other than expected conditions, rather than adjust the policy the normally used policy can be followed through. For example, given lower than expected hay supplies at the start of winter the usual feeding policy can be maintained. This may prove to be successful depending on the winter growth and the cost of alternative feed supplies. In general, such a policy may not be profitable except in areas where feed shortages tend to be only minor, i.e. the probability of severe shortages is extremely low. The costs and return associated with the alternatives must clearly be considered.

PURCHASE FEED To meet feed deficits feed can be purchased during general surplus periods and stored for later use, or purchased during general deficit periods.

If feed costs vary only slightly between periods, similarly the availability, then purchasing when the requirement occurs could be profitable. No more than the actual requirement would be purchased at each point. However, such conditions are unusual so that it usually pays to buy and store. At the other extreme a system that buys and stores sufficient feed to meet all conceivable requirement levels is also unlikely to be optimal. A more likely policy is some combination.

In any area there are a number of alternative feeds including hay, straw, grains, proprietary mixes and meals, silage and some root crops, or some combination. Each will need assessing.

Given feeding is to occur, a decision must be made regarding the ration mix and quantity. This will affect production, so the marginal cost should be related to the marginal return. As a feed shortage period progresses the feeding ration may need to be changed due to price changes, stock condition changes and so on. Three basic ration types can be classified: (i) survival rations; (ii) maintenance rations; and (iii) maintenance plus production rations.

With extensive feed purchasing it may be worth investigating forward contracts both for quantity and price.

PURCHASE GRAZING Some stock classes can be moved to grazing. This might be anything from simple pasture grazing to root crop, and hay and green feed rations, and might be used as a regular policy for which contracts are let every season. More likely, grazing will be purchased when unexpected conditions occur. The costs involved (feed and transport) will determine whether this policy should be used in preference to buying feed or some of the other policies such as simply reducing the feed intake. Questions of whether to feed a sub-maintenance ration must also be answered.

FEED CONSERVATION AND STORAGE It is common to store feed produced in surplus periods. Often more than the expected demand is stored so that a range of feed shortages greater than the expected demand can be satisfied. Given a long series of poor years this may not be possible unless stock numbers are reduced. In such cases feed purchasing may be necessary.

A large range of possible conservation policies exists. Each has a different cost and produces different quality feeds. Some possibilities are hay, straw, silage, haylage, grain, standing pasture, standing fodder crops (e.g. root crops grown in one period and held over, similarly maize produced in early summer for late summer consumption).

SPECIAL PURPOSE PASTURES AND CROPS Feed shortages usually occur due to growth cessation resulting from unfavourable weather conditions, e.g. lack of rain, cold periods. There may be plant species available which tolerate these conditions given special treatment. For example, the cold-tolerant ryegrasses, which will produce in cold periods given adequate nutrients. Such plants tend not to be efficient in other conditions so a decision must be made, for example, whether to reduce feed production in good growth periods to allow this production in other periods.

Special purpose pastures and crops may be used both as a regular policy and as a special policy for seasons in which conditions are 'unexpected'.

IRRIGATION The use of irrigation may be profitable for a number of reasons, one of which is the additional feed conservation made possible. In other cases it can be used directly to reduce feed shortages in water stress periods that would normally limit pasture and/or crop production. For example, irrigation might be used on maize crops in a summer dry period.

DIVERSIFICATION While on average a particular stocking system may be the most profitable alternative, diversifying into an area of cash cropping may provide a lower income variability. In periods of extreme feed shortage the area under cash cropping requires no maintenance expenditure as do stock (feed costs). If a drought is the cause of the extreme feed conditions then neither will cash crops produce any returns. However, once the drought has broken, cash crops provide a quick return at little cost compared with stocking systems, as stock numbers might require building up again.

For weather-induced feed problems, it is possible that sub-regions within the overall region may have different conditions. Through purchasing a number of properties in different regions feed supplies may follow a less variable distribution. Also in some cases seasonal feed shortages may be partially overcome by using blocks of land exhibiting different growth patterns (e.g. an early or late block).

CONCLUDING COMMENTS A wide range of possible policies has been suggested. On any particular farm some may not be feasible, e.g. irrigation and/or spatial diversification. Further, some of the alternatives may require associated conditions. For example, relying on a policy of buying feed in extreme feed shortage situations will require access to liquid funds. Thus, it may be necessary, for example, to borrow funds well before the need arises, or perhaps hold a readily saleable group of animals. Similarly, in considering the possibilities, the opportunity cost of resources (e.g. labour) used must be taken into account to allow for other sections of the whole farm. A complete feed inventory model would take into account all these options.

This discussion also makes it clear primary production decision making is complex. Most decisions have many aspects that must be taken into account, and so often a change to one part of the farm has ramifications in other parts, therefore the need to emphasize that any analysis must consider the 'whole farm' before a final conclusion.

The background to a simple model

Introduction

As noted, the feed inventory problem is being used as an example of farm inventory problems. While the previous section shows many alternative approaches can be used, the symbolic model developed restricts choice to feed storage and purchase options. A reader may need to extend this model if other policies need to be explored.

Thus, assume that a farmer wishes to follow a policy of supplementary feeding in feed shortage periods rather than varying stock numbers (varying demand) or some other combination of possibilities. The problem is to determine the feed inventory level that will minimize the expected cost of feeding all animals at a particular level (possibly maintenance, possibly production).

Factors affecting an optimal solution

The factors which must be considered are listed below.

1. In any one year the quantity of feed required can vary depending on the season. In the extremes, supplementary feeding may be required for the whole year, or not required at all (which is less likely).
2. Each of the possible feed requirement levels has some probability of occurrence. The total feed requirement to maintain production at a particular level is a random variable. The shape of the distribution will vary with the stocking rate, the climatic environment and so on. The shape of the distribution will have an important effect on an optimal policy.
3. The farm can obtain feed supplies from:
- **(i)** making and storing feed supplies in surplus periods (these may be induced by decreasing the per head feed intake);
- **(ii)** buying and storing feed in surplus periods; or
- **(iii)** buying feed supplies when they are required rather than in surplus periods when the requirement will not be known.

4. Each of the alternative feed sources has a different cost.
5. If insufficient feed is obtained and stored prior to a deficit period additional feed will have to be purchased, usually at costs greater than feed costs in surplus periods.

6. If the farmer is not indifferent to risk it will be necessary to consider the variance of alternative policies and possibly other distribution parameters as well.
7. The cost of storing feed may be an important factor. If storage facilities are available there will be repairs and maintenance costs but no capital costs. A major part of storage costs will be the opportunity cost of the capital invested in the stored feed. If funds are not used to purchase feed they can be used in other alternatives. Similarly, if home-produced feed is stored this could have been sold and the funds invested in other alternatives.
8. If too much feed is stored there is a loss in the value of the feed as it will have declined in value through deterioration. To account for this it can be assumed that all feed left at the end of the planning period is sold at a decreased price.

The estimation of the data

Probably the most difficult problem is estimating the alternative possible feed shortages that might occur and their associated probabilities. Subjective estimates can be made but it is helpful to have objective information on which to base the estimates. However, directly applicable historical information is unlikely to be available as the stocking rate, conservation methods, stage of farm development and many other factors have probably varied, nullifying what data may be available.

This means rainfall records and general experience may need to be the basis for the estimates.

For example, given the current stocking rate, stage of development (type and quality of pastures) and 60 years of rainfall records, an adviser may estimate the possible feed shortages to be:

Possible feed shortages (SU)	Probability
1000	0.1
2000	0.2
3000	0.4
4000	0.2
5000	0.1

SU: 1 stock unit is the feed necessary to support a 50 kg sheep

Note that in this example no reference is made to when the feed shortages will occur, and for how long; e.g. a summer shortage of 3 weeks and a winter shortage of 2 months makes up a total shortage of a given stock unit level. Thus, it is assumed that the total feed requirement per year is all that is important. This may not always be the case.

Similarly, estimates of the probabilities attached to the values of all the other random variables are necessary. For example, the cost of purchasing feed during deficit periods is a random variable, so prices and probabilities must be estimated. However, assume that in this example problem the feed requirement is the only random variable.

Another factor which must be considered is the quantity of feed that can be conserved on the farm given the particular stocking rate and stage of farm development. There will be an upper limit to this. The cost of this home-produced feed is the opportunity cost of the resource used in its production. One alternative giving rise to this opportunity cost might be a cash crop. This cost and the quantity produced are two further random variables that will be assumed to be known with certainty in this example.

A symbolic model

A model expressing the expected feeding costs from maintaining a feed inventory policy can be developed and used to compare the expected cost of alternative policies.

Define the following terms:

p_i = the probability of a feed shortage of i feed units occurring. Let a feed unit be 200 SU (the smaller the units used the more calculations will be required);

C_1 = the expected cost of a feed unit (200 SU) of feed produced or purchased *during a feed surplus* period. Preliminary budgeting would indicate whether home-produced or purchased feed should be used, although in some cases a mixture may be necessary as it may not be possible to produce sufficient quantity on the farm. C_1 would include the cost of transport to storage and any other costs;

C_2 = the expected cost of a feed unit (200 SU) of feed purchased *during a feed deficit period*. This information is important, as if insufficient feed is stored extra supplies must be purchased. C_2 would include all transport, feeding out costs etc.;

C_3 = the expected value of a feed unit (200 SU) of feed *at the end of the season*. This may be the sale value or the expected value resulting from its productive use;

R = the opportunity cost of the funds tied up in a feed reserve, expressed as a fraction; i.e. if the opportunity cost is 7%, then $R = 0.07$;

C_k = the expected cost of developing a feed inventory during the major feed surplus period of k feed units; i.e. $k \times 200$ SU;

n = the maximum feed shortage that can occur expressed in feed units of 200 SU.

Using these terms, an expression for C_k can be developed. In this example assume that storage facilities are available for any inventory level that might be envisaged and that these will be maintained whether used or not. These assumptions mean that storage facility costs can be ignored. Thus, the cost of storing k units is given by:

$$C_k = kC_1 + kC_1R + C_2\sum_{i=k+1}^{n}(i-k)p_i - C_3\sum_{i=0}^{k}(k-i)p_i$$

Consider each term in the model:

kC_1 = the cost of making and/or purchasing k feed units during a surplus period;

kC_1R = the opportunity cost of the funds invested in the feed reserve of k feed units.

Note that the terms $kC_1 + kC_1R$ can be combined to form $kC_1(1 + R)$.

$C_2\sum_{i=k+1}^{n}(i-k)p_i$ = the expected cost of buying feed during feed deficit periods given that k feed units are stored. Note that the term

$\sum_{i=k+1}^{n}(i-k)p_i$ estimates the expected feed quantity that must be purchased given that k units are stored.

This is multiplied by the expected cost/unit (C_2).

Consider the $\Sigma(i - k)p_i$ in more detail. The term in brackets $(i - k)$, estimates the possible feed shortages. This is multiplied by the probability for each shortage occurring. The summation is done for only those feed requirement levels that cannot be supplied from the feed stored (k units), i.e. for $i = k+1$ to n.

Possible feed requirements	Probability
5 feed units	0.2
6 feed units	0.5
7 feed units	0.3

If 5 feed units are stored (k=5), the expected cost of feed purchasing will be:

350 [(6 – 5) 0.5 + (7 – 5) 0.3] = \$385 where C_2 is \$350

Note that if k was less than 5, this being the lowest possible requirement, some of the components of the term would be zero as the probability of shortages less than 5 units is zero. Such components would be ignored.

$C_3\sum_{i=0}^{k}(k-i)p_i$ = the expected value of feed left 'on hand' at the end of the period

given k units are stored. The $\Sigma\,(k-i)p_i$ part of the term estimates the expected quantity of feed 'on hand' at the end of the period. For example, using the data above and letting C_3 = \$200, if k = 7 the expected value of feed on hand will be:

200 [(7 – 5) 0.2 + (7 – 6) 0.5 + (7 – 7) 0.3] = \$180

Note that the term in brackets for i values less than 5 can be ignored as they have a probability of zero in this example.

Determining the optimal feed inventory level

The expected cost

The model is used to estimate C_k for all possible k values. The k value giving the lowest C_k is the least-cost inventory level and will be optimal if the farmer is indifferent to risk.

As an example, consider the following simple problem:

Possible feed requirements or shortages (feed units)	Probability
5	0.1
6	0.3
7	0.4
8	0.2

C_1 = \$250
C_2 = \$350
C_3 = \$200
R = 0.10 (10%)

Using this data, the cost of each possible storage level is:

If $k = 0$,

$$C_0 = 0 + 350\,[(5 - 0)\,0.1 + (6 - 0)\,0.3 + (7 - 0)\,0.4 + (8 - 0)\,0.2] - 0$$
$$= \$2{,}345$$

If $k = 1$,

$$C_1 = 250\,(1.1) + 350[(4)\,0.1 + (5)\,0.3 + (6)\,0.4 + (7)\,0.2]$$
$$- 200[(1 - 0)\,0 + (1 - 1)\,0]$$
$$= 275 + 1995 - 0$$
$$= \$2{,}270$$

If $k = 2$, $C_2 = \$2{,}195$
If $k = 3$, $C_3 = \$2{,}120$
If $k = 4$, $C_4 = \$2{,}045$
If $k = 5$, $C_5 = \$1{,}970$
If $k = 6$, $C_6 = (6 \times 250 \times 1.1 + 350[(7 - 6)0.4 + (8 - 6)0.2]) - 200\,[(6 - 5)0.1]$
$= \$1{,}910$
If $k = 7$, $C_7 = \$1{,}895$
If $k = 8$, $C_8 = \$1{,}940$

Thus the optimal (least cost) policy is to store 7 feed units of feed. The chance of requiring 8 feed units is not great enough to warrant storing more.

Note that in this particular example there was no need to consider k values less than 5 units as the cost of feed purchased is less than purchasing the feed during the deficit period. This conclusion can be made as at least 5 units of feed will always be required.

Estimating the cost variance for alternative policies

The least-cost policy may not be optimal where the farmer has a non-linear monetary utility function, in which case the variance of alternative policies needs estimating. In this simple feed inventory, using the general variance formula is possible, although in more complicated models systems simulation may be necessary:

Variance = (possible outcomes − mean outcome)^2
× probability of each outcome occurring

In this problem the variability around the expected or mean cost for a particular policy is made up of two factors: (i) the variability due to the feed reserve being too small to meet the requirement; and (ii) the variability due to the feed reserve being too large. Thus, the variance of a particular inventory policy will be given by:

$$V(Cost_k) = \sum_{i=0}^{k} ((1+R)C_1k + C_3(k-i) - C_k)^2 p_i + \sum_{i=k+1}^{n} ((1+R)C_1k + C_2(i-k) - C_k)^2 p_i$$

where $V(Cost_k)$ = variance if k feed units are stored.

C_k = expected cost of storing k feed units.

The first term gives the variance due to storing more feed than is required, and the second due to storing insufficient feed. The equation can be divided into these two basic components as when k is less than the requirement the term giving value of unused feed will be zero and vice versa.

In order to choose the least variance policy the equation would have to be evaluated for each k value and the one giving minimum variance selected.

Hindsight and an optimal policy

The model will only indicate the expected optimal policy. Given that this policy is followed it may in fact turn out to be sub-optimal in some years, but this knowledge is only available after the end of the season. In the example problem the optimal policy is to store 7 units of feed. If the feed requirement turns out to be 5 units (possibly due to a mild winter) then 2 units of non-required feed have been stored. If it was known before the event that only 5 units would be required then a saving of \$2 × 50 (the difference between purchase and sale values) would have been made. This is a cost of operating in an uncertain environment (the cost of uncertainty).

15.6 Extensions to the Feed Inventory Model

Introduction

As will be evident from the earlier discussion, the inventory model specified has shortcomings. Many of the other factors would only be taken into account in a subjective fashion due to the complexities of incorporating them into symbolic models. To assist in this subjective assessment the main factors are listed below. Before this, however, some comments on re-planning an optimal policy are provided to emphasize the uncertain and dynamic nature of farm planning.

Re-planning in the simple feed inventory problem

Given the optimal feed inventory has been calculated and partially implemented, the farmer should not necessarily proceed to follow the originally estimated policy. As time progresses the expectations regarding feed costs, feed salvage values and the probabilities associated with each possible feed requirement may change. If the changes are large enough the optimal policy may require altering. If purchased feed costs, for example, are currently lower than those originally expected it may pay to purchase and store more feed than envisaged. Similarly, it may even pay to purchase feed during a feed deficit period if the cost is low enough. This could well be optimal where more feed than expected has been used so there might be a reasonable probability of large quantities of feed still being required.

In order to re-determine an optimal policy the simple model should be re-evaluated each month given the new estimates for the variables. In this case not all k values would have to be evaluated as there could already be some feed on hand. The same comment applies when working out an initial optimal policy in that some feed from the previous season may still be on hand. (In some cases the possibility of selling off stored feed during deficit periods may need to be considered.) If the cost and probability estimates have not changed there is no need to recalculate an optimal policy. Recalculation will be easy if the model has been computerized, requiring only the input of the data and selecting a recalculate choice.

The realistic feed inventory problem

A farmer's objective is unlikely to be maximized where it is assumed all stock must be retained and fed at a particular level. Thus, policies such as selling off stock and buying back later should be taken into account and, furthermore, the fact that most of the variables concerned are random variables should be allowed for.

It is clear that a realistic model will be complex. Such a model should include:

1. All the variables included in the simple case together with allowances for:
- **(i)** Feed purchase costs varying with time;
- **(ii)** Feed purchase/production costs being random variables;
- **(iii)** Feed sale prices varying with time and being random;
- **(iv)** Purchase and maintenance of additional feed storage areas thus allowing greater inventory levels. In the simple example it was assumed that any level of feed could be purchased and stored. This is unrealistic. Feed stored in unsuitable facilities will deteriorate. This could also be taken into account.
- **(v)** Limits on the quantity of feed that can be conserved on the farm. Furthermore, this variable is also a random variable. Thus, while plans are made to produce x units of feed, production may in fact be less, or greater.

2. Variables taking into account that the feed demand and supply can be varied through using one or more of the other policies discussed earlier.

Each policy will have a particular effect on the feed demand or supply and certain costs and returns. Each of these effects will also be random variables. For example, some of the factors that would have to be taken into account are:

(i) The opportunity to sell off some stock and re-purchase later at various times, which will reduce the feed requirement. The prices would be random variables;

(ii) The possibility of purchasing off-farm grazing as an alternative source of feed. Similarly, the use of irrigation to increase feed conservation.

It is clear that such a model would be complex and involve considerable costs if a reasonable range of possible alternatives were evaluated. For a totally realistic model it would make sense to use the systems simulation approach and experiment with alternative policies.

The whole farm approach

In theory the inventory problem is not independent of other sectors of the farm. As the farmer's aim is to maximize an objective from the whole farm it is usually necessary to use a whole-farm planning model, which would take into account, for example, the working capital restrictions.

In a whole-farm model, perhaps using linear programming, different feed inventory and associated policies can then be explored and evaluated on the basis of how they affect, say, expected whole-farm profit.

Some of the factors that should be included in a whole-farm profit model with respect to the inventory problem are:

- Allowing for different stocking rates, stocking systems and feeding levels. All these factors will affect costs as well as productivity and thus returns.
- Constraints on the possible inventory policies due to the available working capital, machinery capacity to handle fodder conservation and the like.
- The effect of shearing at different times of the year on feed demand and on costs and returns.

As noted, allowing for all these factors is complex. One approach is to accept the mistakes inherent in solving part-farm situations as independent problems, and given the results using the simple feed inventory model, subjectively assessing factors such as working capital limitations. This is where the experience of an analyst and adviser becomes very important.

Inventory analysis and dynamic programming

As inventory problems involve several time periods (inventory levels are built up during particular periods for use in later periods) they are readily amenable to solving using dynamic programming. Many examples of inventory analysis given in the literature use dynamic programming as the solving technique. The most important state variable is the current level of stored feed.

15.7 Replacement Analysis

The general problem

Introduction

Replacement analysis works out the cost-minimizing time to replace assets that provide services for creating profit. With use and time, an asset's output and costs change. Eventually an asset costs more to maintain than a replacement and, usually, productivity declines relative to new models.

Information on the optimal replacement time is used to indicate when assets should be replaced and is, therefore, part of the information necessary to decide whether to sell an existing asset. The optimal replacement time information relates to assessing the profitability of an asset as it determines its least cost. To decide whether to invest in, or sell off, an asset the optimal return obtainable from the asset must be known.

Types of replacement problems

There are two basic types of replacement problems.

1. Problems in which the productivity of the asset declines with use and/or age and, possibly, the maintenance and running costs vary. An example of this type of problem is the animal replacement problem. There is an optimal time at which to replace e.g. ewes or cows as their productivity declines with age (wool production declines from approximately 2 years of age, the lambing percentage increases with age but eventually declines). Furthermore, the maintenance costs increase in that they require more attention, better feed and have a greater death rate. The net return per ewe/cow eventually declines to the point where it pays to sell and replace with younger stock. Another example of this general type is the pasture renewal problem.

2. Problems in which the productivity of the asset remains relatively constant but the running and maintenance costs increase with use and/or time. The objective is to determine the replacement time at which average costs will be minimized. For example, a header/harvester can provide much the same service as its use increases provided it is adequately serviced. The costs of maintaining the header increase with use and age so eventually these costs will rise to the extent that it will be cheaper to replace the machine, although there may, of course, be other reasons why the machine should be replaced. For example, the farmer may be increasing the quantity of cash crop and requires a header with greater capacity.

Both of the above general problem types can be divided into two sub-types based on the value of the assets: (i) assets of appreciable value, which can be repaired and maintained with continued good output; and (ii) assets of little value, which cannot be repaired or are not worth repairing. Assets such as headers, tractors and dairy sheds are in the first group, whereas assets such as light bulbs and spark plugs are not worth repairing. It can be cheaper to replace them all at periodic intervals compared with individual testing at more frequent intervals. This second group is only of small significance on farms, so that it will not be discussed further.

The case of tractor replacement

To demonstrate the principles involved in solving replacement problems a simple case of tractor replacement is considered.

Assume that the tractor is maintained so its productivity remains constant. The problem is to determine the replacement time so that annual costs are minimized. It is necessary to determine all the costs of running and holding the tractor and, using this information, determine the number of periods the machine should be held so that the *average cost per period* is minimized. It is assumed the hours of use per period are constant. If not, the period costs will need assessing on an hourly basis.

The cost categories involved in purchasing and running a tractor include:

- the initial purchase outlay;
- repairs and maintenance costs required to maintain the tractor in an efficient working condition;
- running costs (fuel, oil, etc.);
- overhead costs (registration, insurance);
- the opportunity cost of the funds sunk in the tractor both by way of initial outlay and repair costs etc.

In practice, not all of these costs will markedly change with time (or per hour of use). *If the costs do not vary they do not need to be considered in the analysis of optimal replacement time*, though including them provides a full costing. In the extreme case, if none of the costs varied from period to period it would never pay to replace the tractor as it would continue to provide the same service for the same cost. This situation is, of course, unrealistic.

As constant costs can be ignored, consider each of the categories. If it is assumed the tractor is well maintained it is likely that the running costs will not increase much, so ignore them for this analysis (this assumes the use of *real* values so that inflation is taken into account). Similarly, it can be assumed that the overhead costs remain relatively constant and can be ignored. The repairs and maintenance, and the opportunity costs, all vary as with time the total investment in the tractor increases due to the increasing repairs and maintenance bills each period. The opportunity cost per dollar invested probably will not, however, vary. Similarly, the initial outlay must be considered.

The holding cost for a tractor

Basic model

Ignoring constant costs, at any time the sum of all costs incurred by the tractor is referred to as the *holding cost*. Thus, in the tractor case:

Holding costs = initial outlay + all repairs and maintenance + all opportunity costs

To estimate the optimal replacement time this holding cost must be estimated for each alternative time for which the tractor can be held.

However, when the tractor is replaced, a trade-in price is received, or else it is sold on the open market (salvage value). This leads to a net holding cost for each period for which the tractor might be kept. This is the relevant cost figure for decision making. The net holding cost is given by:

Net holding cost = holding cost − salvage value

With time, the net holding cost increases at an increasing rate as the holding cost increases and the salvage value declines. The net holding cost will probably have the form shown in Fig. 15.1.

The time period giving the minimum net holding cost is the optimal replacement time and is calculated through:

$$\text{Average net holding cost for } x \text{ periods} = \frac{\text{total net holding cost for } x \text{ periods}}{x}$$

where x can also equal hours worked.

Graphically, the optimal replacement time will be the point at which a straight line drawn from the origin to the curve is a tangent to the curve. This is shown in Fig. 15.2.

Of course, the average holding cost is given by the slope of the hypotenuse of a right-angled triangle (like OAB) formed using the origin as one apex, the time for which the average holding cost is to be estimated as the right-angled corner, and the curve as the other apex. The slope is the average holding cost as the height represents the total net holding cost and the base represents total time. The triangle with the hypotenuse having the smallest slope of all triangles touching the curve represents a replacement time with the smallest average net holding cost.

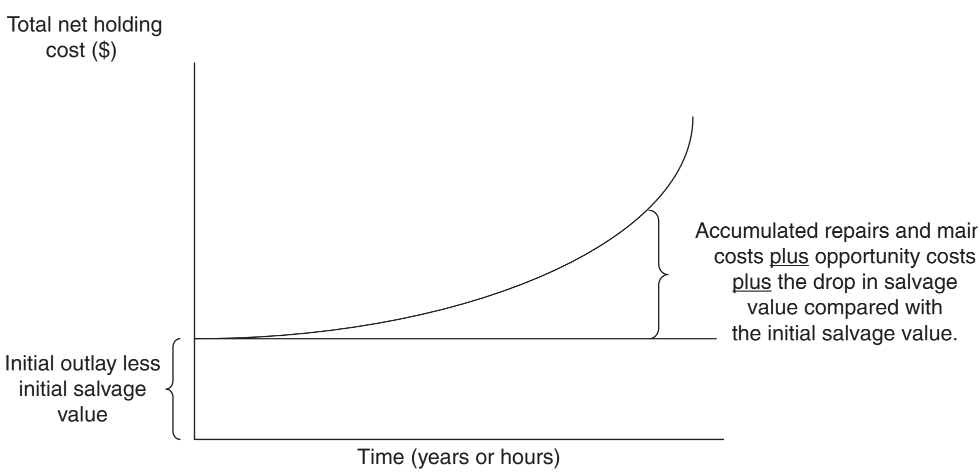


Fig. 15.1. Net holding cost graph for a typical machinery asset.

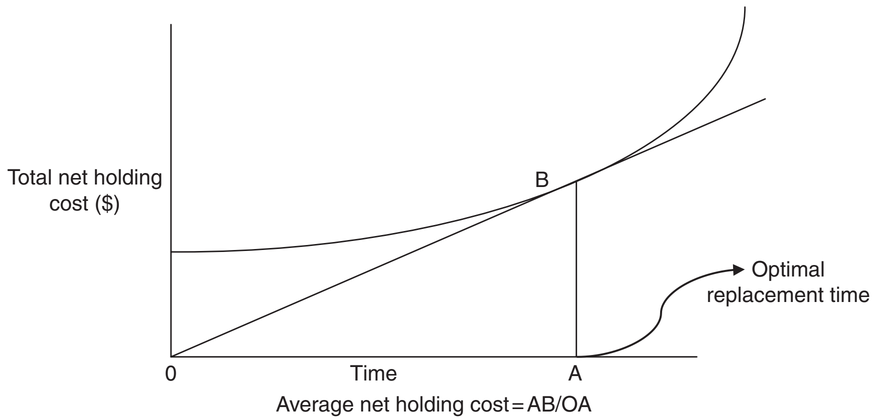


Fig. 15.2. The total net holding cost for an asset and its optimal replacement time.

Further variables which must be considered

These include the following.

TAXATION Some of the expenditure is tax deductible, so reducing the net holding cost. Further, while depreciation should not be directly considered in the cash analysis above, it has to be calculated in order to determine the tax deduction. This also reduces the net holding cost as without the machine the taxation paid would be greater. Thus, the *reduction* in net holding cost due to taxation is:

the sum of tax deductible expenditure (including depreciation)
× the tax rate in $

Note that in calculating depreciation it may be necessary to allow for any special and investment allowance depreciation incentives a government might be using. (It is interesting to note any investment allowance has the effect of reducing the initial net holding cost and therefore reduces the optimal replacement time.)

When the asset is sold there may need to be a further taxation adjustment depending on whether the sale price is less than (the difference is deductible) or greater than (taxable income) the book value.

UNCERTAINTY Many of the variables are random variables. Thus, if the farmer's monetary utility function is not linear it may be important to calculate the cost variance of alternative holding times as well as the expected cost. This, however, involves considerable work so that it should only be attempted in cases where the investment is very large and the farmer is an extreme risk averter or preferer. Alternatively, subjective estimates should be made. For example, if the farmer is a risk averter, replace somewhat sooner than the time giving the minimum expected net holding cost. For very large assets, creating a systems simulation model may be appropriate.

OBSOLESCENCE New models periodically appear that are technically improved. These models should produce greater output for much the same cost. Thus, by continuing to hold an obsolete tractor the farmer is losing increased production leading to an opportunity cost. If this opportunity cost could be measured it should be added to the net holding cost of the existing tractor. It might be estimated, for example, that a new model can be expected every 3 years. However, the estimation of the expected production increase given a new model will probably be difficult. For most farm situations this aspect should be subjectively considered. Given that obsolescence has the effect of increasing the net holding cost and therefore making the total net holding cost curve steeper, it will reduce the optimal holding time. This is demonstrated in Fig. 15.3.

EFFICIENCY In most cases the efficiency of a tractor declines with use, so its productivity declines. This loss of productivity should be added to the net holding cost as it is an opportunity cost compared with replacing the tractor with a new one of the same type.

Similar to the obsolescence case it is difficult to estimate this opportunity cost. This should be subjectively accounted for by replacing the tractor somewhat sooner than the optimal time, as with the greater opportunity cost the net holding cost curve is steeper.

UPDATING THE OPTIMAL REPLACEMENT TIME ESTIMATE With time many of the item and opportunity cost estimates may need revising leading to an updated replacement time. Understanding the basic structure of the problem and how the various variables affect the net holding cost curve, it should be easy to assess whether a particular cost estimate change should bring forward, or delay, the optimal replacement time. For example, consider the effect of a lower salvage value than originally expected. This will alter the net holding cost curve's shape so that it pays to replace sooner than originally planned. This is shown in Fig. 15.4.

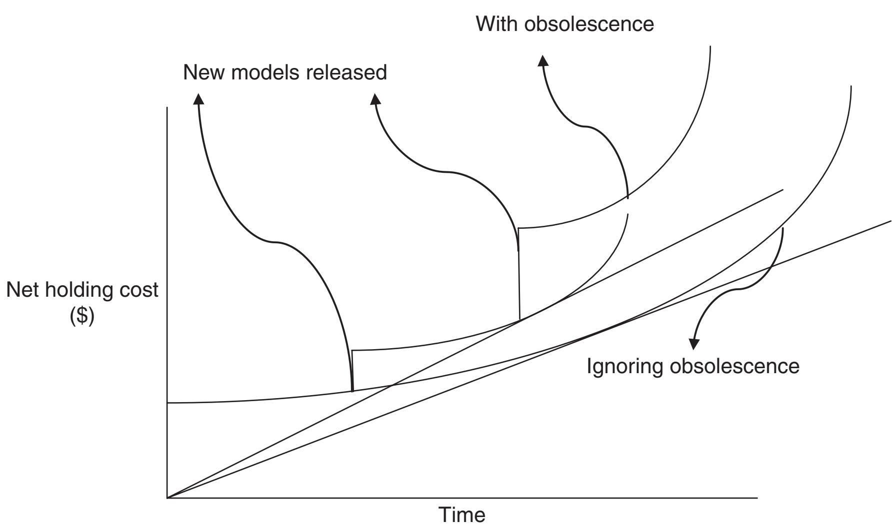


Fig. 15.3. The impact of asset obsolescence on net holding cost and optimal replacement.

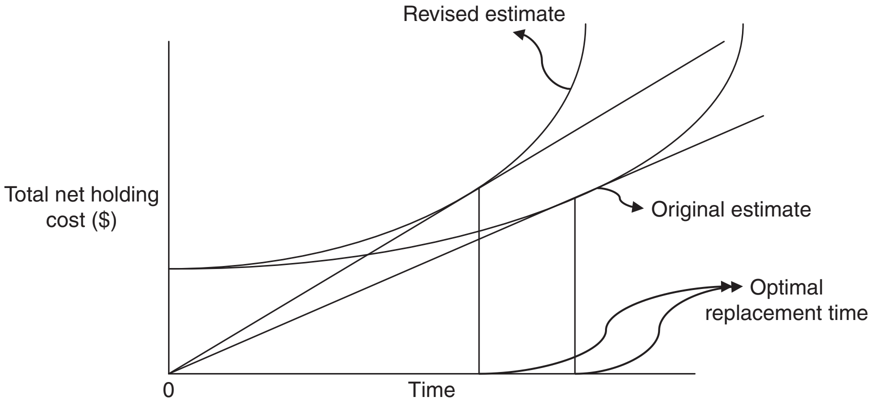


Fig. 15.4. Updating the optimal replacement time as conditions change – decreased salvage value.

A model of the tractor replacement problem

To develop an arithmetic model for tractor replacement, and also allow comparing second-hand and new tractors, assume that:

- the running costs remain constant;
- all overhead expenses remain constant;
- the efficiency of the tractor does not decline; and
- new models are not being introduced, or if they are, the increased efficiency is offset by the increased costs.

If any of these assumptions are violated the model outlined below will have to be adjusted.

Define the following terms:

Let C_q = the total net holding cost when the machine is held for q time periods;

M_i = the repairs and maintenance costs during the ith period. Assume these are paid at the beginning of the period (this assumption affects the opportunity cost);

subscript i ranges from 1 to q;

K = the initial outlay;

S_q = the salvage value of the machine at *the end* of the qth time period;

T_s = the saving in tax occurring due to the depreciation investment allowance in the year of purchase. Assume the benefit accrues at the beginning of the period in which the machine was purchased.

T_i = the tax saved in the ith period as a result of:

(a) depreciation;
(b) repairs and maintenance;
(c) running costs and overheads if these are not constant.

Assume that the benefit accrues at the beginning of each period:

T_x = the tax saved *or* tax paid as a result of the salvage value being different from the book value;

r = the opportunity cost of capital (expressed as a fraction) for *one* time period. Thus, if 10% per annum is the opportunity cost, for 6-monthly periods use 0.05. Given these terms, the total net cost can be estimated for alternative holding time using the following relationship:

$$C_q = (K - T_s)(1+r)^q + (M_1 - T_1)(1+r)^q + (M_2 - T_2)(1+r)^{q-1} + \ldots +$$
$$(M_q - T_q)(1+r) - S_q + T_x$$
$$= (K - T_s)(1+r)^q + \sum_{i=1}^{q}(M_i - T_i)(1+r)^{q+1-i} - S_q + T_x$$

The $(1 + r)$ terms are the compound amount factors that account for the opportunity cost of the capital. The $(K - T_s)$ term is the net initial outlay while the $(M_i - T_i)$ terms are the net outlays per period.

In order to *solve the problem*, this model is used to estimate C_q for all alternative holding times. Then, using the C_q estimates, determine the average net holding cost C_q/q for each q.

The optimal time is that q giving the smallest C_q/q

In solving a replacement problem it must be remembered that this is a part-farm problem and cannot necessarily be solved in isolation. For example, the analysis may suggest that the tractor should be replaced every 3 years but other development work may not leave sufficient cash to purchase the new tractor. If the development work is more profitable than the cost savings obtained by optimal tractor replacement, the tractor should be held for a longer period. Thus, the whole-farm considerations must be taken into account. Conceptually, the replacement model should form part of a whole-farm model, which is solved simultaneously together with all the other problems. In practice, such factors may have to be subjectively taken into account.

An example for tractor replacement

Assume a farmer is about to purchase a small tractor for \$4000 and wants to know the optimal replacement time so that he will know the minimum average holding cost. Assume that the farmer is paying tax at the rate of 40 cents per dollar and that this rate will hold for all income levels being considered in this example. Assume that tractor depreciation is calculated on the basis of 20% DV (diminishing value). Further, an investment allowance of 10% is tax deductible. The opportunity cost of capital is 10%. Assume that the farmer has estimated that the repairs and maintenance, and salvage values will be:

	Year				
	1	2	3	4	5
Repairs & maintenance	\$200	\$300	\$500	\$800	\$1000
End of year salvage value	\$3000	\$2500	\$2000	\$1700	\$1500

Given this information, $C_1, C_2 \ldots C_5$ can be estimated. Then the average net holding cost for each holding time can be estimated and the minimum selected. Consider C_1:

1. $T_s = (4000 \times 0.10)\ 0.4 = \$160$
(i.e. (initial outlay × investment depn.) × tax rate)
$\therefore (K - T_s)(1 + r)^1 = (4000 - 160)(1.1) = \$4{,}224$

2. In order to estimate $(M_1 - T_1)(1 + r)^1$, T_1 must first be estimated:
$\mathrm{T}_1 = ((4000 \times 0.2) + 200)\ 0.4 = \$400$
(i.e. (book value × depn. rate) + R & M tax rate)
$\therefore (M_1 - T_1)(1+r)^1 = (200 - 400)(1.1) = -\$220$
(the negative cost means, in effect, a net tax saving)

3. $S_1 = 3000$

4. In order to estimate T_x the sale price of \$3000 must be compared with the book value of the tractor at the end of the year. This will be \$4000 – (4000 × 0.2) = \$3200. Thus,
$T = (3000 - 3200)\ 0.4 = -\$80$

5. $\therefore C_1 = 4224 - 220 - 3000 - 80 = \$924$
$\therefore$ Average net holding cost = 924/1 = \$924

Consider C_2:

1. $(K - T_s)(1 + r)^2 = (4000 - 160)(1.1)^2 = \$4646.4$

2. **(i)** $(M_1 - T_1)(1 + r)^2 = -\$242$
(ii) To estimate $(M_2 - T_2)(1 + r)^1$, T_2 must first be estimated. To estimate T_2 the book value of the tractor must be calculated for the end of year 1 (or start of year 2).
Book value = (4000 – (4000 × 0.2)) = 3200
$\therefore T_2 = ((3200 \times 0.2) + 300)\ 0.4 = 376$
$\therefore (M_2 - T_2)(1 + r)^1 = (300 - 376)(1.1) = -\$83.6$

3. $S_2 = 2500$

4. $T_x = (2500 - 2560)\ 0.4 = -\$24$
$\therefore \mathrm{C}_2 = 4646.4 - 242 - 83.6 - 2500 - 24 = \$1796.8$
$\therefore$ Average net holding cost = 1796.8/2 = \$898.4

Consider C_3 and C_4:

Using the same process as above, these two costs would be estimated and from them the average net holding costs.
Consider C_5:

1. $(K-T_s)(1+r)^5 = (3840)(1.1)^5 = 6184.3$

2. $(M_1-T_1)(1+r)^5 = (-200)(1.1)^5 = -322.1$
$(M_2-T_2)(1+r)^4 = (-76)(1.1)^4 = -111.27$
$(M_3-T_3)(1+r)^3 = (95.2)(1.1)^3 = 126.71$
$(M_4-T_4)(1+r)^2 = (316.2)(1.1)^2 = 382.6$
$(M_5-T_5)(1+r)^1 = (1000 - 531.07)(1.1) = 518.82$

3. $S_5 = 1500$

4. $T_x = (1500 - 1310.8)\ 0.4 = 75.68$
$\therefore C_5 = \$5{,}354.74$
$\therefore$ Average net holding cost = 5354.74/5 = \$1070.95

The Optimal Replacement Time:

To determine this the C_q/q estimates would be compared and the minimum selected. This cannot be completed in this example as C_3 and C_4 have not been estimated. You might like to complete the calculations to see whether replacement every 2 years is in fact the least cost time.

Further analytical questions

Solving using calculus

If the total net holding cost curve can be adequately represented by a continuous function of the form:

Net holding cost = f(age (or hours of use), initial cost, tax rate, etc.)

then calculus can be used to determine the optimal replacement time through setting up the equation giving the average net holding cost, differentiating and setting equal to zero, and then solving for the replacement time giving the minimum average net holding cost.

Where it is necessary to calculate each C_q before the function can be estimated, it will be easier to use the numerical approach given above rather than using calculus. This will be the usual case.

Parameter variations

To explore the effect of parameter variations on an optimal holding time the net holding cost curves need to be estimated for a range of parameter values. Where farmers in an area have different opportunity costs, different tax rates, different levels of machinery use per year, it will be important to determine an optimal solution for a range of parameter values. As noted, a knowledge of the structure of the problem will enable further extrapolation from the parametric results. For example, it is possible to deduce that the higher the opportunity cost the sooner a tractor should be replaced as the net holding cost will be steeper. This is shown in Fig. 15.5.

Another important example is the effect of varying the number of hours worked per time period. The more hours that are worked, the greater will be the repairs and maintenance per time period so that the net holding cost will be steeper. Thus, it will pay to replace the tractor sooner.

Asset investment decisions

Introduction

The discussion has shown that estimating the minimum average cost per period for an asset is dependent on the optimal replacement time. Thus, decisions on whether to buy or sell assets require this estimate of the optimal replacement. Part of this problem involves whether to purchase new or second hand.

Purchasing new or second–hand

If it is assumed that both new and second-hand assets have equal productivity, the choice depends on which asset has the *lowest* average net holding cost assuming both are replaced at their optimal time. For example, if the net holding costs were those shown in Fig. 15.6, it would pay to purchase a new asset.

The curves may well have different shapes for different farms, so in some cases a second-hand asset will be optimal.

Where the productivities are not equal, the holding cost curves have to be adjusted to account for the opportunity cost. This would be added to the second-hand costs as potential productivity is lost. However, it would be difficult to accurately estimate the opportunity cost, so subjective estimates will be necessary.

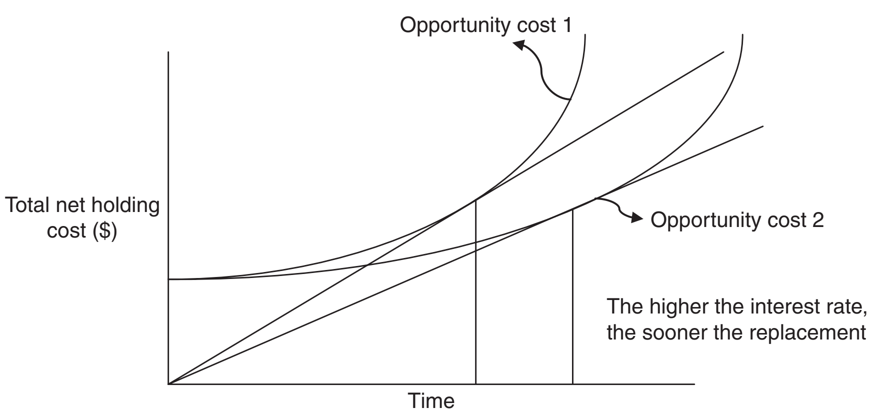


Fig. 15.5. Optimal replacement time with an increasing cash opportunity cost (interest rate).

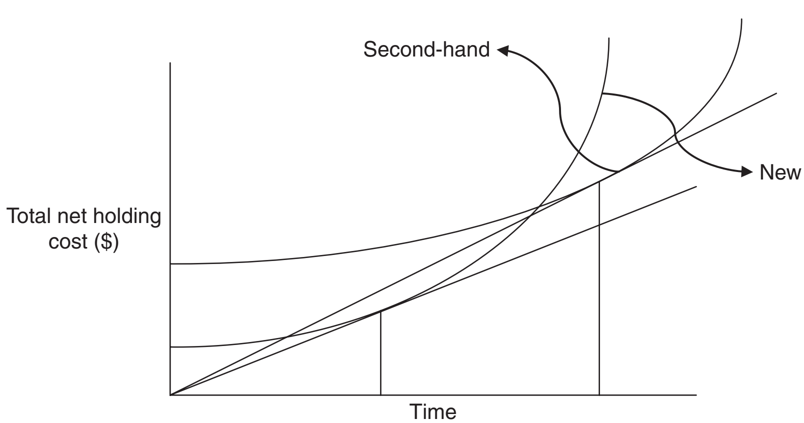


Fig. 15.6. Holding cost variation with a second-hand machine.

The different holding cost curves indicate, in part, why a second-hand asset (machinery) market exists. Holding costs will differ due to the different number of hours the machine is used, differences in the opportunity cost of capital, differences in the farmer's ability to maintain the machine and therefore the repairs and maintenance charges, and the marginal tax rate. Another factor is the availability of cash, with cash shortage possibly precluding a new tractor.

The decision on whether to invest in an asset

The discussion has assumed that it is profitable to hold the asset and the only question is the optimal replacement time. As noted, a decision to invest cannot be made until the cost of holding the asset is known.

Given the minimum net holding cost, the steps involved are:

1. Estimate the increased return which would result from having the asset.
2. Compare the minimum net holding cost (this could be for a new or a second-hand asset depending on the cost) with the cost of employing a contractor to provide the same services.
3. Compare the least cost alternative in (**2**) with the expected increase in return.
4. If necessary, repeat the above analysis for alternative types and sizes of the assets.

The major difficulty is the estimation of the expected increase in return. In many cases 'considered estimates' may be the only data available (linear programming could be used, however, to provide an estimate of the Marginal Value Product of an asset).

A somewhat similar problem is whether to give a machine a major overhaul. An analysis of this type could be put into a similar framework. Similarly, the same approach can be applied to deciding whether to sell an existing asset – is the net holding cost greater than the marginal return from the asset?

15.8 Critical Path Analysis

Introduction and flow diagrams

Introduction

Critical path analysis (CPA) is about project sub-job sequencing. The emphasis is on ensuring the project will be completed in as short a time as possible with the cost being minimized, though there may be a trade off between time and cost. A cropping farmer, for example, should organize crop harvesting so the overall job is carried out efficiently and, consequently, might well use CPA to achieve this. The results will tell which sub-jobs will be critical to the completion of the whole process, when various spare parts must be ordered, trucks ordered for grain cartage and so on.

Methods and flow diagrams

CPA proceeds through creating a flow diagram of the project and determining the time taken to perform each sub-job or step in the overall task. From this information the series of sub-jobs or steps which limit the earliest time the overall job can be completed are determined. This is referred to as the critical path. The critical path, together with other information contained in the diagram, is then used in planning and carrying out the project.

To show how a flow diagram is constructed, consider a simple example of building a hay barn. Assume that the sub-jobs that must be performed, together with the sub-jobs that must precede before each sub-job can be started, are as follows:

Sub-job	Estimate time required to complete (days)	Jobs which must be completed before the sub-job can be commenced
Ordering and obtaining framing materials	6	Nil
Ordering and obtaining foundation materials	2	Nil
Ordering and obtaining roofing materials	1	Nil
Prefabricating frame	2	Ordering and obtaining framing materials
Constructing foundations	4	Ordering and obtaining foundation materials
Prefabricating the roof	3	Ordering and obtaining roofing materials
Putting the frame onto the foundations	½	Prefabricating the frame AND constructing the foundations
Putting the roof on the frame	1	Prefabricating the roof AND putting the frame up

To determine this information a list of all the sub-jobs making up an overall job is made and then the relationship between each sub-job determined.

This leads to a flow diagram (Fig. 15.7) showing which sub-job must precede others, and which are independent. The job time information is also included.

Each arrow represents a particular sub-job and the figure beside it the time taken to complete each sub-job. The nodes (boxes) represent the completion of one or more sub-jobs. A sub-job represented by an arrow leading from a node *cannot* be commenced until all the sub-jobs represented by arrows leading into the node have been completed. Thus, for example, the sub-job 'construct foundation' must await the arrival of the materials. The nodes are given a code for identification purposes. For convenience, it is logical to number from left to right.

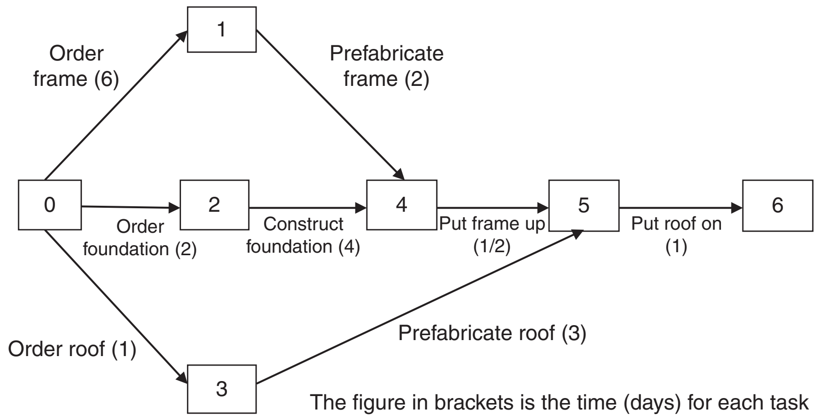


Fig. 15.7. Job flow diagram for an example task (hay barn construction) – jobs, times and completion sequence.

Determining the critical path

To determine the critical path the flow or network diagram is used to calculate:

- The earliest time each sub-job can be commenced. This is dependent on what sub-jobs must precede it and the earliest time they, in turn, can be commenced; and
- The latest time each sub-job can be commenced without holding up the final completion date of the overall job. Whether a particular sub-job can be delayed without affecting the overall completion date depends on whether there are other sub-jobs that take longer than the particular one under consideration.

The critical path is determined by examining each sub-job to see whether there is a difference between their earliest and latest commencement times. If there is no slack or surplus time (i.e. earliest = latest time) the sub-job forms part of the critical path. There will be a connected set of sub-jobs from start to finish with no slack time. *This connected set is then the critical path*. If one of the critical sub-jobs should be delayed, the completion date of the overall project will be delayed. On the other hand, if a sub-job for which there is some surplus time is delayed, this may not delay the completion time of the job depending on the length of the delay.

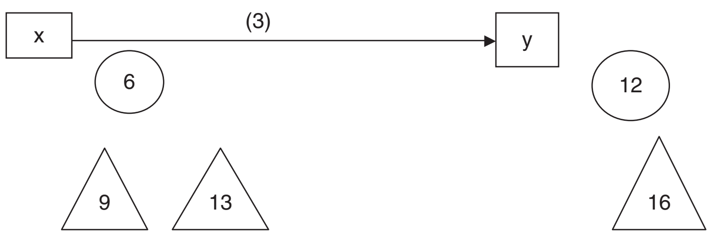
To reinforce the above discussion, consider the example problem. Firstly calculate the earliest commencement times by starting at node 0 and initially determining the longest time it takes to get to each of the nodes connected to node 0. Where there is only one arrow leading to a node, the longest time is simply the time to complete this sub-job. Where there are two arrows (or more) leading into a node, the longest time of these gives the earliest time at which any sub-job emanating from the node can be commenced. This process is then continued throughout the network for other nodes. Beside each node is placed a figure representing the earliest time at which sub-jobs emanating from the node can be commenced. These data are used in calculating the earliest time for succeeding sub-jobs. Carrying out this process for the example gives the data shown in Fig. 15.8.

Secondly, calculate the latest commencement times by starting at the node representing the completion of the overall job (node 6 in this example) and working backwards through the network. The latest commencement time for each node preceding (call it node B) a particular node (call it node A) is:

Latest commencement time for node B
= earliest commencement time for node A *minus* the time to complete the sub-job depicted by the arrow joining the nodes.

Carrying out this process for the example gives the data shown in Fig. 15.9.

There are only two nodes in which the earliest commencement time does not equal the latest commencement time. In realistic examples these could well be *sequences* of nodes in which earliest commencement times are not equal to latest commencement times. In such cases there are two latest commencement times for a node. For example:



Node × has two latest commencement times depending on whether node y is completed in 12 or 16 time units.

Having calculated the earliest and latest commencement times the surplus, or slack time (sometimes called float time), is calculated by subtracting the

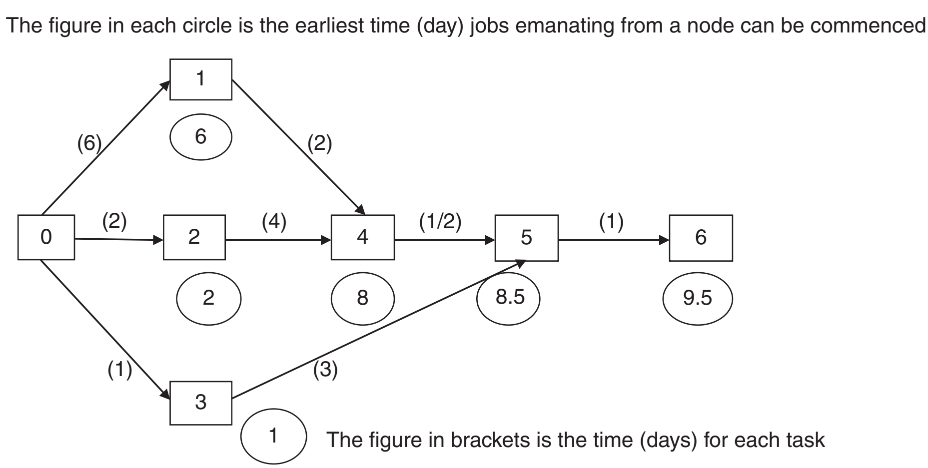


Fig. 15.8. Earliest time to complete each sub-job in the hay barn example job flow diagram.

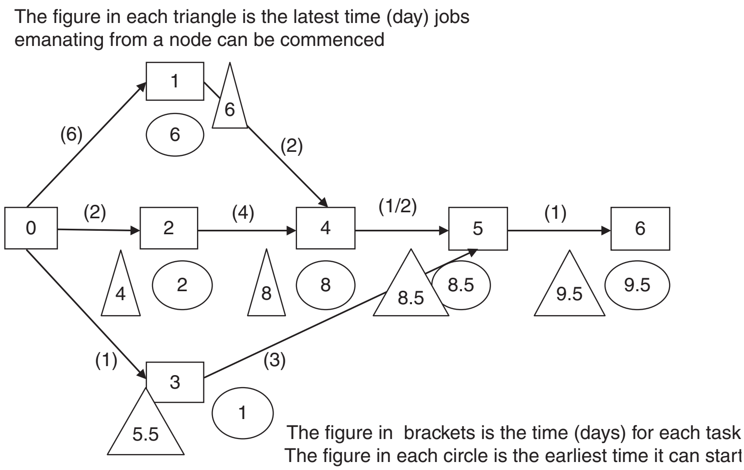


Fig. 15.9. Latest time to complete each sub-job in the hay barn example without causing a delayed ending.

earliest time from the latest time. (Note that where there is more than one latest commencement time for a node there is more than one surplus time estimate.) For example, given:

the surplus time for x is 13 – 6 = 7, and the surplus time for y is 16 – 12 = 4.

Carrying this out for the example problems gives the data shown in Fig. 15.10.

The critical path is now self evident. This is the path joining the nodes that have zero surplus time. In the example this is the path:

$$0 \rightarrow 1 \rightarrow 4 \rightarrow 5 \rightarrow 6$$

In realistic problems there may be more than one critical path. Note that sub-jobs emanating from nodes 2 and 3 can be delayed 2 and 4½ days respectively without affecting the completion date of the overall job.

Using information obtained from the flow diagram

Besides the critical path, other uses of CPA are listed below.

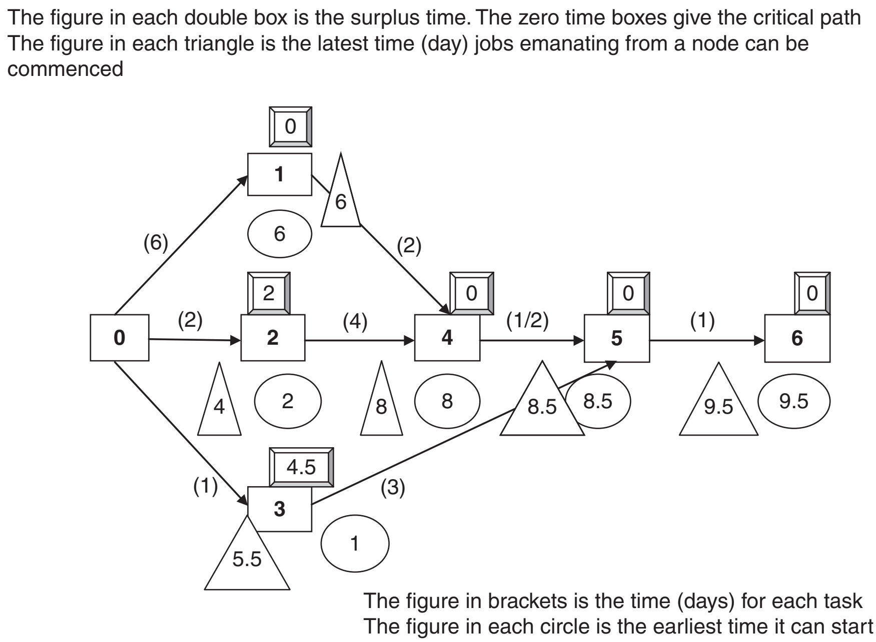


Fig. 15.10. The critical path in the hay barn example, showing surplus (or slack) time.

The commencement date necessary

CPA indicates the shortest time a job is going to take. If, therefore, a job must be completed by a certain date the CPA will indicate the latest date by which it must be started. There is often a loss of profit in finishing jobs too late, so it is important to start on time. For example, if the wheat is not harvested on time the yield will be reduced. It also indicates the latest date by which each sub-job must be started if the project is to be completed on time.

Ordering materials

CPA emphasizes the importance of ensuring that all materials are ordered and obtained in reasonable time. It clearly indicates when each item must be obtained if job completion is to be on time.

Re-scheduling resources

Given the nodes at which surplus time exists, and the critical path, the overall completion time can be shortened by re-scheduling men and machines from jobs where there is slack time to jobs on the critical path. The critical path will be shortened and men and machines which might otherwise be idle are utilized more efficiently. This will mean that the non-critical sub-jobs will take longer, but this is of little concern. This re-scheduling of men and machines is important where the completion time affects profitability and there are limits to how soon the overall job can be commenced.

Altering the critical path

Besides re-scheduling, the critical path can be reduced by employing additional resources (men and machines) or using overtime hours. These services are obtained at a cost, so the expected increase in returns from earlier job completion must be compared with these marginal cost increases.

Choosing between alternative job methods

In some cases there could be a number of alternative ways of performing a sub-job. For example, in the hay barn building example it was assumed the roof was prefabricated before lifting into place. An alternative may be to construct the roof 'in place'. CPA enables comparing alternatives, thus allowing for the, usually, smaller cost of slower methods. If surplus time exists, the less expensive technique can be used without affecting the completion date.

Multi-job analysis

On most farms a number of main jobs are usually being carried out concurrently. There are often opportunities to 'dove tail' jobs so all jobs are completed on time. The example looked at a single job but CPA could be used to examine the completion of several jobs. The link between the different jobs is the use of surplus time. Machines and men which would normally be working on a particular sub-job can be diverted to a sub-job on another job if surplus time exists in one and not the other. Whether this rescheduling between main jobs is worthwhile depends on the expected return from each as a result of earlier completion.

Sequential control

Sub-job completion is unlikely to occur precisely as predicated. As a job progresses it is often necessary to re-calculate the CPA flow or network diagram and update all the sub-job plans, material ordering times and so on. This is the same procedure used in budgetary control. In fact, updating the CPA diagram is a small section of the overall budgetary control process.

(Note that large industrial firms would be regularly updating their critical path estimates and thus hiring additional men, and other resources, as required to fulfil contracts. Because of the repetitive nature of the process the procedure is readily computerized.)

Uncertainty

The analysis method explained has largely ignored uncertainty. In reality the sub-job completion times, the ordering times and the cost will all be random variables. Thus, each node will have an *expected* earliest and latest commencement time, and also a time variance. The estimated critical path/s will be the *expected* critical path, so that in practice some other path may turn out to be the critical one. These uncertainty factors can be taken into account in CPA but this obviously adds to the complexity. In general this is not worthwhile in farming studies. In their place *subjective hedges* against uncertainty could be included. For example, completion times which are very uncertain could have a 'hedge factor' of a number of days added to them.

Application in farming

Farm problems should in theory be solved using the principles outlined above. However, most will be solved mentally in contrast to a formal analysis as most jobs are not that complex, with sub-jobs seldom being heavily inter-related. The flow diagrams are simple with at most one or two concurrent sets of nodes. Consequently the cost of setting up a formal CPA analysis is seldom warranted. An adviser/consultant, or a farmer, should, however, base decisions on the CPA principles with, usually, only a few simple calculations performed.

On large farms employing many men where complicated development work is being carried out, a formal analysis may be worthwhile. Formal CPA may also be warranted on some intensive farms such as a large horticultural unit producing crops at all times of the year and utilizing large machinery investments.

15.9 Queuing Analysis

Background

Queuing or waiting line analysis covers problems where there is, usually, a fluctuating demand for a service, so a decision on the quantity of the service facility is required. The traditional example is checkout points in a supermarket. The checkout and associated attendants provide a service but the fluctuating demand from the varying number of customers arriving can cause waiting. If the queues are too long customers will go to other shops. The analysis must determine the optimal number of checkout points to give maximum profit. The more checkout points the greater the cost, but the greater the number of customers happy to shop.

Queuing problems are characterized by four major factors: (i) the form of the demand for the service; (ii) the form of the supply of the service; (iii) the cost of the service provided; and (iv) the effect on profit of letting a queue develop. The queue is a function of the demand and supply of the service.

The 'service' in queuing problems can be extremely varied. For example, it can range from the service provided by the local medical doctor to the service provided by a lubrication bay in a garage. All such problems are amenable to queuing analysis. In most cases all the variables are stochastic (random variables), so the queue length at any one time will be a random variable. The loss in profit will, in turn, be a random variable, and so on. Given a linear utility function the problem is to determine the number of service people, points or facilities so the *expected marginal cost* of additional facilities equals the *expected marginal return* resulting from the additional facilities (assuming capital is not limiting so that there is no restriction on the number of facilities).

While the demand for the service is usually a random variable, the supply of the service is often known with certainty. For example, the number of cars a vehicle testing station can handle with x attendants will tend to be known with certainty.

Farm examples

There are a large number of queuing problems on any farm. Any system in which a service is required is a queuing problem. Some examples are:

- The use of a welding plant and of an experienced welder in the farm workshop. The demand fluctuates as breakdowns are a random variable. At certain times of the year, on large farms, a queue or backlog may develop, especially where the labour has other fixed jobs to do as well.
- The use of drafting facilities in a set of stockyards. At shearing, or perhaps weaning, more than one set of facilities may be useful. The return from a second set might be the saving in overtime payment to employees where there is only one set.
- The size and type of milking shed (parlour). The demand for milking facilities varies at different times of the year, so with a small shed the milking staff may have to spend more time milking at peaks and less working on the farm.

An off-farm agricultural example is providing unloading facilities at public stockyards. If an agency does not provide sufficient unloading points, farmers and carriers might use other agents. A similar example is how many unloading facilities to provide at a railway station, or airport.

Solving method

The procedure is to compare the costs and return of providing alternative quantities of the service. In some cases it will also be necessary to consider the variances.

Consider a simple example. A cropping farmer wants to know whether to employ contract header harvesting services to supplement his own harvester. (He could also consider purchasing a larger header, or even a second header.) In this example the header provides the service ‘heading’, i.e. removing crop heads and separating them from other plant material. The quantity of ‘heading service’ provided per day can be regarded as a random variable due to the possibility of breakdowns, problems of getting spare parts and so on. The demand for the ‘heading service’ comes from the crops. This is also a random variable. The area of crop becoming ready for harvesting per day varies according to the weather conditions and other factors. The queue, or waiting line, is the area of crop waiting to be harvested. The queue length is also a random variable, not only because the arrival rate (crops becoming ready) and the service rate are random variables, but also because the weather affects whether a crop remains a member of the queue. The costs are providing additional headers, larger headers, or contractors, or some combination. The saving in profit is from preventing crops, which are ready for harvesting, standing in the paddock reducing in yield and/or quality.

Assume the farmer has a header which is capable of harvesting, on average, 15 ha of wheat per day. Assume that only 150 ha of wheat is grown and that on average 20 ha arrive in the queue every day (i.e. 20 ha ripen/day). Assume that once in the queue a ha stays in the queue (unrealistic).

After 1 day there will be 5 ha outstanding. After 2 days there will be 10 ha in the queue and so on. The alternatives available are: (i) to have the ripe crops standing until the farm header can eventually harvest all the wheat; or (ii) employ a contractor.

Assume that for every day 1 ha of wheat is left standing after ripening there is a loss in profit of $40. It costs $170/ha for contract harvesting and this can be carried out as soon as the crop is ready. To solve the problem the total number of 'hectare standing' days must be estimated. (Note that in some problems the loss per 'standing day' may not be a constant as assumed here. In these cases an assumption is necessary on which hectares in the queue are harvested each day. A common assumption is that those waiting longest are harvested first. This may not be the most profitable. Technically, this assumption on service priorities is called the 'queue discipline assumption'.)

To determine the total number of 'hectare standing' days, it is necessary to count from day to day the number of hectares left un-harvested, given that 20 ha arrive per day and 15 ha can be harvested. Thus:

Day number	1	2	3	4	5	6	7	8	9	10
Hectares ready	20	25	30	35	40	45	50	45	30	15
Cumulative total becoming ready	20	40	60	80	100	120	140	150	–	–
Hectares harvested	15	15	15	15	15	15	15	15	15	15
Hectares not harvested	5	10	15	20	25	30	35	30	15	0
Cumulative total harvested	15	30	45	60	75	90	105	120	135	150

where hectares ready = 20 + ha not harvested from previous day. Once all hectares are ready the 20 is dropped from the calculation.

Thus, the total number of 'hectare standing' days is the sum of the hectares not harvested now. This is 185. The loss in profit is therefore 185 × $40 = $7400.

Alternatively, a contractor could be employed each day to harvest the outstanding 5 ha (although in reality this is doubtful, so there would have to be some waiting until a reasonable area was ready.) If the crop is harvested at the rate at which it is ready, 7½ days elapse. Thus, a contractor would be required on 7 days to do $7 \times 5 = 35$ ha. The cost would be $35 \times \$170 = \$5950$. Thus, by employing a contractor the queue length is maintained at zero level so that the \$7400 profit loss due to standing is prevented.

In this example many simplifying assumptions have been made. It is clear that if all these were removed the analysis would become complex.

Concluding comments for farming

Any farm has many queuing problems. If their stochastic nature is recognized the analysis becomes complex. However, the economic significance of the problems is usually not sufficient to warrant this complex analysis. Most problems must be solved using simple estimates and, often, subjectively. The importance of studying the structure of queuing problems is to provide the framework within which the subjective, simplified estimates can be made.

Further Reading

Hillier, F.S. and Lieberman, G.J. (2005) *Introduction to Operations Research*, 8th edn. McGraw Hill, Boston (for inventory and queuing problems).

Operational Research Society (2001) *Critical Path Analysis in Practice*. Tavistock Publications, reprinted by Routledge, London (for critical path analysis).

16 Concluding Comments – Review and Summary

Every farmer is constantly making resource allocation decisions even if the decision is to simply follow what the neighbours are doing! However, most farmers are more active than this, and many carry out simple calculations, sometimes on paper, sometimes in their head, to help with the decisions. Seldom, however, do they create and use economic models. Some farmers review what the commentators and researchers are writing on optimal decisions, and subsequently come to a conclusion, often through intuition. Indeed, research has shown the better farmers tend to avidly read what is on offer to aid their intuition and decision making. They also attend talks, field days and workshops. This means the results of farming systems analyses that are reported in various media, and demonstrated, can have a major impact on good decision making in any primary producing community. This book has been about methods of carrying out these analyses and is therefore useful to existing and potential advisers/consultants and research analysts. The results of analyses may also have value in developing government policies for primary production. Work on representative farms, and surveys, allows assessing the value of suggested policies.

While some analytical methods are not reported here (e.g. Markov Chains), most of value have at least been introduced. Indeed the book must be regarded as introductory in some respects, as there is certainly much more to be learned. In any area that needs further development the general agricultural economics and operations research literature should be searched. Similarly, the farm management literature should be researched, though in recent years there has been less work being published in this area partly as new analytical methods have not been developed. What has appeared, however, is an interest in learning about farm managers' characteristics and abilities with a view to improving their managerial skill (see, for example, Nuthall, 2009, 2010). For some reason the manager as a skilled decision maker has been ignored as a research focus in the last few decades with preference being given to researching farm systems analytical models, and the systems themselves. This book, together with the two mentioned above, reverses this trend as they cover both areas including the human factor and methods of rational analysis, thus forming a complete package. It seems totally logical to study the farmer decision maker in his own right, for the farmers are clearly the key people to making the correct investment and operational decisions. Indeed, the majority of farmers worldwide rely on intuitive decision making so studying, and subsequently, developing their inherent skills can have major payoffs.

For analytical methods, as previously stressed, the framework, or blueprint, for studying farming systems is provided by production economics. Many of the important components of this theory have been summarized in an appendix so students can remind themselves of this basis. However, for most practical analyses production economics is not suitable due to the complexity of the maths once realistic production economics models are created. The consequence is that study and conclusions on improved farming systems must be made from either observing the real world (surveys), or creating realistic models that can be used for experimentation.

Once surveys and/or model experiments are carried out it is important to be sure the results are robust. Thus any analysis must include appropriate statistical analyses to ensure conclusions are not due to chance effects. The real world is made up of random variables, so any observation might simply be from the extreme of a variable distribution and seldom repeatable; similarly with the results of experiments. Consequently, any analyst must have a reasonable working knowledge of statistical theory. While reference is made in the book to various statistical questions and factors, details of the procedures are not covered. Many statistical texts are available to provide this theory, and even some have been written specifically for economic analysis. As an aside, it is interesting to note that the first statistician of real note was involved in assessing agricultural experiments so, if you like, agriculture is the father of statistical method, which is now used in all areas of human endeavour (R. Fisher was the early mathematician who studied at Cambridge and then, after a range of short-term appointments, worked at Rothamsted Agriculture Research Station in England where he developed the techniques to analyse experimental results that checked whether apparent differences between treatments were real and significant).

Thus, to become a good analyst of farming systems any student must start with a working knowledge of both production economics and statistical analysis. But, furthermore, equally as important is developing a good knowledge of analytical models and methods. This is where this book comes in – it provides an introduction to the common methods. If for no other reason, a good knowledge of the methods enables an analyst to select which technique to use when faced with a need to analyse farming systems. Often an analyst is familiar with only a limited range of methods, so the results can be less than desirable if the wrong analytical technique is selected as well it may if the analyst is not reasonably familiar with the range available. The remainder of this chapter, therefore, reviews the methods available and provides contextual comments.

Returning to analytical methods, the available approaches can be divided into two streams. Faced with a research problem the analyst can obtain a conclusion from *either* observing the real world (a survey) *or* creating some kind of model of the problem, allowing experimentation. Sometimes a mixture might be appropriate. For example, before developing a linear programming model of a liquid milk-supplying farm a small survey might be conducted of existing farms to discover the general nature of the management systems that need exploring.

These two approaches are referred to as *ex post* and *ex ante* studies. Borrowing these terms from Latin enables pointing out that the survey approach is analysing what has happened in the past (ex post, 'after the event'), even if the very recent past in many cases. In contrast, the modelling approach is concentrating on the future (ex ante, 'before the event'), even if the basis of the models is grounded in the past.

This book has covered the common methods in both these research 'arms'. Sometimes a researcher has a choice of which system to use, in others not. Clearly if you wish to know what is happening in the community some kind of survey is necessary, but if you wish to explore improved systems you have a choice of either method, or some combination. The latter assumes there is sufficient information available to create the model. If not, the research must start with some kind of survey and also, usually, a literature search.

As pointed out, surveying can be either *quantitative* or *qualitative*. If facts and figures are necessary then you will be using a quantitative survey. For example, comparing the profitability of irrigated farms with those not irrigating. Sub-problems include the level of irrigation and the crops to be irrigated. Where opinions and ideas are to be explored then a qualitative survey may be appropriate. It is also said that qualitative surveys enable inquiring with detail and depth into problems. A smaller number of farmers are interviewed so it is possible to spend time prompting detailed and encompassing replies, which are usually electronically recorded for later analysis. So, for example, if the researcher is interested in obtaining details of how farmers gather and analyse information leading to decisions, a qualitative survey is likely to be necessary. Further, qualitative surveys often use the subjects to suggest ideas in free-flowing interviews. In contrast a quantitative survey requires a questionnaire of some kind, so the researcher has already decided on the information required having set up a hypothesis to be tested. In most qualitative surveys the researcher tries to approach the situation with an open mind and lets the interviewees give rise to the hypotheses, which can later be related to the literature. On the other hand, quantitative surveys would normally be started with a literature review to obtain what is already understood about the problem before deciding on the information to collect, and subsequently, further develop the ideas and theories already in existence. Overall, however, the problem should dictate which approach is used.

In summarizing surveys, it should also be noted that the resolution of some problems can be helped through a literature review. If the survey is about factors and ideas that have previously been considered, the body of literature is clearly the first place to look. Young researchers are often quick to collect new information before checking out studies that others have carried out. The saying 'there is nothing new in the world' should be respected.

The book contains descriptions of the methods for analysing survey information. Qualitative surveys lead to sorting out ideas and opinions that summarize what the respondents have had to say. Often this is a tedious transcript review, though software has been created to assist in sorting out the basic ideas and the subsidiary sub-ideas that follow in a kind of tree structure.

For quantitative surveys, it is apparent that a whole range of analytical approaches can be taken. These start with simple tables of the information collected, to the creation of production functions using regression analysis, and to more sophisticated models using, for example, structural equation modelling. The book introduced most of these methods. There was also an extensive description of benchmarking and comparative analysis in which survey information is used to divide the sample into a group of high-achieving farmers relative to the remainder. Comparisons are used to suggest successful farming systems and standards to aim for. Students should fully understand this approach, for it has common appeal. An emphasis must be placed on ensuring robust statistically sound results, for there is a temptation to simply make comparisons between groups without checking whether differences are causal and other than chance effects.

For ex ante studies, the book has sections explaining the common analytical methods. These start from simple budget studies, including enterprise gross margins, which all rely on the budgeter's skill in estimating the output relative to the input levels and system configurations. The calculations give the profit levels for each proposed system to enable comparisons. A sophistication of budgeting is formularizing the budget to create a parametric budget that enables quickly exploring the impact of critical parameter variations (such as product prices and yields). Farmers frequently create budgets for their farms, and these often get converted into cash flows in which the monthly cash, both in and out, gets related to the starting bank balance, thus creating a forecast of the monthly balance relative to any overdraft limits. These budgets also form the basis of the management cycle of planning–execution–control. As the year progresses the actual cash balances are compared with the forecast balances, allowing the farmer to make changes to overcome undesirable deviations.

Consultants and advisers frequently help farmers with all their budgeting and monitoring work. In contrast, an analyst or researcher may be involved in exploring farming systems for a whole area. The results of such studies then get used by farmers and consultants in the area. In this work the case-farm unit is often selected to represent as many properties as possible. Thus, there is a section in the book about selecting representative farms, for it makes sense to base studies on situations that will have wider appeal than a specific individual farm situation.

Then, of course, there are the more sophisticated analytical methods, which are used by researchers and the better trained consultants. The most commonly used, and very useful, is linear programming, which is used to explore many factors about improved farming systems. Two chapters were devoted to linear programming (and an appendix on the solving method) as it determines a farming system that maximizes a quantified objective, often profit, subject to the restrictions imposed by the resource levels available. The computer algorithm necessary to solve real problems, the simplex method, is readily accessible for most computers. The approach embodied in linear programming was introduced through describing gross margin analysis and programme planning. The latter provides a method of maximizing the return to the most limiting resource. The first step is to work out which resource is in fact the most limiting, and then to select the production processes that give the greatest return per unit of this most limiting resource. Programme planning requires considerable skill from the analyst to ensure the objective is achieved. Today, this technique is not practical given the general availability of computers thus leading to the use of linear programming, which has the same objective but uses a solving method that guarantees the optimal solution is found. In this respect it will be recalled that the features of the general farm decision problem can be utilized to create a very efficient solving process (an optimal solution will be a corner point on the production possibility curve – something suggested by production economics. Thus, in obtaining a solution only the corner points need examining, which provides an enormous computational advantage. Furthermore, there is a one to one correspondence between production economics and linear programming).

While linear programming is a very efficient analytical system, it is not always a good model of farm decision problems, particularly where non-certainty is a major factor, and similarly multi-period decision problems that reflect the ability of farmers to constantly re-plan as conditions change, which frequently they do in the non-certain world of weather and markets. Consequently, chapters were included in the book explaining both Bellman's dynamic programming and general systems simulation. The latter approach requires a specific symbolic model to be constructed to represent each decision problem being studied. Once developed the model is used to conduct paper experiments using high-speed computers. As the model does not have to conform to any pre-specified relationships (as in linear programming where the relationships must all be linear), non-linear and stochastic situations are readily modelled. The downside is such models are costly to develop and experiment with, and require well-trained researchers and computer programmers.

Dynamic programming similarly requires a specifically designed computer program for each situation, though computer packages containing the building blocks are available. They are also available for creating simulation models. Dynamic programming is a model that is almost a perfect reflection of real-world farm decision problems by encompassing their non-linear, multi-period stochastic nature. So often in life, however, there has to be a trade off – in this case, while conceptually the model is perfect, in reality the computational burden is very high so in many cases only simple farm problems, or part-farm problems, can be solved with dynamic programming.

Finally, the book had sections on four specific part-farm problems: namely inventory, asset replacement, critical path and queuing analyses. The first two are common problems for which simple models were explored, whereas for the latter two only the general outline of the problem structure was explored for the significance of the problems in farm situations only justifies subjective decision making.

Overall, any analyst must weigh up the problem to be analysed by considering its importance and the potential improvement in outcomes if optimal systems could be found. The analyst must also initially examine the problem to discover its structure so it can be compared with the assumptions each analytical model contains. A conclusion is then reached as to the most appropriate model and approach. The result may be to conduct a survey of some kind, or it might be to create a model enabling experimentation. To enable a robust decision the researcher must have a good knowledge of all techniques, something which this book helps provide.

This commentary and review has not mentioned the contents of all chapters. In particular, the chapters explaining non-certainty and cost–benefit/investment analysis. An understanding of these topics, which can impinge on all analytical techniques, is important in deciding which of the associated methods need introducing in the model or approach selected. In addition, the discussion on objectives needs to be noted. Farmers have complex objectives, as do their families and these may be just as relevant as the farmer's. As the goal of any analysis must be to find systems that encompass the farm unit's objectives, any study must start with assessing these objectives. This is not always easy due to the farmers' inability to enunciate just what they want. Often astute observation is required as past decisions will reflect, to a certain extent, the objectives. The other difficulty is that most farmers', and farm families', objective set will be unique. The consequence is a researcher working on, say, representative farm models, must experiment with a wide range of objectives to enable individual farmers to isolate the systems that will best fit their requirements.

All farm decision problems have risk and uncertainty as components. In simple techniques such as budgeting, the analyst must subjectively allow for non-certainty using various approximations such as downgrading a price assumption if the price is deemed to be particularly risky, and as another example, ensuring high feed reserves in a variable feed-production climate. Systems simulation and dynamic programming cater for non-certainly particularly well as would have been noticed, so simplifications are seldom necessary when using these models.

Where non-certainty is regarded as being particularly important these models should be favoured. Where the base problem is relatively simple, dynamic programming can be used and is preferred. But for the more complex situations, due to the experimental approach used with each unique model, systems simulation will be a more realistic option. This does not mean, however, that linear programming should be discarded in such cases for there are a wide range of adjustments that can be made to the model to allow for non-certainty. These have not been covered in detail in the text, but a search of the literature will quickly unearth possible approaches such as, for example, minimizing the profit standard deviation subject to a minimum expected profit. Many other possibilities exist. As always, however, the more sophistication is added to the models the larger and more difficult to analyse they become. It is always a trade off.

For time-dependent analyses (involving time preference), any model encompassing many time periods must include the concepts of net present value, discounting and, possibly, the internal rate of return. Thus, for example, if a survey is used to explore, ex post, forestry methods and species, the objective must be to find the system that maximizes the net present worth. This is an extension of a simple budgeting analysis. Similarly it is possible to create a multi-period linear programming model involving, say, a programme to develop a block of land over many years. This kind of model will probably need to include time analysis principles to maximize the net present value, though in some cases the objective might be to maximize the internal rate of return. The reader, however, will remember the discussion on whether the IRR is an appropriate objective. And of course, dynamic programming needs to have time preference principles included where the period spans are appreciable.

It must always be remembered that no matter how carefully constructed an analytical model might be, its successful use will largely depend on the accuracy of the data used in the model. For example, if the price assumptions are wrong the results will most likely be totally misleading, and similarly for all the technical assumptions included. For price and cost data the critical factor is to ensure their relativities are accurate in contrast to their absolute values, though if both are correct the results will be of greater use. The ability to forecast prices is a study in itself and relies on creating supply and demand models from historic information and their use with the relevant and up-to-date data. While most farm management analysts rely on forecasts from experts, some students may wish to study this area and consequently should search out a reputable text on price analysis and forecasting, or attend a suitable course which will no doubt be associated with a suitable text. Price analysis relies on collecting time series data and analysing them using, probably, some form of regression analysis, though other alternatives exist.

Overall, besides a good knowledge of the analytical methods outlined in this book, an analyst must view any work with a critical mind. This means the researchers must be critical of the method selected, of the data collected, of the results obtained, but above all, be critical in the review of past studies that relate to the problem to hand. In essence the researcher must be able to justify to other professionals and farmers the totality of the work and defend the selections made. Thus, good reasons must be given for the selection of the method, the data used, the experiments carried out and the conclusions. In this critical view of analyses, it is useful to consider what many regard as the criteria for assessing research quality. The questions to be asked include (based on Marsland *et al*., 2001):

1. Can you have confidence in the *truth* of the findings? (internal validity)
2. Can you apply the findings to other contexts or with other groups of people? (external validity)
3. Would the findings be repeated if the enquiry was replicated with the same or similar objectives? (reliability)

4. Can you be certain that the findings come from the subjects, or the results of the model experiments, rather than reflecting the biases, motivation and perspectives of the investigators? (objectivity)

These questions provide cause for reflection. Remember, farming is a complex operation involving most aspects of human endeavour. In this sense getting the farming systems right is an exciting and complex challenge. Many farmers do manage to achieve efficient and profitable production by learning the lessons of experience through a reasonable intelligence and careful observation as well as anticipation of future scenarios. However, even more farmers are not this able and will benefit from the outcome of a wide range of farm management research and analyses. This book has provided, primarily, potential consultants and researchers, but some farmers too, the means of carrying out positive studies of improved farming systems. Students who learn well, gain experience from trying the techniques and procedures, will reap the rewards of high achievement both in satisfaction and employment.

References

Marsland, N., Wilson, I., Abeyasekera, S. and Kleih, U. (2001) *Combining Quantitative(formal) and Qualitative(informal) Survey Methods*. National Resource Institute, University of Greenwich, UK.

Nuthall, P.L. (2009) *Farm Business Management – the Human Factor*. CAB International, Wallingford, UK.

Nuthall, P.L. (2010) *Farm Business Management – the Core Skills*. CAB International, Wallingford, UK.

Appendix 1: A Synopsis of Production Economics

Introduction

To make clear conclusions on useful analytical methods, guidelines are required. For farming systems the theoretical base is provided by 'production economics'. This is the study of the decision rules, which give optimal decisions for a measurable objective function where physical production relationships, prices and costs are known with certainty. In reality this is not the case, but at least this situation provides a framework from which to conclude on appropriate analytical techniques.

The real world has uncertainty and multi-component objective functions. For example, most farmers and their families are interested in both cash and leisure, and probably many more 'outputs' as well. They are probably also interested in operating a long-run sustainable system. Modern texts on production economics, of which a search of the World Wide Web will come up with many, usually expand the base theory to include both uncertainty and multiple objectives. For the moment, this synopsis merely summarizes production economics without these complications. It will be clear from the chapters in the book how some of the complexities can be analysed. Furthermore, this synopsis only covers the key components that are important in this book; for example, detailed cost relationships are not considered.

In effect, this synopsis serves as a reminder of the basics for students who have already studied production economics and, for those who have not, it provides a brief introduction sufficient to understand most references to production economics used in this book. For more detail on the basic concepts see, for example, Barnard and Nix (1979, last reprinted 1999), and for a detailed treatise see, for example, Beattie *et al.* (2009).

The Decision Problem

Most farmers are involved in production using a limited set of resources. They have a bank balance (sometimes negative), various quantities of different fertility land together with its attributes of slope, rainfall and other production-defining climate components, a labour force with various attributes, a set of machinery and buildings, stocks of inputs (e.g. stored feed) and a defined managerial ability. In a nutshell, there is a set of fixed resources from which the farmer must produce output that maximizes, or satisfies, various objectives.

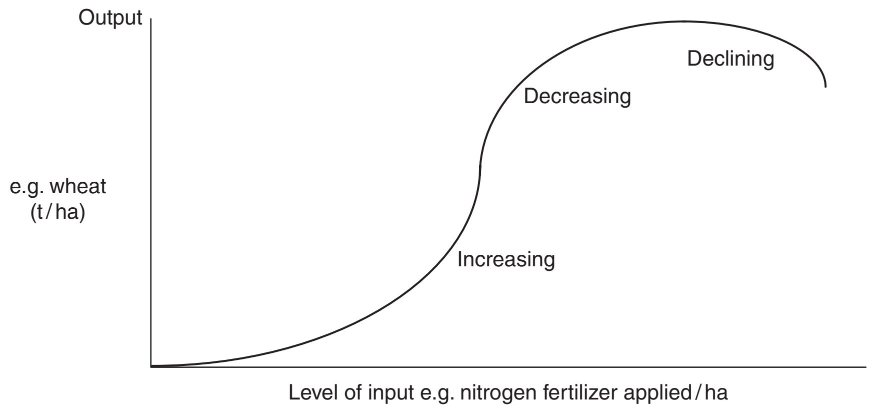
In combining and using this limited supply of resources the farmer must find a solution to three basic questions. These are:

1. The combination of products to produce (which might end up being only one, e.g. milk, or a mixture of products, e.g. one-third of the farm devoted to wheat production, a third to clover seed and a third to meat and wool from sheep).
2. The level of output of each product (either in total, or on a technical unit basis, e.g. per hectare) as, depending on the intensity of production and input levels used (e.g. fertilizer), the production levels can vary.
3. The combination of inputs to use in producing each product, e.g. large quantities of irrigation water and little fertilizer, or vice versa, or some combination. The objective is to find the combination that creates the defined level of output per technical unit (e.g. 1 ha) at least cost.

These three basic questions are not independent and must be solved as a total entity with due regard to the set of fixed resources. The optimal combinations depend on resource use, output, prices and costs. The least cost production methods will most likely depend on the total production level possible with the given resources, as will the output per technical unit. Note, however, that while production economics provides the theory behind what defines an optimal system, seldom can the theory be used in its entirety to solve complex farming problems as the maths involved becomes too complex. Thus, for the most part, practical representations of the basic production economics theory must be devised and used except in very simple 'one product/limited variable input' situations. In this case production economics can be used with the help of calculus to find the optimal points.

Production economics usually assumes the physical production possibilities follow a sigmoid curve so that as more inputs are added production eventually starts to decline. For example, as massive quantities of nitrogen fertilizer are added to a hectare of wheat, the crop will start lodging (falling over) and yield is lost and/or toxic levels of N will be reached in the soil. Just about every form of output follows this sigmoid type production relationship as demonstrated by the thousands of plot trials that have been carried out and by the vast quantity of animal feeding experiments over the decades.

Thus, the typical *production function*, as it is called, exhibits the following curve:



Output initially increases at an increasing rate (area of increasing returns), then continues to increase but at a declining rate of increase, then eventually total output declines. In some cases there might be an area of constant returns between the increasing and decreasing areas. This would be reflected by a straight line joining the two sections.

Algebraically (there is always a one to one correspondence between a graph and an equation), this production function is represented by:

$$Y = f(X_1, X_2, X_3,\ldots X_n/X_{n+1,}\ldots)$$

where f represents that output is some function of the inputs;

Y = the output per technical unit (hectare, or perhaps per animal);

X_1 to X_n the level of each input per hectare that is farmer controlled;

X_{n+1} … the level of non-farmer controlled inputs (e.g. rainfall).

For example, the function might be

$$Y = 34.56 + 3.2X_1 - 0.67X_1^2 + 6.5X_2 + 58X_3 - 1.2X_3^2$$

In the two-dimensional graph only one input is varied, so the curve represents what happens as all inputs but one are held constant. In reality, the situation is multi-dimensional so a production surface, in contrast to a line, exists.

The Factor–Product Problem

First consider the problem of deciding on the optimal output per technical unit. This is question (**2**) posed above. To find an optimal point an objective to be maximized must be defined. While any objective that can be quantified can be used, for this discussion assume the objective is maximum profit as this is largely universal. Thus, the cost of inputs and the price of the output must be known.

Before considering how to determine the optimal point it is possible to make some preliminary conclusions. If the costs and prices are such that at least some production is profitable, it can be concluded that production should *not* occur in either the area of increasing returns nor the area of declining returns. That is, if production is worthwhile at all, moving beyond increasing returns is profitable as for each additional input the output increases, and so profit increases. And clearly beyond the decreasing returns area additional expense is only decreasing the return, so it never pays to go beyond the maximum production point.

To move to the next conclusion consider what is called the marginal product (MP). This variable is defined as the increase in output per unit increase in the input. That is:

$$\text{MP} = \frac{\text{increase (change) in output}}{\text{increase (change) in input}}$$

In the area of increasing returns the MP is positive and increasing, in the decreasing area MP is positive but decreasing, and in the final section of the production curve, MP is negative.

Note that the MP will be constantly changing as the slope of the curve is constantly changing at different input levels, and consequently could be graphed on the production function diagram (and, therefore, has its own algebraic expression). Graphically, MP is the slope of a tangent at any point on the production function curve.

As will be shown below, the optimal level of production is given where the input is increased to the point where

$$\text{MP} = \frac{P_x}{P_y}$$

where P_x is the cost per unit of input;

P_y is the price per unit of output.

Where MP is given as

$$\frac{\text{Change in } Y}{\text{Change in } X}$$

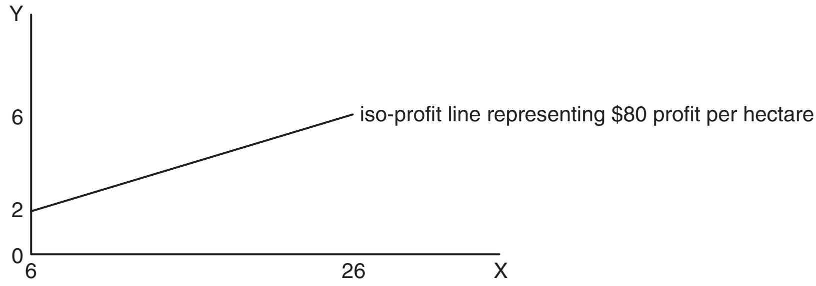
then the optimal point is where

$$\frac{\text{Change in } Y}{\text{Change in } X} = \frac{P_x}{P_y}$$

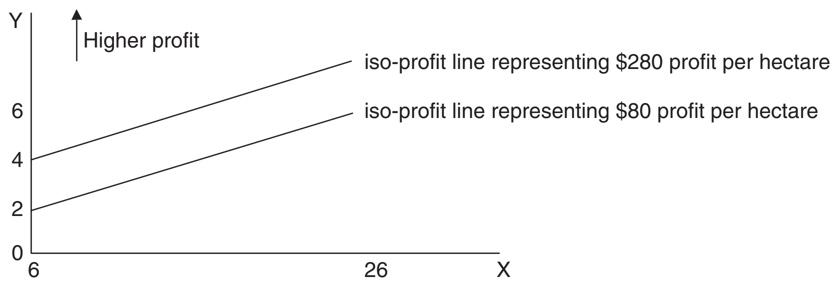
or Change in $Y \times P_y$ = Change in $X \times P_x$

The left-hand-side value is called the *marginal return* (MR) and the right-hand side the *marginal cost* (MC). Thus, the optimal point is where the MR equates to the MC. This is logical. If you go beyond this point there is a loss in profit for the increase in cost will be greater than the increase in return, and if you do not quite reach this point it pays to add more input for the increase in return is greater than the cost increase. Thus, you move to the point where they are equal. Any other point has less profit.

Now consider how this same conclusion can be reached using a graphical representation. Superimpose on the graph lines that represent all points giving a particular level of profit. For example, where the price of the output is \$100 per unit, and the cost of a unit of input is \$20, then a profit of, say, \$80 is represented by all points on the line on the graph below:

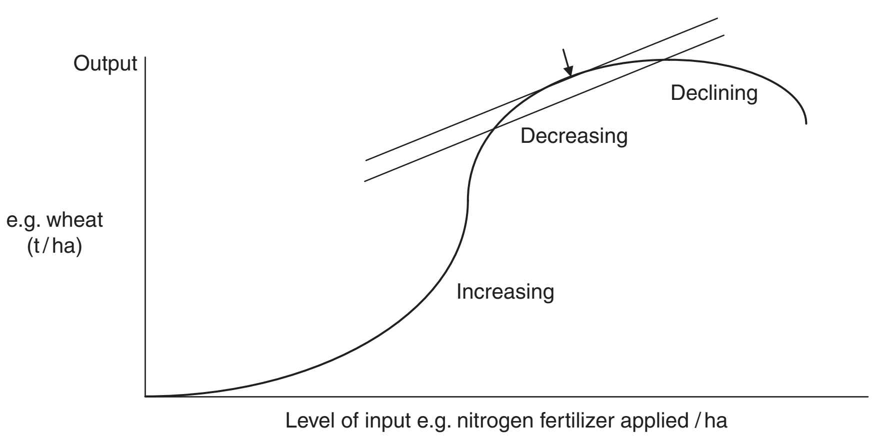


Now consider adding a line for a profit of $280 per hectare (assuming the technical unit is a hectare). This graph will be:



Similarly a whole series of parallel lines can be drawn, each representing an increasing profit. The slope of the line reflects the difference in the price of the output relative to the cost of the input. Thus, as output increases by one unit ($100), the input increases by five units ($100/$20) if the line is to represent all points with the same profit.

As the objective is to maximize profit, the producer must work on the highest iso-profit line that is technically feasible. To find this point, superimpose the production function (production possibility curve in effect) on the graph of iso-profit lines. Thus:



The highest profit point is arrowed, and is where the production function is just tangent to the highest iso-profit line that can be reached by the production curve. Any other point on the curve will be on a lower profit line, or beyond what is technically feasible.

As noted before, the tangent to the curve reflects the slope of the curve, or the marginal product (MP). Now, it will be noted that the slope of the iso-profit line is given by the ratio of the output price to the input cost thus:

$$\text{Slope of iso-profit line} = \frac{P_x}{P_y}$$

i.e. it reflects the number of output units in an input unit with respect to their values.

The optimal point is, therefore, where the slope of the iso-profit line is equal to the slope of the production curve provided it is the point on the highest iso-profit line reached by the production function. Thus, the optimal point is where:

$$MP \frac{\text{(change in output)}}{\text{(change in input)}} = \frac{P_x}{P_y}$$

or change in output × P_y = change in input (usually one) × P_x

Remembering that the MP is the increase in output for a unit increase in input, we have the additional decision criterion that the *marginal return must be equal to the marginal cost* where the marginal cost is the increase in cost (change in input × input price; where the change in input is one unit, the marginal cost is P_x) and the marginal return is the change in output as input is increased by one unit multiplied by the price of a unit of output, i.e. output change * P_y.

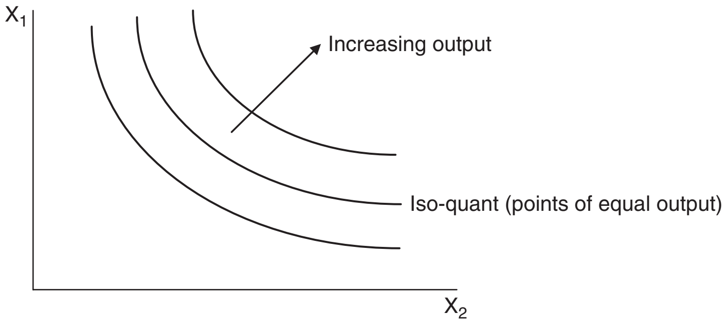
The Factor–Factor Problem

This is problem (**3**) outlined in 'The Decision Problem' above, i.e. finding the combination of inputs that produces a defined level of output at least cost. Ignoring for the moment restrictions placed on output levels due to the resource limitations, the solution to the factor–product problem just outlined gives the profit-maximizing level of output to produce per technical unit. This exposition only looked at one input with the assumption that all others were held constant, whereas in reality many inputs are involved and each can be varied. The problem then becomes multidimensional, but the same principles apply. The factor–factor problem looks at the combination of these many inputs to use in maximizing profit.

To outline this problem it is only possible to graph two dimensions representing two possible inputs. In reality, the problem becomes multidimensional but with the same principles applying. But returning to the beginning, imagine a two-dimensional production function. That is:

$$Y = f(X_1, X_2 / X_3, X_4, \ldots X_n)$$

From this function it is possible to work out the combination of the two inputs that give rise to any defined level of output. Plotting the levels for each output level produces what are called iso-quants, or curves representing combinations of inputs producing equal output. There are a whole series of these curves (iso-quants), each representing a higher level of output. Thus:



The slope of any iso-quant (as given by the slope of a tangent to any point) is given by

$$\text{Marginal rate of substitution (MRS)} = \frac{\text{Change of input } X_1}{\text{Change of input } X_2}$$

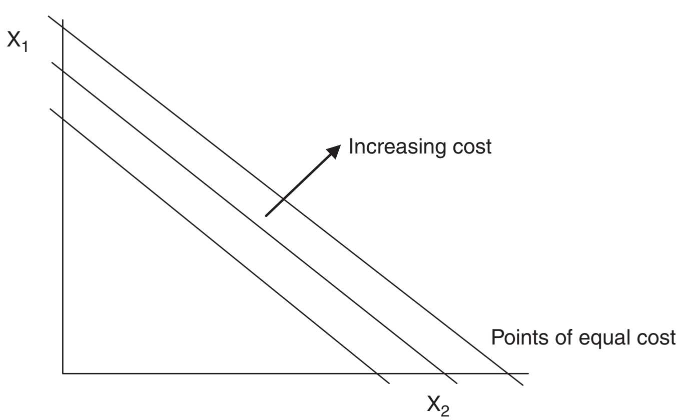
to maintain the same output.

Note that the MRS will change depending on the section of the iso-quant at which it is taken.

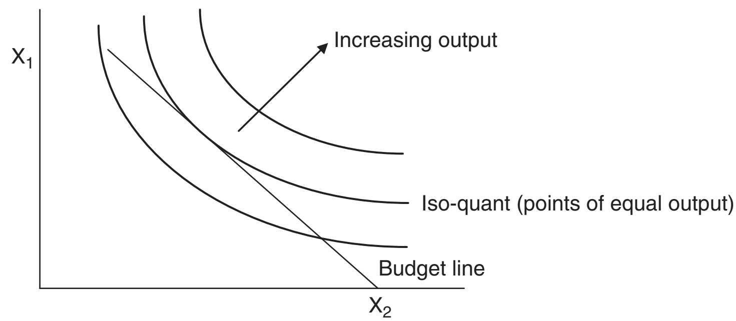
To determine the least cost point the costs of the inputs must now be introduced. These can be used to plot an iso-cost line on the graph (sometimes simply called a budget line). The slope of the line will depend on the relative cost of each input (which could potentially vary depending on the level of output as there could be cost reductions for increased purchase volume. But assume here that one farmer is not sufficiently large to influence the costs). The iso-cost line's slope is given by

$$\text{Slope of budget line} = \frac{P_{x2}}{P_{x1}}$$

For any given level of budget available, a series of budget lines can be graphed thus:



To determine the combination of inputs to produce a particular level of output the budget line must be superimposed on the iso-quant graph. Thus:



The least cost point of production is where the budget line is just tangent to the iso-quant as no other point on the iso-quant will have lower cost. All other points are higher up on the graph and thus reflect greater cost. This point does not, however, tell you the optimal output level, just the least cost combination of inputs in producing that particular output level. The previous discussion gave the optimal output level (but not the combination of inputs for least cost).

In algebraic terms, the least cost point is where:

MRS = the slope of the budget line

or

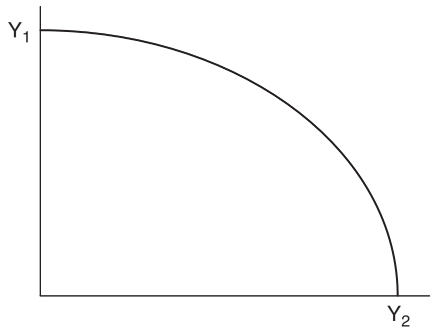
$$\text{MRS} = \frac{P_{x2}}{P_{x1}}$$

In words, the optimal point decision rule (for least cost production) is to produce where the marginal rate of substitution is equal to the input cost ratio. In a multi-input situation the graph is multidimensional but the same principle applies. That is, the MRS must equate the cost ratios for all combinations.

The Product–Product Problem

This final problem is problem (**1**) listed in 'The Decision Problem' (above) and occurs where a farmer can produce more than one product. The objective is to find the combination that maximizes profit assuming production occurs at the optimal output level for each product (problem (**2**) and uses the least cost input combinations (problem (**3**)).

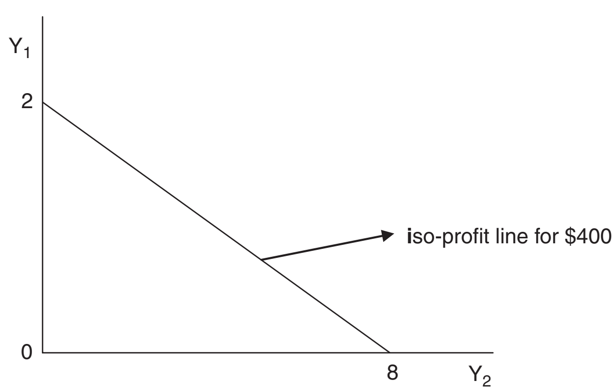
To determine the decision rule that will ensure profit is maximized, consider what is called the production possibility curve. Given the set of resources available to the farmer, and for the moment assuming only two products are possible (to enable two-dimensional graphs), if all resources are devoted to product 1 (e.g. wheat (Y_1)) this will enable output to be at the maximum possible for this product. Similarly with respect to product 2 (e.g. meat (Y_2)). In between these extremes resources can be devoted, in varying proportions, to both Y_1 and Y_2, in which cases there will be output of both products but at lower levels than the maxima possible. Graphically this can be shown as:



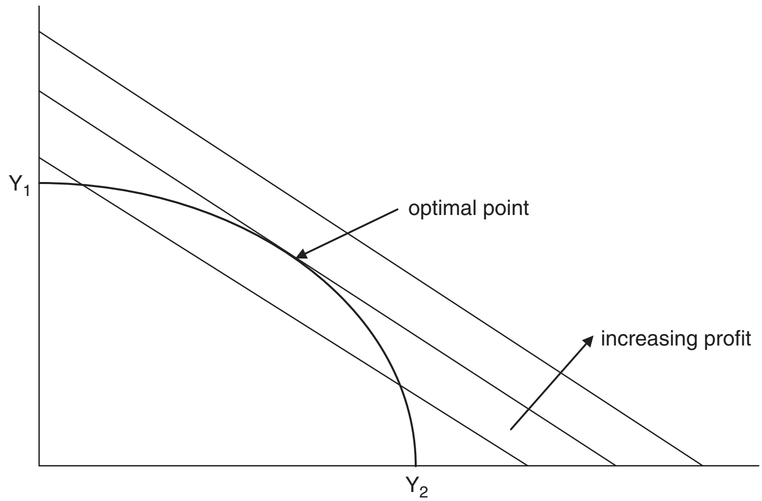
This graph could be for one technical unit, or for the whole farm. It does not matter which as one is just a microcosm of the other assuming resources are split equally between hectares. For a different farm that has different levels of fixed resources, the curve will most likely be in a different position as, for example, the labour levels may not allow reaching these output quantities.

It is also worth noting that this *production possibility curve*, as it is called, should represent output levels using the most efficient methods. Any inefficient farmer will be producing at some point below the curve, in which case an examination of the methods used may enable changes allowing production to lie on the curve. If it is a matter of managerial ability this may take time and training to correct, but if it is simply a technical matter (e.g. time of applying fertilizer) this can be simply corrected.

It is possible to imagine how to determine the optimal combination of products to maximize profit using the same logic as used in working out the least cost combination of inputs. Assuming production is indeed worthwhile such that the prices will more than cover the input costs, to determine the optimal point consider creating a line representing points of equal profit. This line will represent the relative prices of the products. Thus, for example, if P_{y1} is \$200, and P_{y2} is \$50, then four units of Y_2 is equivalent to Y_1 giving a slope of 1/4. Graphically:



A whole series of these lines representing points of equal profit can be drawn and then superimposed on the production possibility curve. Thus:



The objective is to produce on the highest iso-profit line that still has contact with the production possibility curve. This is the arrowed point at which an iso-profit line is a tangent to the production possibility curve (as can be seen the slope is negative).

As with the least cost input, the *marginal rate of substitution* between the outputs can be defined as

$$\text{MRS} = \frac{\text{Change in } Y_1}{\text{Change in } Y_2}$$

This MRS varies with the position on the possibility curve, and is in effect the slope of the graph at each point (the slope of a tangent to the curve). As with the iso-profit line, it will be negative as it is backward sloping.

As the optimal point is where the price ratio line is also a tangent to the possibility curve, the algebraic statement of the point is where the MRS is equal to the price ratio. Thus, for optimality the farmer should move round the possibility curve until:

$$\text{MRS} = \frac{P_{y2}}{P_{y1}}$$

or

$$\left(\frac{\text{Change in } Y_1}{\text{Change in } Y_2}\right) = \frac{P_{y2}}{P_{y1}}$$

or

$$\text{change in } Y_1 \times P_{y1} = \text{change in } Y_2 \times P_{y2}$$

In production economic terminology, the terms on the left and on the right are called the *marginal value product* of each product, respectively. Clearly, as production is moved round the possibility curve one product is substituted for the other with the rate of substitution depending on the resources released by one product as less is produced, and how much of the second can be produced with these released resources. When these quantities are combined with the product prices you get the value of product given up and the value produced by the substitute. If the value given up is greater than the value created, then it does not pay to make this move; similarly in the opposite direction. Thus, for optimality, move round the curve until the change is neutral, i.e. move to where the $\mathrm{MVP}_1 = \mathrm{MVP}_2$.

Where many products can be produced creating a multidimensional production possibility surface, the optimal point is where the MVP of all products is equal. Any shift from this point will decrease profit as the point is the position on the surface just tangent to the highest iso-profit surface possible.

The production possibility curve drawn represents what are called *competitive* products as they compete for the same resources (labour perhaps, and probably land). Other relationships are possible. One possibility is where increasing one also increases the other: such products are known as *complementary* products. For example, increasing clover seed production can increase the quantity of wheat produced as the legume increases the fertility of the soil. And then it is possible for two products to be *supplementary*, which is where they are independent so increasing one does not decrease the other. Often two products will have areas of each relationship. For example, while clover increases wheat output, eventually there will be competition, for example, the harvesting equipment may become limiting to both as there is a limit to the hours it can work. And, for example, honey production and clover production could be complementary for some quantities, but adding more bees may eventually have no impact on the clover as there are more than sufficient to pollinate the clover.

For complementary products, if it pays to produce at all, it pays to produce quantities that move the relationship into the competitive area. For supplementary relationships the optimal production levels will depend on whether each is profitable, for they do not compete.

Summary Comments

Given a multi-product multi-input situation there is, theoretically, a production function for each product. Given the equation for each and the expected prices and costs a profit function can be produced. Thus, given two outputs and three inputs, for example, the profit function will have the general form:

$$\text{Profit} = Y_1 P_{y1} + Y_2 P_{y2} - X_1 P_{x1} - X_2 P_{x2} - X_3 P_{x3}$$

given that output depends on the level of inputs the Y values can be substituted within the function determining output (the production function). Thus:

$$\text{Profit} = F_1(X_1, X_2, X_3)P_{y1} + F_2(X_1, X_2, X_3)P_{y2} - X_1P_{x1} - X_2P_{x2} - X_3P_{x3}$$

Using calculus (the first derivative being set to zero) the equation of the first derivative can be solved to find the values of each of the inputs allotted to each product that maximizes the profit. The problem is if the functions are quite complex, and many more products and inputs are the reality, the maths becomes very difficult. Thus, more practical methods are used as discussed in the various chapters in the book.

If a real case had available all the data and a solution was practical, the optimal point would have to be determined after also allowing for the resources available (e.g. the working capital available both on hand and borrowed). This further complicates the maths by using additional constraint equations. If all but land was non-limiting, then the optimal point, as shown in the discussion, would be where:

Marginal cost = marginal return for all inputs with respect to each product;
MRS for each input pair = the cost ratios;
MVP was equal for all products.

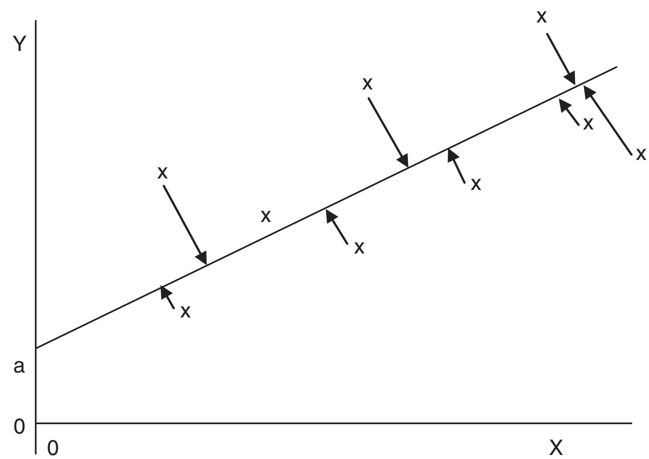
It must be noted, however, that the farmer's resource base may not allow reaching MR = MC in all cases, though resource hiring should be considered in such cases.

Obtaining the Data

The discussion assumes a knowledge of all the production functions. This is seldom the case. Perhaps for a few farms an extensive research station is nearby that has the same climate and soils on which extensive trials have been held, thus enabling the functions to be calculated. Such cases are rare. Consequently extrapolation is necessary from any evidence that is available. Furthermore, sometimes extensive farm survey data is available providing physical input–output information, which can then be used to estimate the functions. Most farms will be using different input levels, so each farm becomes a point on a multidimensional function. In other cases it will be necessary to use 'best estimates', either from experienced researchers, or from farmers themselves, or perhaps a combination of people. It is almost always possible to come up with reasoned estimates, though there will be cases of new products in new areas where the estimates might be quite incorrect. Such cases must wait for some experience or trials to enable reasoned estimates to allow decisions to be made.

Where data are available on input and output levels this must be analysed to come up with the most likely production function. This requires the use of appropriate statistical methods to produce the 'most likely' function. Invariably the analysis will involve the use of regression analysis to estimate the best fit equation. For details and proofs, a modern statistical methods book should be consulted (e.g. Hair *et al*., 1998).

Regression analysis provides the best least-biased estimates of the parameters of a proposed equation together with goodness of fit statistics. It is a procedure which finds the estimates by finding the parameters of an equation which best fits the plot of the observed data. The procedure minimizes the sum of the squared differences between the line and the observed points. This can be demonstrated by the graph below:



In this example the 'x's are the observed points for, say, a fertilizer experiment (X) on wheat yields (Y). A straight line is assumed, so the equation will have the form:

$$Y = a + bX$$

where a is the constant (production is, with zero fertilizer, at 'a' on the graph), and b the parameter representing the change in Y as X is increased (the slope of the equation). This parameter does not change as X increases, so it is a straight-line equation. The arrows represent the distance between the observed points and the proposed equation. The regression procedure finds the parameters a and b that minimize the sum of squares of the distances between the observed points and the equation. The squared values are used as this gives more weight to the 'further away' points. The derivation of the regression method gives rise to equations for solving for 'a' and 'b'. Most statistical packages have these routines, so obtaining the equation is just a matter of entering the data, the proposed equation form (through defining appropriate variables), and the package comes up with the parameters and significance statistic values (an important one is what is known as R^2 and gives the amount of the variation explained by the calculated equation on a scale of 0 to 1 with $R^2 = 1$ indicating a perfect fit of the equation to the data, and vice versa).

The same procedure is used for non-linear equations, but in these cases the equation will have a different form. For example, a decreasing returns function could be a quadratic of the form:

$$Y = a + bX - cX^2$$

In this case the regression procedure finds the parameter values a, b and c that minimize the sum of squared differences. In the computer package the user enters, for each observed Y, the associated value of X and also the squared value of X so you end up with three columns of data. The maths of the system does not understand that the two independent variables are related (X and X^2), just that the three parameters that minimize the sum of the squared differences must be found. The value of 'c' will be much smaller than 'b' so for small values of X the third term will have little impact, but as X increases it will come into play thus bending the increasing line down. Similarly, various other functions can be used depending on the nature of the plotted points.

Concluding Comments

The logic presented shows that production economics provides the theory for working out which products to produce, which inputs to use in producing each product, and the level of production per technical unit. Once risk and uncertainty are also included in the analysis it becomes rather complicated, though where the objective is the maximization of expected profit, expected values of output and prices can be used. When, however, random impacts mean the expected optimal points are no longer feasible (perhaps a lower yield and price than expected occurs which means, e.g. exceeding the bank's limits on working finance), a new production function comes into play as does a new optimal point. Theoretically, the whole system could be solved for a range of possible 'states of nature', thus coming up with a range of optimal solutions each with a probability giving rise to an expected optimal profit. While this approach would be manageable with modern computers, the assumption is that each state is predictable. In reality this is not the case. Thus, more analytically practical methods must be used. And when the constraints imposed by the farmer's resource set and ability are added, the complexity of the maths in solving constrained production economic problems becomes too difficult for practical application except in the most simple of farm situations.

However, no matter what situation a farmer finds himself in, production economics indicates any move that will increase profit is clearly worthwhile. Thus, moving around the production function to give MR = MC should occur, and similarly for the other decision rules. Once the farmer reaches a production point from which profit (or whatever the objective set is) will decrease if he moves from it, the best farming system has been found. In this dynamic world, this point is always changing.

Most farmers operate by making small increments in moving to the optimal point. Thus, what is thought to be the best system is used each year, and after reviewing the outcomes and new evidence a new plan is devised which is, in most cases, a small, but beneficial, move. Thus, marginal changes are constantly being made rather than a radical re-organization. Farmers 'feel' their way to improvements, particularly where knowledge of the production functions and markets is not certain, as is the case for the majority of farmers.

References

Barnard, C.S. and Nix, J.S. (1979) Basic principles and concepts of planning. In: *Farm Planning and Control*. Last reprinted 1999. Cambridge University Press, Cambridge.

Beattie, B.R., Taylor, C.R. and Watts, M.J. (2009) *The Economics of Production*. Krieger Publishing Company, Malabar, Florida.

Hair, J.F., Anderson, R.E., Tatham, R.L. and Black, W.C. (1998) *Multivariate Data Analysis, with Readings*. Prentice Hall, New Jersey.

Appendix 2: Example of the Output From an Individual Farm 'End of Year' Analysis

(Note a DSE refers to a Dry Stock Equivalent, i.e. namely the feed needed to maintain a dry sheep. The monetary unit is $10.)

PRELIMINARY ANALYSIS OF INDIVIDUAL FARM PERFORMANCE FOR 2025 – 2026

FINANCIAL SUMMARY

NET WORTH STATEMENT*

	OPENING VALUE	PURCHASES	SALES	CLOSING VALUE
ASSETS	$	$	$	$
Land and improvements	160,000	0	0	165,000
Plant	40,000	2,000	0	36,000
Livestock	29,620	1,100	10,200	28,710
Other assets	4,630			3,850
Total	234,250	3,100	10,200	233,560
LIABILITIES	20,000			7,000
Equity	214,250			226,560
Equity %	91.5			97.0

*This Statement reconciles by columns but not by rows

PROFIT STATEMENT	$	CAPITAL GAIN STATEMENT	$
Total farm income	61,557	Closing value of land, plant and improvements	201,000
Less total operating costs	33,337	Add depreciation	7,400
Less operator labour	2,400	Add sales	0
operating return	25,820	Adjusted closing value (A)	208,400
Less rent and interest	1,000	Opening value of land plant and improvements	200,000
business return	24,820	Add capital costs	500
Operating return/av total assets (%)	11.0	Add purchases	2,000
Business return/av equity (%)	11.3	Adjusted opening value (B)	202,500
		Capital gain (A – B)	5,900
		Capital Gain/Av Total Assets (%)	2.5

SUMMARY OF PHYSICAL PRODUCTION

LIVESTOCK

SHEEP PRODUCTION	MERINOS	CATTLE PRODUCTION (CONT.)	DAIRY
Attributed grazed acres	600.0	Overall mortality by balance (%)	4.5
Av number of breeding ewes	1,000.0	Total butterfat (lb)	26250.0
Breeding ewes/adult sheep (%)	56.2	Milk cow (gal)	525.0
Lambs marked/ewes joined (%)	95.0	Butterfat/cow (lb)	262.5
Lambs survived/lambs marked (%)	94.7	Butterfat/grazed acre (lb)	65.6
Overall mortality by balance (%)	4.2	Average value cattle sold ($/HD)	30.0
Wool cut/grown sheep (lb)	10.7		
Wool cut/lamb (lb)	2.2	PIG PRODUCTION	PIGS
Wool cut/use (dry stock equiv) (lb)	8.2	Number of sows	20.0
Wool cut/grazed acre (lb)	30.0	Piglets born/sows farrowed	10.5
Wool produced/use (lb)	9.1	Piglets born/farrowing	6.4
Wool produced/grazed acre (lb)	33.3	Farrowing per sow/year	1.6
Average wool price/lb (cents)	36.1	Piglets weaned/born (%)	85.7
Average value sheep sold ($/HD)	6.0	Average value pigs sold ($/HD)	30.0
CATTLE PRODUCTION	DAIRY		
Attributed grazed acres	400.0		
Av number of breeding cows	100.0		
Breeding cows/adult stock (%)	82.6		
Calves marked/cows mated (%)	90.0		
Calves survived/calves marked (%)	100.0		

PASTURE & CROPS*

	GRAIN OR SEED		HAY		OTHER PRODUCE	
	Total produced	Per acre harvested	Total produced	Per acre harvested	Total produced	Per acre harvested
Pasture	0	0	0	0	150.0	1.5
Wheat	25,000.0	25.0	0	0	0	0
Lucerne	0	0	225.0	1.1	0	0

* Units as in Input Sheets

FINANCIAL ANALYSIS OF LIVESTOCK ENTERPRISES

GROSS MARGINS CALCULATIONS	MERINOS	DAIRY	PIGS	PER DSE COST ANALYSIS	SH'P	D'RY
	$	$	$			
Add closing inventory	9,750	17,780	1,180	1. Animal health	0.23	0.09
Add livestock sales	4,500	2,400	3,300	2. Purchased feed (Sheet IV)	0.09	0
Add rations	80	50	50			
Less opening inventory	10,500	18,000	1,120	3. Services and labour	0.40	0.12
Less livestock purchases	400	500	200	4. Pasture costs – attrib to l/stock	0.05	0.10
TRADING PROFIT	3,430	1,730	3,210			
Add produce sales	6,500	11,100	0	5. Sundry expenses	0.23	0.10
GROSS INCOME	9,930	12,830	3,210	6. Fuel, oil, grease and elect	0.01	0.01
Less home-grown feed (Sheet V)	223	463	1,250			
Less purchased feed (Sheet VIII)	577	422	0	7. Repairs and maint to plant	0.03	0.04
Less variable cash costs (Sheet IV)	2,300	610	460			
SHORT TERM GROSS MARGIN	6,828	11,330	1,500	8. Repairs and maint to structures	0	0.01
Less allocable depreciation	400	300	200			
Less allocable permanent labour	500	400	500	13. Sharefarmers part payment of costs	–0	–0
LONG TERM GROSS MARGIN	5,928	10,633	800			
				Home grown feed (Sheet V)	0.10	0.19

Continued

Continued.

GROSS MARGINS CALCULATIONS	MERINOS	DAIRY	PIGS	PER DSE COST ANALYSIS	SH’P	D’RY
GROSS MARGIN RATIOS						
A. Short term				Purchased feed (Sheet VIII)	0.26	0.26
$/DSE (Dry Stock Equiv)	3.10	7.02	0			
$/Grazed acre	11.38	28.33	0	TOTAL SHORT TERM COSTS	1.40	0.92
$/Man month	1,365.74	2,266.76	300.00			
$/$100 capital	61.38	60.00	36.14	14. Allocable depreciation	0.18	0.19
				15. Allocable permanent labour	0.23	0.25
B. Long term						
$/DSE (Dry Stock Unit)	2.69	6.59	0			
$/Grazed acre	9.88	26.58	0	TOTAL LONG TERM COSTS	1.63	1.36
$/Man month	1,185.74	2,126.76	160.00			
$/$100 capital	53.29	56.29	19.28			

FINANCIAL ANALYSIS OF LIVESTOCK ENTERPRISES

AVERAGE PRICES RECEIVED FOR LIVESTOCK SALES

	$
1. Sheep	Merinos
Store lambs	6.40
Fat lambs	0
Ewes 1–2 yrs	0
Wethers 1–2 yrs	0
Cull ewes	0
Cull wethers	0
CFA ewes	4.25
Sundry stock	5.00
Rams	0
All sheep	6.00
2. Cattle	Dairy
Calves	25.79
Yearlings	0
Breeding heifers	0
Stores 1–2 yrs	0
Fats 1–2 yrs	0
Stores over 2 yrs	110.00
Fats over 2 yrs	0
Sundry stock	0
Bulls	0
All cattle	30.00
3. Pigs	Pigs
Weaners and slips	12.00
Porkers	28.50
Baconers	41.20
Brood sows	0
Sundry stock	14.50
Boars	0
All pigs	30.00

FINANCIAL ANALYSIS OF PASTURE AND CROP ENTERPRISES

GROSS MARGINS CALCULATIONS	PASTURE	WHEAT	LUCERNE
	$	$	$
Add closing invt	300	1,150	1,400
Add sales	0	30,000	4,000
Add seed used this yr	0	1,150	0
Add grain and fodder fed	687	0	100
Less opening invt	300	230	2,000
Less purchases	150	920	0
Trading profit	537	31,150	3,500
Less seed used for crop	150	1,150	0
Less var cash costs	1,400	6,900	1,580
Short term GM	–1,013	23,100	1,920
Less allocable Dpn	0	2,000	500
Less allocable perm lbr	0	1,000	500
Long term GM	0	20,100	920

PER ACRE COST ANALYSIS	PAST	WHEAT	LUC
5. Sundry exps	0.06	0.75	1.75
6. Fuel, oil, grease and elec	0.12	0.40	0.40
7. R & M to plant	0.06	0.55	0.80
8. R & M to structures	0.00	0.15	0.20
9. Fert	1.50	2.25	1.50
10. Contr, ins, harvesting etc.	0.00	0.30	1.50
11. Chems, twine, bags etc.	0.00	2.50	0.75
12. Irrigation charges	0.00	0.00	1.00
13. Share farmers part payment of costs	–0.00	–0.00	–0.00
Seed used	1.19	1.15	
Total short term costs	1.93	8.05	7.90
14. Allocable dpn	0.00	2.00	2.50
15. Allocable perm lbr	0.00	1.00	2.50
Total long term costs	0.00	11.05	12.90

GROSS MARGIN RATIOS

A. Short term			
$/Sown acre	–1.08	23.10	9.60
$/Man month	0	3,850.00	640.00
$/$100 capital	0	77.00	19.20
$/Acre foot	0	0.00	4.50
B. Long term			
$/Sown acre	0	20.10	4.60
$/Man month	0	3,350.00	306.67
$/$100 capital	0	67.00	9.20
$/Acre foot	0	0.00	2.05

INCOME, COSTS AND GROSS MARGINS FOR ALL ENTERPRISES

ENTERPRISE	GROSS INCOME	% OF TOTAL	SHORT TERM VARIABLE COSTS	% OF TOTAL	SHORT TERM GROSS MARGINS	% OF TOTAL
	$		$		$	
Merinos	9,930	16.2	3,100	17.7	6,828	15.6
Dairy	12,830	21.0	1,497	8.6	11,333	26.0
Pigs	3,210	5.3	1,710	9.8	1,500	3.4
All livestock	25,970	42.5	6,307	36.1	19,661	45.0
Pasture	537	0.9	1,550	8.9	–1,013	– 2.3
Wheat	31,150	50.9	8,050	46.0	23,100	52.9
Lucerne	3,500	5.7	1,580	9.0	1,920	4.4
All pasture and crop	35,187	57.5	11,180	63.9	24,007	55.0
All enterprises	61,157	100.0	17,489	100.0	43,668	100.0

RECONCILIATION STATEMENTS

FARM INCOME		OPERATING COSTS	
	$		$
Enterprise income	61,157	Enterprise ST var costs	17,487
Other farm income	400	Alloc depr and perm labour (Sheet 11)	6,250
		Overhead (excl operator labour)	9,600
Total farm income	61,557	Total operating costs	33,337
		FARM PROFIT	
			$
		Total farm income	61,557
		Less total operating costs	33,337
		Less operator labour	2,400
		Operating return	25,820
		Less rent and interest	1,000
		Business Return	24,820

Appendix 3: The Simplex Method of Solving Linear Programming Problems

Introduction

In Chapter 11 a method of solving linear programming (LP) problems was outlined. It was not, however, particularly efficient. This discussion develops a more efficient and robust method, though identical in principle. The difference is a set of defined rules is used to adjust the equations to create the desired canonical form rather than a trial and error method. The method is referred to as the *simplex method*. In a practical sense users of LP models do not need to understand solving as they will use computer packages. However, an understanding helps in model creation, overcoming problems as well as providing a better understanding of the wealth of information available from solutions.

The Example Problem

To demonstrate the simplex method, consider a modified version of the example problem. That is, find a set of x_j which:

max. $Z = 100x_1 + 200x_2 + 0x_3 + 0x_4 + 0x_5$

and satisfy:

$200 = 1x_1 + 1x_2 + 1x_3 + 0x_4 + 0x_5$ (land equation)
$200 = 0.5x_1 + 4x_2 + 0x_3 + 1x_4 + 0x_5$ (labour equation)
$60 = 0x_1 + 1x_2 + 0x_3 + 0x_4 + 1x_5$ (max. potatoes equation)
$x_j \geq 0$, all j

where x_1 = hectares of wheat, x_2 = hectares of potatoes, x_3 = hectares of land non-use, x_4 = hours of September labour non-use, x_5 = non-use of max. potatoes constraint.

The Simplex Method

Introduction

The best way to describe the method is to list and demonstrate the steps involved. The reason for each step should be clear given an understanding of the solving demonstration in Chapter 11.

The simplex tableau

The first step is to set up the simplex tableau (or table). This contains the first solution to the problem and room for calculating further solutions. This first solution is always assumed to have the disposal activities at a maximum and all real activities at zero. This solution is immediately available from the constraint equations without the need for any calculations and provides a convenient starting point for the iterative process.

Subsequent solutions are written below the first on the same sheet of paper. Each sub-tableau represents one solution and contains the adjusted equations.

The first tableau is set up as:

Table A3.1. The first simplex sub-tableau.

			C_j	\$100	\$200	0	0	0	R
C_i	P_i	b_i	P_j	P_1	P_2	P_3	P_4	P_5	Ratio
0	P_3	200		1	1	1	0	0	
0	P_4	200		0.5	4	0	1	0	
0	P_5	60		0	1	0	0	1	
	Z_j	0		0	0	0	0	0	subsequent solutions
	$Z_j - C_j$			–100	–200	0	0	0	follow below:

Within the main body of the table are found the constraint equations without the variables included. Rather than write in the variables, each column is headed with the relevant variable code. Thus, under P_1 appear the coefficients for x_1 from each equation. For convenience in making calculations, the C_j for each P_j is listed above the relevant column.

Under the column headed b_i are listed the constants from the equations. As these constants represent the levels of the basic variables this column will contain the solution to each iteration of the solving process and will eventually contain the optimal solution. As the equations are adjusted to form new sub-tableaux, these values will, of course, change.

Under the b_i column in the Z_j row is recorded the profit made from the solution which the particular sub-tableau represents. For the first solution, profit equals zero as the disposal activities have C_j values = 0.

Under the column headed P_i is listed the variable code (or its name might be used) of the *i*th basic variable. For example, the P_3 activity (land disposal) is the first basic activity in the first solution, P_4 is the second, etc. Thus the code number appearing beside an entry in the b_i column indicates which basic activity has that particular value.

Under the column headed C_i are listed the gross margins of the basic activities.

The Z_j, $Z_j - C_j$ rows and the column headed '*R* Ratio' will be discussed in the next sections.

Determining the variable (activity) to enter the next basic solution

In Chapter 11 it was shown that the new basic activity is determined by calculating the $Z_j - C_j$ for each non-basic activity and selecting the activity with the most negative $Z_j - C_j$. This activity will increase Z more than any other activity. Thus, the next step is to calculate Z_j and $Z_j - C_j$ and enter them in the table. It was shown in Chapter 11 that Z_j is given by:

$$Z_j = \sum_{i=1}^{m} r_{ij} C_i$$

where i represents a particular row number and they are numbered 1 to m starting from the top of the table.

Thus:

$$Z_j - C_j = \sum_{i=1}^{m} r_{ij} C_i - C_j$$

(it can now be seen why the C_i column is included in the tableau).
For example:

$$Z_1 = r_{11}\, C_1 + r_{21}\, C_2 + r_{31}\, C_3$$
$$\therefore Z_1 = (1 \times 0) + (0.5 \times 0) + (0 \times 0) = 0$$
$$\therefore Z_1 - C_1 = 0 - 100 = -100$$
$$Z_2 = (1 \times 0) + (4 \times 0) + (1 \times 0) = 0$$
$$\therefore Z_2 - C_2 = 0 - 200 = -200$$

In Table A3.1 all the $Z_j - C_j$ are listed for this first solution. These are easy to calculate as $C_i = 0$ for all i. (These marginal net return figures will, of course, change for subsequent solutions as the coefficients, the r_{ij} values will change as will the C_i.)

The most negative $Z_j - C_j$ is the −200 for P_2. Thus, P_2 is selected as being the activity to enter the next basic solution (if two or more activities have the lowest $Z_j - C_j$, an arbitrary choice can be made).

Determining the variable (activity) to be removed from the old basic solution

If a new basic solution is calculated with one new variable, *one* of the old variables must be removed (driven to zero) if the solution is to be a basic solution (contains m variables at a positive level). It was shown in Chapter 11 that an optimal solution will have just m variables positive where m is the number of equations.

Increasing a variable above zero requires resources, but the current solution is a set of activity levels accounting for all the resources available. Thus, as one activity is increased above zero level, the other variables' levels must change to release, or produce, the necessary resources. If the new activity is increased to a high enough level, one of the other activities will be driven to zero. Once this point has been reached the new activity cannot be increased to a greater level as there are insufficient resources available (unless considering a solution with less than m activities. But it is known such a solution will not be optimal).

Thus, if the maximum level at which the new activity can be brought into a new basic solution is determined, this indicates which activity should be removed from the old basic solution (i.e. the one that becomes zero). Recall that the r_{ij} of the first tableau, and of subsequent tableaux, are the MRS between the basic activities (which may represent resource disposals as is the case in the first solution) and the non-basic activities.

Thus, if the jth non-basic activity is increased above zero level, it will force the ith basic activity to change by r_{ij} for each unit above zero it is increased. Thus, the maximum value x_j (the value of the jth non-basic activity) can take on *with respect to the ith basic variable* is given by:

$$R = \frac{b_i}{r_{ij}}$$

where b_i = level of the ith basic activity (in the first solution, b_i = resource levels).

Note that if r_{ij} is negative, the ith basic activity will increase, rather than decrease, as x_j is increased. Note that if x_j was made equal to R, *one of the other basic variables might* be driven to a negative level. This would be infeasible. Thus, to determine the maximum value x_j can take on, follow the steps below:

1. Calculate:

$$R = \frac{b_i}{r_{ij}} \text{ for each } i$$

provided $r_{ij} > 0$ (if $r_{ij} \leq 0$, do not calculate as the activity supplies, rather than uses, the resource).

2. Select the smallest of the R. This value indicates the maximum value x_j can take. Furthermore, the row (that i) which gives the smallest R indicates that the ith basic activity of the old solution is dropped from the new solution.

At the end of the simplex tableau is provided a column for recording the R ratio. Thus, after selecting the activity to be introduced, the R ratio is calculated for each i in the jth column and recorded in the R column (not for 0 or −ve r_{ij} values).

The smallest R is selected to indicate the basic activity to be dropped from the new solution. If there are two R values that are equally as small, an arbitrary choice can be made.

The *R* ratio in the example problem and some conventions

The R entries in the first tableau are (recall that P_2 has been selected as the new activity, thus $j = 2$):

$$R_1 = \frac{b_1}{r_{12}} = \frac{200}{1} = 200$$

$$R_2 = \frac{b_2}{r_{22}} = \frac{200}{4} = 50$$

$$R_3 = \frac{b_3}{r_{32}} = \frac{60}{1} = 60$$

Thus, the activity that will be driven to zero in the next basic solution is the second basic activity as the smallest R ratio occurs in row 2. That is, P_4 (labour disposal) *will leave the solution* and P_2 will take its place.

Knowing which activity is to leave and which is to enter the new solution, the new set of equations, which are required to be in canonical form with respect to the new set of basic activities, can be determined. The next section will show how this is done.

Before doing this, define some terminology. To aid this discussion, the initial simplex tableau is re-presented but now with the R ratio figures included. This tableau can be taken as being typical of the sub-tableaux which will follow:

Table A3.2. The initial simplex tableau again (slightly simplified).

C_i	P_i	b_i	P_1	P_2 ↑	P_3	P_4	P_5	R
0	P_3	200	1	1	1	0	0	200
0	← P_4	200	0.5	$\boxed{4}$	0	1	0	50*
0	P_5	60	0	1	0	0	1	60
	$Z_j - C_j$		–100	–200	0	0	0	

Some terminology defined:

- The outgoing column: the column representing the activity that has been selected to enter the next basis (solution) is called the *outgoing column*. For identification purposes, an 'outward' arrow can be placed beside the relevant P_j. See Table A3.2.
- The outgoing row: the row giving the smallest R ratio is referred to as the *outgoing row*. For identification purposes an asterisk (and an arrow beside the relevant P_i) can be placed beside the smallest R ratio.
- The pivot: the element (coefficient, r_{ij}) at the intersection of the outgoing column and the outgoing row is referred to as the *pivot*. This *element* is

important in calculating the new equations and thus a box is placed around it.

- The incoming row: this is the row whose left-hand-side constant (b_i) gives the level of the new basic activity in the *new* solution. This row will appear in the *next* tableau. It appears in all tableaux except the initial tableau (Table A3.2). To indicate the incoming row an inwards arrow is placed beside the relevant P_i. (This does not appear in Table A3.2 as it is the initial tableau.)

Calculating the new tableau

To obtain the new equations (the new tableau) the following rules are used. These rules *ensure that the equations end up in the desired canonical form*:

1. To calculate the equation (row) representing the level of the new basic activity and the new MRSs (incoming row), divide the corresponding element of the outgoing row by the pivot.

Thus, in the example problem, the incoming row is:

	C_i	P_i	b_i	P_1	P_2	P_3	P_4	P_5
	200	P_2	$\frac{200}{4}$	$\frac{0.5}{4}$	$^4/_4$	$^0/_4$	$^1/_4$	$^0/_4$
This gives		$\rightarrow$						
	200	P_2	50	0.125	1	0	0.25	0

Note that the x_2 coefficient is the desired 1.

Note that under the P_i and C_i columns for this new row *are placed* P_2 and 200, the code and gross margin for the new basic activity.

2. *To calculate* the elements of the new tableau (includes both the r_{ij} and b_i) other than those in the incoming row (these have been calculated above), take the corresponding element in the old tableau and subtract from it the product of:

(i) The element at the intersection of the same row as the element being operated on *and* the outgoing column (i.e. the element that is in the same row as the element being operated on and is above or below the pivot, as the case may be), and

(ii) The element at the intersection of the column in which the element being operated on is found *and* the incoming row. (Thus, this element comes from the new tableau.)

Thus, for the example problem, *assuming* the incoming row is placed in the first row of the new tableau, the elements for the row representing activity P_3 will be (let this row appear in row 2 of the new tableau):

$b_2 = 200 - (1 \times 50) = 150.0$

$r_{21} = 1 - (1 \times 0.125) = 0.875$

$r_{22} = 1 - (1 \times 1) = 0.0$
$r_{23} = 1 - (1 \times 0) = 1.0$
$r_{24} = 0 - (1 \times 0.25) = -0.25$
$r_{25} = 0 - (1 \times 0) = 0.0$

Thus, the second row of the new tableau is:

C_i	P_i	b_i	P_1	P_2	P_3	P_4	P_5
0	P_3	150	0.875	0	1.0	–0.25	0

Notice that this equation is in the desired canonical form. That is, x_3 has a unit coefficient and x_2 and x_5, the other two basic activities for this solution, have zero coefficients. The rules have been devised to ensure this occurs.

Similarly, the other equation from the old tableau would be adjusted. This gives the third equation of:

C_i	P_i	b_i	P_1	P_2	P_3	P_4	P_5
0	P_5	10	–0.125	0	0	–0.25	1

Using these rules the equations are adjusted so that, given a particular equation from the old tableau, the new form gives adjusted coefficients representing the new value of the *i*th basic activity, and the new MRS figures. This holds for all equations except for the outgoing row and the resultant incoming row. This equation, of course, is the equation whose left-hand side represents the level of the new basic activity and was calculated by dividing through the old equation by the pivot.

The completed first and second tableaux are shown in Table A3.3.

Table A3.3. The first and second tableaux.

		C_j / P_j	100	200	0	0	0	R
C_i	P_i	b_i	P_1	P_2 ↑	P_3	P_4	P_5	
0	← P_3	200	1	1	1	0	0	200
0	P_4	200	0.5	4	0	1	0	50*
0	P_5	60	0	1	0	0	1	60
	$Z_j - C_j$	0	–100	–200	0	0	0	
0	→ P_2	50	0.125	1	0	0.25	0	
0	P_3	150	0.875	0	1	–0.25	0	
0	P_5	10	–0.125	0	0	–0.25	1	
	$Z_j - C_j$	10,000						

Completing the problem

The entire simplex method has now been discussed. To complete the problem the new $Z_j - C_j$ must be estimated. The activity with the most negative $Z_j - C_j$ is selected as the activity to enter the next solution. The R ratio is calculated and the outgoing row and the pivot determined. Then, the new tableau representing the next basic solution is calculated using the rules. This process is repeated until a solution is obtained in which all $Z_j - C_j$ are greater than or equal to zero. This solution is the optimal solution. Table A3.4 gives the entire set of simplex tableaux (note that the solution is the same as that obtained by the graphical method).

Table A3.4. The completed simplex table.

			100	200	0	0	0	
C_i	P_i	b_i	P_1	P_2	P_3	P_4	P_5	R
0	P_3	200	1	1	1	0	0	200
0	P_4	200	0.5	4	0	1	0	50*
0	P_5	60	0	1	0	0	1	60
	$Z_j - C_j$	0	–100	–200	0	0	0	
0	P_2	50	0.125	1	0	0.25	0	1600.0
0	P_3	150	0.875	0	1	–0.25	0	171.43*
0	P_5	10	–0.125	0	0	–0.25	1	–
	$Z_j - C_j$		–75	0	0	50	0	
100	P_1		1	0	1.14	–0.29	0	
200	P_2		0	1	–0.14	0.29	0	
0	P_5		0	0	0.14	–0.29	1	
	$Z_j - C_j$	22,857	0	0	86	29	0	

Optimal solution:

$x_1 = 171.43$ = hectares of wheat
$x_2 = 28.57$ = hectares of potatoes
$x_5 = 31.43$ = max. potatoes constraint disposal

Summary of the Simplex Method

1. Calculate:

$$Z_j - C_j = \sum_{i=1}^{m} r_{ij} C_1 - C_j \text{ for all } j$$

2. Select the activity with the most negative $Z_j - C_j$ as the activity to enter the new basis.
If all $Z_j - C_j \geq 0$, the optimal solution has been attained.
Let k = the value of the j index of the selected activity.

3. Calculate

$$R = \frac{b_i}{r_{ik}}$$

for all i except for any $r_{ik} \leq 0$.

4. The smallest R indicates the outgoing row (the activity to be dropped from the new basic solution).
$\therefore$ Let this row = S.

5. Calculate the new tableau:
Let b_i and r_{ij} = old coefficients, let $\hat{b}_i$ and $\hat{r}_{ij}$ = new coefficients.
(i) The incoming row (that representing the new basic activity) is given by (assuming this is placed in the first row of the new tableau so that $i = 1$):
$\hat{b}_i = b_s / r_{sk}$
$\hat{r}_{ij} = r_{sj} / r_{sk}$
(ii) The other rows are given by (each represents the ith basic variable):
$\hat{b}_i = b_i - (r_{ik} \times \hat{b}_1)$ = level of ith basic variable
$\hat{r}_{ij} = r_{ij} - (r_{ik} \times \hat{r}_{1j})$ = MRS between ith basic variable and the jth non-basic variable

where **(a)** a new equation will 'move down' a row in the new tableau if it is above the outgoing row in the old tableau (i.e., new i = old $i + 1$)
(b) new i = old i if the equation is below the outgoing row in the old tableau.

6. Proceed back to Step (**1**) again and repeat until all $Z_j - C_j$ are greater than, or equal to, zero.

Also note that:

- The r_{ij} represent the MRS between the ith basic activity and the jth non-basic activity for each tableau.
- The $Z_j - C_j$ represent the marginal return per unit of producing the jth non-basic activity, given that the plan represented by the particular tableau is put into operation.

Further Reading

Beneke, R.R. and Winterboer, R. (1973) Linear programming procedures. In: *Linear Programming Applications to Agriculture*. Iowa State University Press, Ames.

Vanderbi, R.J. (2008) *Linear Programming. Foundation and Extensions*, 3rd edn. Springer, New York.

Appendix 4
An Example of a Schematic LP Matrix for a Simple Lamb-Producing Farm

		Lucerne 1st year	Lucerne 2nd year	Lucerne 3rd year	Lucerne 4th year	Make hay	Turnips	G.F. rye	Buy hay (winter)	Buy hay (summer)	Buy straw (summer)	Buy sheep nuts (winter)	Buy sheep nuts (summer)	Corriedale ewe X Corr. ram	Corriedale ewe X B.L. ram	X Bred ewe X fat lamb ram	Rear Corr. replacements	Buy Corr. 2 tooths	Rear X bred ewes	Sell Corr. ewe lambs	Sell Corr. wether lambs	Sell X bred ewe lambs	Sell X bred wether lambs	Sell light fat lambs	Sell heavy fat lambs	Sell X bred hoggets	Cows	Breed replacement cows	Buy replacement cows	Sell weaners	Sell 18-month cattle	Trans. surplus females
		$-C_j$	$-C_j$	$-C_j$	$-C_j$	$-C_j$	$-C_j$	$-C_j$	$-C_j$	$-C_j$	$-C_j$	$-C_j$	$-C_j$	$+C_k$	$+C_k$	$+C_k$	$-C_k$	$-C_j$	$-C_j$	$+C_k$	$+C_k$	$+C_k$	$+C_k$	$+C_k$	$+C_k$	$+C_k$	$-C_j$	$-C_j$	$-C_j$	$+C_k$	$+C_k$	
Land	300 ≥	1	1	1	1		1	1																								
Lucerne rec.	0 ≥	–1	1																													
Lucerne rec.	0 ≥		–1	1																												
Lucerne rec.	0 ≥			–1	1																											
Summer feed	0 ≥	$-r$	$-r$	$-r$	$-r$	$+r$				$-r$	$-r$		$-r$	$+r$	$+r$	$+r$	$+r$		$+r$	$+r$	$+r$	$+r$	$+r$	$+r$	$+r$		$+r$	$+r$		$+r$	$+r$	
Winter feed	0 ≥	$-r$	$-r$	$-r$	$-r$	$-r$	$-r$	$-r$	$-r$			$-r$		$+r$	$+r$	$+r$	$+r$	$+r$	$+r$							$+r$	$+r$	$+r$	$+r$		$+r$	
Corriedale replacement reconciliation	0 ≥													$+r$	$+r$		$-r$	$-r$														
X bred replacement reconciliation	0 ≥															$+r$			$-r$													
Corriedale ewe lamb reconciliation	0 ≥													$-r$			$+r$			$+r$												
Corriedale wether lamb reconciliation	0 ≥													$-r$							$+r$											
X bred ewe lamb reconciliation	0 ≥														$-r$				$+r$			$+r$				$+r$						

Continued

Continued.

		Lucerne 1st year	Lucerne 2nd year	Lucerne 3rd year	Lucerne 4th year	Make hay	Turnips	G.F. rye	Buy hay (winter)	Buy hay (summer)	Buy straw (summer)	Buy sheep nuts (winter)	Buy sheep nuts (summer)	Corriedale ewe X Corr. ram	Corriedale ewe X B.L. ram	X Bred ewe X fat lamb ram	Rear Corr. replacements	Buy Corr. 2 tooths	Rear X bred ewes	Sell Corr. ewe lambs	Sell Corr. wether lambs	Sell X bred ewe lambs	Sell X bred wether lambs	Sell light fat lambs	Sell heavy fat lambs	Sell X bred hoggets	Cows	Breed replacement cows	Buy replacement cows	Sell weaners	Sell 18-month cattle	Trans. surplus females
		$-C_j$	$-C_j$	$-C_j$	$-C_j$	$-C_j$	$-C_j$	$-C_j$	$-C_j$	$-C_j$	$-C_j$	$-C_j$	$-C_j$	$+C_k$	$+C_k$	$+C_k$	$-C_k$	$-C_j$	$-C_j$	$+C_k$	$+C_k$	$+C_k$	$+C_k$	$+C_k$	$+C_k$	$+C_k$	$-C_j$	$-C_j$	$-C_j$	$+C_k$	$+C_k$	
X bred wether lamb reconciliation	$0 \geq$														$-r$								$+r$	$+r$								
Finished lamb reconciliation	$0 \geq$																								$+r$							
Male calf reconciliation	$0 \geq$																										$-r$			$+r$	$+r$	-1
Female calf reconciliation	$0 \geq$																										$-r$	$+r$				$+1$
Cow replacement reconciliation	$0 \geq$																										$+r$	$-r$	$-r$			

Many more activities could be added to account for labour, capital, machinery (e.g. limits on haymaking), etc. The general coefficient r has been used as the specific coefficients will depend on the particular farm.

Index